The Phenomenology of Rare $b \rightarrow s\ell^+\ell^-$ Decays

Danny van Dyk

Institute for Particle Physics Phenomenology, Durham

Seminar, University of Warwick, Nov 23rd 2023
deviations between measurements and Standard Model (SM) predictions requires careful interpretation
1. QED: mismatch between predictions and measurements, particularly in differential observables
   - unlikely explanation
   - “dangerous hard-collinear logarithms cancel at the differential level in the currently used experimental treatment
   - not further discussed here

[Isidori/Nabebaccus/Zwicky 2009.00929]
1. QED: mismatch between predictions and measurements, particularly in differential observables
   - unlikely explanation
   - “dangerous hard-collinear logarithms cancel at the differential level in the currently used experimental treatment
   - not further discussed here

2. QCD: we lack the correct understanding of the Standard Model long-distance dynamics, which mimic beyond the Standard Model (BSM) effects
   - quantify potential hadronic and BSM effects (within the Weak Effective Theory)
   - topic of this presentation
1. **QED**: mismatch between predictions and measurements, particularly in differential observables
   - unlikely explanation
   - “dangerous hard-collinear logarithms cancel at the differential level in the currently used experimental treatment
   - not further discussed here

2. **QCD**: we lack the correct understanding of the Standard Model long-distance dynamics, which mimic beyond the Standard Model (BSM) effects
   - quantify potential hadronic and BSM effects (within the Weak Effective Theory)
   - topic of this presentation

3. **BSM**: do we see genuine BSM effects in the data?
   - interpret potential BSM effects qualitatively
   - task for model builders (i.e.: not me!)
Interpretation within the Weak Effective Theory
Weak Effective Theory

- widely used tool of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM

\[ b \rightarrow s \ell\ell \]
\[ 10 \text{ semileptonic} \]
\[ 4\text{-quark} \]

\[ E_{\text{LHC}} \]
\[ M_t \]
\[ M_W \]
\[ M_B \]
\[ M_p \]

\[ \gamma/Z \]
\[ \ell^- \]
\[ \ell^+ \]

\[ b \rightarrow u/c/t \]
\[ s \]
Weak Effective Theory

- widely used tool of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM
- replaces dynamical degrees of freedom (here: $t, W, Z$) with coefficients $C_i$ and static structures in local operators (here: $\Gamma_i$)
Weak Effective Theory

- widely used tool of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM
- replaces dynamical degrees of freedom (here: $t, W, Z$) with coefficients $C_i$ and static structures in local operators (here: $\Gamma_i$)
- local operators must respect remaining $U(1)_{\text{EM}} \times SU(3)_C$ symmetry

\[
\sum_i C_i \times [\bar{s} \Gamma_i b] \times [\bar{\ell} \Gamma'_i \ell]
\]

\[
\begin{align*}
E_{\text{LHC}} & \quad \text{Energy} \\
M_t & \\
M_W & \\
M_B & \\
M_p & \\
\end{align*}
\]
Weak Effective Theory

- widely used tool of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM
- replaces dynamical degrees of freedom (here: $t$, $W$, $Z$) with coefficients $C_i$ and static structures in local operators (here: $\Gamma_i$)
- local operators must respect remaining $U(1)_{EM} \times SU(3)_C$ symmetry
- for $b \to s\ell\ell$ we find in general
  - 10 semileptonic $[\bar{s}\Gamma b] [\bar{\ell}\Gamma' \ell]$ ops
  - 20 four-quark $[\bar{s}\Gamma b] [\bar{c}\Gamma' c]$ ops
  - …

\[ \sum_i C_i \times [\bar{s} \Gamma_i b] \times [\bar{\ell} \Gamma'_i \ell] \]
Weak Effective Theory: $b \rightarrow s \ell \ell$ SM operators

In the SM, only the following set of $D = 6$ effective operators contributes:

$$\mathcal{L}_{\text{eff}}^{\text{SM}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}} \left[ \lambda_t \sum_{i=3}^{10} C_i \mathcal{O}_i + \lambda_c \sum_{j=1}^2 C_j^c \mathcal{O}_j^c + \lambda_u \sum_{k=1}^2 C_k^u \mathcal{O}_k^u \right]$$

with $\lambda_q \equiv V_{qb} V_{qs}^*$

Semileptonic

$$\mathcal{O}_9 = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10} = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Radiative

$$\mathcal{O}_{7'} = \frac{e}{16\pi^2} m_b (\bar{s} \sigma^{\mu\nu} P_{R(L)} b) F_{\mu\nu}$$

$$\mathcal{O}_{8'} = \frac{g_s}{16\pi^2} m_b (\bar{s} \sigma^{\mu\nu} P_{R(L)} T^A b) G^{A}_{\mu\nu}$$

Four-quark current-current ($q = c, u$)

$$\mathcal{O}_{1q} = (\bar{q} \gamma_\mu P_L b)(\bar{s} \gamma^\mu P_L q)$$

$$\mathcal{O}_{2q} = (\bar{q} \gamma_\mu P_L T^a b)(\bar{s} \gamma^\mu P_L T^a q)$$

Four-quark QCD penguins

$$\mathcal{O}_{3,5} = (\bar{s} \Gamma_\mu \bar{P}_L b) \sum_q (\bar{q} \Gamma_\mu q)$$

$$\mathcal{O}_{4,6} = (\bar{s} \Gamma_\mu T^A P_L b) \sum_q (\bar{q} \Gamma_\mu T^A q)$$

SM contributions to $\mathcal{C}_i(\mu_B)$ known to high accuracy (NNLL) [Bobeth,Misiak,Urban ‘99; Misiak,Steinhauser ‘04, Gorbahn,Haisch ‘04]

[Gorbahn, Haisch, Misiak ‘05; Czakon, Haisch, Misiak ‘06]
Wilson coefficients $C_i$ can be computed in perturbation theory at some high energy scale $m_b \ll M_W \sim \mu_0$

however, matrix elements of operators are evaluated (e.g. from lattice QCD) at some low energy scale $\Lambda_{\text{had}} < \mu_1 < m_b$

mismatch must be resolved to obtain reliable predictions

Renormalization Group Equations (RGEs) provide means to evolve both the Wilson coefficients and the matrix elements from their respective intrinsic scales to one common scale

$\Rightarrow$ RGE-improved perturbation theory
Tangent 1: Renormalization Group Equations (RGE)

- RGE for multiplicatively-renormalizing quantities:

\[
\frac{\mu}{d\mu} C(\mu) = \gamma(\alpha_s(\mu))C(\mu) \quad \text{and} \quad \frac{d}{d\mu} \alpha_s(\mu) = 2\beta(\alpha_s(\mu))
\]

\[
\gamma = \gamma(0) \frac{\alpha_s}{4\pi} + O\left(\alpha_s^2\right) \quad \text{and} \quad \beta = \beta(0) \left(\frac{\alpha_s}{4\pi}\right)^2 + O\left(\alpha_s^3\right)
\]

Solution

\[
C(\mu_1) = C(\mu_0) \left[\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right] \left(\frac{\gamma(0)}{2\beta(0)}\right) + O\left(\alpha_s^{n+1}(\mu_0) \ln^n\left(\frac{\mu_1}{\mu_0}\right)\right)
\]

\sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right) \left(\frac{\gamma(0)}{2\beta(0)}\right) = 1 - \gamma(0)\alpha_s(\mu_0) \ln \left(\frac{\mu_1}{\mu_0}\right) + O\left(\alpha_s(\mu_0)^2 \ln^2 \left(\frac{\mu_1}{\mu_0}\right)\right)
\]

(*) resums all leading-logarithmic (LL) terms $\alpha_s^n(\mu_0) \ln^n \left(\frac{\mu_1}{\mu_0}\right)$ via

\[
\sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right) \left(\frac{\gamma(0)}{2\beta(0)}\right) = 1 - \gamma(0)\alpha_s(\mu_0) \ln \left(\frac{\mu_1}{\mu_0}\right) + O\left(\alpha_s(\mu_0)^2 \ln^2 \left(\frac{\mu_1}{\mu_0}\right)\right)
\]
Tangent 2: Role of $sbcc$ Operators at One Loop

- $sbcc$ 4-quark operators yield UV divergence
  - must be renormalized
  - require $s\bar{b}ll / s\bar{b}\gamma$ counterterm ($C_9 / C_7$)

- SM operator basis renormalizes multiplicatively
  - $\gamma$ is promoted to a matrix $\gamma_{ij}$
  - operators mix under RGE

- phenomenologically important
  - SM $sbcc$ operators contribute $\sim 50\%$ of $C_9^{SM}(\mu_b)$ at NNLL
Weak Effective Theory: $b \rightarrow s\ell\ell$ BSM Operators

- in the presence of NP effects

$$\mathcal{L}_{\text{BSM}}^\text{eff} = \mathcal{L}_{\text{SM}}^\text{eff} + \frac{4G_F}{\sqrt{2}} \left[ \lambda_t \sum_i c_i \mathcal{O}_i \right]$$

semileptonic

$$\mathcal{O}_9' = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_S = \frac{\alpha}{4\pi} (\bar{s}P_R b)(\bar{\ell}\ell)$$

$$\mathcal{O}_P = \frac{\alpha}{4\pi} (\bar{s}P_R b)(\bar{\ell}\gamma_5\ell)$$

$$\mathcal{O}_T = \frac{\alpha}{4\pi} (\bar{s}\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\ell)$$

$$\mathcal{O}_{10'} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_{S'} = \frac{\alpha}{4\pi} (\bar{s}P_L b)(\bar{\ell}\ell)$$

$$\mathcal{O}_P' = \frac{\alpha}{4\pi} (\bar{s}P_L b)(\bar{\ell}\gamma_5 \ell)$$

$$\mathcal{O}_{T5} = \frac{\alpha}{4\pi} (\bar{s}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5 \ell)$$

- regularly considered in the literature!
Weak Effective Theory: $b \rightarrow s \ell \ell$ BSM Operators

- in the presence of NP effects

$$\mathcal{L}^\text{eff}_{\text{BSM}} = \mathcal{L}^\text{eff}_{\text{SM}} + \frac{4G_F}{\sqrt{2}} \left[ \lambda_t \sum_i C_i O_i \right]$$

- add further $2 \times 18$ current-current operators with $q = c, u$
- add further $3 \times 16$ QCD-penguin operators with $q = d, s, b$
- these operators are routinely ignored in the literature!

[except by Jäger,Kirk,Lenz,Leslie '17]

- for a truly model-independent analysis of data, would need to fit coefficients of all 114 operators!
  - if we ignore tiny contributions due to $V_{ub} V_{us}^*$, reduces to 94 operators
  - if we focus on resonantly enhanced contributions due to intermediate $\bar{c}c$ states, reduces to 34 operators somewhat more manageable!
WET makes calculations in the SM possible in the first place
  - separates long-distance from short-distance physics (C from ops)

“divide and conquer”
  - SM WET contributions under excellent theory control
  - precision of SM predictions hinges on accurate control of hadronic matrix elements

accounts transparently and model-independently for the effects of physics beyond the SM
  - treat Wilson coefficients as generalized couplings and fit from data
  - excellent interface to model builders
From the WET to the Observables
Anatomy of exclusive $b \to s \ell^+ \ell^-$ decay amplitudes

\[ A_{\lambda}^\chi = N_{\lambda} \left\{ (C_9 \mp C_{10}) F_\lambda(q^2) + \frac{2m_bM_B}{q^2} \left[ C_7 F_T^\lambda(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} \]

nomenclature of the essential hadronic matrix elements

- $F_\lambda$ local form factors of dimension-three $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu \gamma_5 b$ currents
- $F_T^\lambda$ local dipole form factors of dimension-three $\bar{s}\sigma^{\mu\nu} b$ currents
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

$q^2 = m_{\ell\ell}^2$

all three needed for consistent description to leading-order in $\alpha_e$
Local Form Factors

- local form factors are conceptually “easy”
  - yet a substantial source of uncertainties
- lattice QCD provides results typically at large $q^2$
  for $B \rightarrow K$, $B \rightarrow K^*$, $B_s \rightarrow \phi$
  - caveat: $K^*$ is broad state, non-zero width can
    have $O(10\%)$ effects
    [Descotes-Genon, Khodjamirian, Virto ‘19]
  - new lattice results down to $q^2 = 0$ for $B \rightarrow K$ form
    factors
    [HPQCD ‘22]
- light-cone sum rules provide anchor points at
  small $q^2$
  - caveat: systematic uncertainties hard to quantify
Local Form Factors

- Local form factors are conceptually “easy”
  - Yet a substantial source of uncertainties
- Lattice QCD provides results typically at large $q^2$
  - For $B \rightarrow K, B \rightarrow K^*, B_s \rightarrow \phi$
    - Caveat: $K^*$ is broad state, non-zero width can have $O(10\%)$ effects
    - New lattice results down to $q^2 = 0$ for $B \rightarrow K$ form factors
- Light-cone sum rules provide anchor points at small $q^2$
  - Caveat: Systematic uncertainties hard to quantify
- IPPPP group recently revisited dispersive bounds for all local $b \rightarrow s$ form factors
- global analysis finds good compatibility between LCSR and lattice QCD results
- dispersive bounds have been split for the first time by polarization state
  - remove spurious theory correlations between different form factors
  - reduces extrapolation error
- Global analysis finds good compatibility between LCSR and lattice QCD results.
- Dispersive bounds have been split for the first time by polarization state:
  - Remove spurious theory correlations between different form factors.
  - Reduces extrapolation error.
- Commonly used BSZ parametrization surprisingly efficient:
  - Dispersive bound and BSZ very compatible for \( q^2 \geq 0 \), no need to swap params as of yet.
  - For non-local form factors, we will require \( q^2 < 0 \), where BSZ underestimates uncertainties.

[Gubernari, Reboud, DvD, Virto '23]
Non-Local Form Factors: Spectrum

\[ \mathcal{H}_\lambda = P(\lambda) \mu \langle H_s \mid \int d^4x \ e^{iq \cdot x} \ T \{ J_{\text{em}}^\mu(x), [C_1 O_1^C + C_2 O_2^C](0) \} \mid H_D \rangle \]

source of dominant systematic uncertainties in theoretical predictions!
Non-Local Form Factors: Spectrum

\[ \mathcal{H}_\lambda = P(\lambda) \mu \langle H_s | \int d^4x \ e^{i q \cdot x} \ T \{ J_{\text{em}}^\mu(x), [C_1 O_1^c + C_2 O_2^c](0) \} | H_D \rangle \]

- leading contributions expressed through local form factors \( F_\lambda \)
- correction suppressed by \( 1/(q^2 - 4m_c^2) \) can be systematically obtained

\[ O_{1,2}^c \sim [\mathcal{G}b] [\bar{c}\Gamma'c] \]
\[ \mathcal{H}_\lambda = P(\lambda) \mu \langle H_s | \int d^4x \ e^{iq\cdot x} \mathcal{T} \{ J_{em}^\mu(x), [C_1O_1^c + C_2O_2^c](0) \} | H_D \rangle \]

- $O_{1,2}^c \sim [\mathbb{S} \Gamma b] [\bar{c}\Gamma' c]

- for $q^2 = M_J^2/\psi$ and $q^2 = M_{\psi(2S)}^2$, spectrum dominated by hadronic decays

- experimental measurements provide additional information about $\mathcal{H}_\lambda$
Non-Local Form Factors: Spectrum

\[ \mathcal{H}_\lambda = P(\lambda)_{\mu} \langle H_s | \int d^4x \ e^{iq\cdot x} \ T\{ J^\mu_{\text{em}}(x), [C_1 O_1^c + C_2 O_2^c](0) \} | H_B \rangle \]

[Graph showing energy and mass distributions with labels for QCDF, OPE, photon pole, narrow c\bar{c} resonances, broad c\bar{c} resonances, and narrow pole-photon interference.]

- New strategy
  - Compute \( \mathcal{H}_\lambda \) at spacelike \( q^2 \)
  - Extrapolate to timelike \( q^2 \leq 4M_D^2 \) using suitable parametrization
  - Include information from hadronic decays to narrow charmonia \( J/\psi \) and \( \psi(2S) \)
the literature frequently discusses “the QCDF“ approach to the non-local form factors
  - more correctly labelled: 1-loop, perturbative approach to non-local form factors
QCDF predicts ratios of local form factors, and ratios of some contributions to non-local form factors
  - QCDF is not predictive by itself, requires lattice QCD or light-cone sum rules as inputs
  - QCDF is not “dealing” with the charm loop contributions; it is agnostic of their treatment
slightly more technical
  - QCDF is used to express exclusive form factors for small $q^2$ in terms of nonlocal $B$ and $K^{(*)}$ matrix elements (LCDAs)
  - this calculation encounters universal divergences $\Rightarrow$ not predictive for an individual form factor
  - universal divergences cancel in ratios
Preparing $b \rightarrow sll$ predictions for the era of the High-Luminosity LHC
Reduce systematical theory uncertainties

- **check previous computations** of the nonlocal form factors at subleading power
  - previous results incomplete, missing terms cancel known contributions
  - subleading-power terms are negligible at spacelike $q^2$

- **improve the parametrization** to control the extrapolation error
  - use dispersively-bounded parametrization for both local and non-local form factors

- **challenge implicit theory assumptions** in the nonlocal form factors
  - determine WET Wilson coefficients of $sbcc$ operators from data

---

[Gubernari,DvD,Virto '20; Gubernari,Reboud,Virto '22; Gubernari,Reboud,Virto '23]

[Kirk,McPartland,Reboud,DvD,Virto]
Compute Light-Cone OPE

\[ 4m_c^2 - q^2 \gg \Lambda_{\text{hadr}}^2 \]

- expansion in operators at light-like distances \( x^2 \sim 0 \)
  
  [Khodjamirian, Mannel, Pivovarov, Wang 2010]

- employing light-cone expansion of charm propagator
  
  [Balitsky, Braun 1989]

\[ \int d^4x \ e^{iq\cdot x} \mathcal{T} \{ J_{\text{em}}^\mu(x), [C_1 O_1^c + C_2 O_2^c](0) \} \]

\[ q^2 \ll 4m_c^2 \quad \left( \frac{C_1}{3} + C_2 \right) g(m_c^2, q^2) [\bar{s} \Gamma b] + \cdots \]

\[ \text{coeff #1} \]

\[ + (\text{coeff #2}) \times [\bar{s}_L \gamma^\alpha (i n_+ \cdot D)^n \bar{G}_\beta \gamma^\nu b_L] \]
**Compute Light-Cone OPE**

\[ 4m_c^2 - q^2 \gg \Lambda_{\text{hadr}}^2 \]

- expansion in operators at light-like distances \( x^2 \sim 0 \)
  
  [Khodjamirian, Mannel, Pivovarov, Wang 2010]

- employing light-cone expansion of charm propagator
  
  [Balitsky, Braun 1989]

\[ b \xrightarrow[t]{s} 0 \xrightarrow[\gamma]\sim \gamma \]

\[ b \xrightarrow[s]{uX} 0 \xrightarrow[\gamma]\sim \gamma \]

\[ \Rightarrow H_\lambda = \text{coeff } #1 \times F_\lambda + H^{\text{spect.}}_\lambda + \text{coeff } #2 \times \tilde{V} \]

- **leading** part identical to QCD Fact. results
  

- **subleading** coefficient computed previously
  
  [Khodjamirian, Mannel, Pivovarov, Wang ’10]

- we find **full agreement**, also cast result in convenient form
  
  [Gubernari, Virto, DvD ’20]

- next step: determine “subleading form factor” \( \tilde{V} \)
## Soft gluon matrix elements

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\mathcal{V}(q^2 = 1,\text{GeV}^2)$</th>
<th>GvDV2020</th>
<th>KMPW2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to K$</td>
<td>$\AA$</td>
<td>$(+4.9 \pm 2.8) \cdot 10^{-7}$</td>
<td>$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}_1$</td>
<td>$(-4.4 \pm 3.6) \cdot 10^{-7}$ GeV</td>
<td>$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}$ GeV</td>
</tr>
<tr>
<td>$B \to K^*$</td>
<td>$\mathcal{V}_2$</td>
<td>$(+3.3 \pm 2.0) \cdot 10^{-7}$ GeV</td>
<td>$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}$ GeV</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}_3$</td>
<td>$(+1.1 \pm 1.0) \cdot 10^{-6}$ GeV</td>
<td>$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}$ GeV</td>
</tr>
<tr>
<td>$B_s \to \phi$</td>
<td>$\mathcal{V}_1$</td>
<td>$(-4.4 \pm 5.6) \cdot 10^{-7}$ GeV</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}_2$</td>
<td>$(+4.3 \pm 3.1) \cdot 10^{-7}$ GeV</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}_3$</td>
<td>$(+1.7 \pm 2.0) \cdot 10^{-6}$ GeV</td>
<td>—</td>
</tr>
</tbody>
</table>

Reduction by a factor of $\sim 200$

- **new structures** in three-particle LCDAs account for factor 10 (due to cancellations!)
- **updated inputs** that enter the sum rules account for further factor 10
- **similar relative uncertainties**, but **absolute uncertainties reduced by $\mathcal{O}(100)$**
ongoing project at IPPP to compute **leading non-local** contributions for full BSM basis of sbcc operators

- first step to full control of non-local form factors in the WET
- we plan to also leverage measurements of $\bar{B} \rightarrow K\eta_c$ and $\Lambda_b \rightarrow \Lambda\eta_c$ decays

ongoing project in Siegen to better classify non-local operators

- of particular interest: contributions with hard-collinear gluon
- relevant to “internal” charm loop
Taylor expand $H_\lambda$ in $q^2/M_B^2$ around 0

- simple to use in a fit
- incompatible with analyticity properties, does not reproduce resonances
- expansion coefficients unbounded!

[Ciuichini et al. ‘15]
Taylor expand $\mathcal{H}_\lambda$ in $q^2/M_B^2$ around 0

- simple to use in a fit
  - incompatible with analyticity properties, does not reproduce resonances
  - expansion coefficients unbounded!

use information from hadronic intermediate states in a dispersion relation

$$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(s_0) = \frac{q^2-s_0}{\pi} \int ds \frac{\text{Im} \mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \ldots$$

- reproduces resonances
  - hadronic information above the threshold must be modelled
  - complicated to use in a fit, relies on theory input in single point $s_0$
Extrapolate Parametrisations

- Taylor expand $\mathcal{H}_\lambda$ in $q^2/M_B^2$ around 0
  - simple to use in a fit
  - incompatible with analyticity properties, does not reproduce resonances
  - expansion coefficients unbounded!

- use information from hadronic intermediate states in a dispersion relation
  \[ \mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(s_0) = \frac{q^2-s_0}{\pi} \int \frac{ds}{(s-s_0)(s-q^2)} \text{Im} \mathcal{H}_\lambda(s) + \ldots \]
  - reproduces resonances
  - hadronic information above the threshold must be modelled
  - complicated to use in a fit, relies on theory input in single point $s_0$

- expand the matrix elements in variable $z(q^2)$ that develops branch cut at $q^2 = 4M_D^2$
  - resonances can be included through explicit poles (Blaschke factors)
  - easy to use in a fit
  - compatible with analyticity properties
  - expansion coefficients unbounded!
map $q^2$ to new variable $z$ that develops branch cut at $q^2 = 4M^2_D$

[Bobeth/Chrzaszcz/DvD/Virto '17]
map $q^2$ to new variable $z$ that develops branch cut at $q^2 = 4M_D^2$

- branch cut is mapped onto unit circle in $z$
- real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued $z$
- data and theory live inside the unit circle
map $q^2$ to new variable $z$ that develops branch cut at $q^2 = 4M_D^2$

- branch cut is mapped onto unit circle in $z$
- real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued $z$
- data and theory live inside the unit circle
- expand in $z$
  + resonances $J/\psi, \psi(2S)$ can be included (via explicit poles/Blaschke factors)
  + easy to use in a fit to theory and data
  + compatible with analyticity
  - expansion coefficients unbounded!
matrix elements $\mathcal{H}^{(\lambda)}$ arise from nonlocal operator

$$\mathcal{H}^\mu \sim \langle K | O^\mu(Q; x) | B \rangle \quad O^\mu(Q; x) \sim \int d^4 y \ e^{iQ \cdot y} T \{ J^\mu_{\text{em}}(x + y), [C_1 O_1 + C_2 O_2](x) \}$$

construct four-point operator to derive a dispersive bound

- define matrix element of “square” (i.e., hermitian) operator

$$\int d^4 x \ e^{iQ \cdot x} \langle 0 | T \{ O^\mu(Q; x) O^\dagger_{\nu}(Q; 0) \} | 0 \rangle \equiv \left[ \frac{Q^\mu Q^\nu}{Q^2} - g^\mu\nu \right] \Pi(Q^2)$$

- $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}, c\bar{c}c\bar{c}, \ldots$)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}, b\bar{s}g, b\bar{s}c\bar{c}, \ldots$)
Extrapolate Cuts of $\mathfrak{N}$

unflavoured states ($c, c, c, \ldots$)

b-flavoured states ($b, bs, bs, bs, \ldots$)
Extrapolate Cuts of $\Pi$

- unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, \ldots)

![Diagram with arrows and nodes representing unflavoured states and b-flavoured states.](image-url)
Extrapolate Cuts of $\Pi$

- **unflavoured states** ($c\bar{c}, c\bar{c}c\bar{c}, \ldots$)

- **$b\bar{s}$-flavoured states** ($b\bar{s}, b\bar{s}g, b\bar{s}c\bar{c}, \ldots$)
Extrapolate

Lay of the Land

light-cone OPE

SL phase space

$J/\psi, \psi(2S)$

$\bar{s}b$ cut
Extrapolate Dispersion relation for $\Pi$

dispensable representation of the $b\bar{s}$ contribution to a derivative of $\Pi$

$$\chi(Q^2) \equiv \frac{1}{2!} \left( \frac{d}{dQ^2} \right)^2 \frac{1}{2i\pi} \int_0^{\infty} ds \frac{\text{Disc}_{b\bar{s}} \Pi(s)}{s - Q^2} > 0 \quad \text{if } Q^2 < 0$$

- $\text{Disc}_{b\bar{s}} \Pi$ can be computed in the local OPE
  $$\to \chi^{\text{OPE}}(Q^2)$$

- $\text{Disc}_{b\bar{s}} \Pi$ can be expressed in terms of the nonlocal form factors $|\mathcal{H}_\lambda|^2$
  $$\to \chi^{\text{had}}(Q^2)$$

- global quark hadron duality suggests that $\chi^{\text{OPE}}(Q^2) = \chi^{\text{had}}(Q^2)$

- parametrize $\mathcal{H}_\lambda \propto \sum_n a_{\lambda,n} f_n$ with orthonormal functions $f_n$
  $$\Rightarrow \text{dispersive bound: } \chi^{\text{OPE}} \geq \sum_n |a_{\lambda,n}|^2$$

- first application of such a bound to nonlocal form factors

- technically more challenging than for local form factors
Extrapolate New parametrisation w/ dispersive bounds

- expand in $z$
  - $f_n(z)$ orthogonal on arc
  - accounting for behaviour on arc produces dispersive bound on each parameter ✓

- turns (so far!) hardly quantifiable systematic theory uncertainties into parametric uncertainties

- implemented in EOS
  - open source software at github.com/eos/eos
  - Python 3 interface, available via pip as eoshep
> predictions mutually compatible; slight change to the slope in $B_s \to \phi$ due to local FFs

> our uncertainties larger, but systematically improvable
SM Predictions: Challenging Data

- substantial tensions in $\mathcal{B}(B \to K\mu^+\mu^-)$ and $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$, lower in $\mathcal{B}(B \to K^*\mu^+\mu^-)$
substantial tensions in $\mathcal{B}(B \to K\mu^+\mu^-)$ and $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$, lower in $\mathcal{B}(B \to K^*\mu^+\mu^-)$

tension in angular distribution in $B \to K^*\mu^+\mu^-$ remains
Updated BSM Interpretation

- no global fit yet
  - large # of nuisance params makes global fit difficult
  - instead, three individual fits
  - mutually compatible results!
  - compatible with previous analyses

- fits use all available data, incl. angular obs.

- substantial tensions in $B \to K$ and $B_s \to \phi$, slightly lower in $B \to K^*$
Summary
phenomenology of rare $B$ decays is a complicated business
  - WET under good control
  - local form factors see revitalized interest from lattice QCD
  - non-local form factors now under reasonable theory control

new approach to (B)SM predictions corroborates earlier results qualitatively
  - larger uncertainties reduce significance of the anomalies somewhat
  - uncertainties very conservative and systematically improvable

still: a lot to do for phenomenologists, amongst others:
  - performing a truly global fit in the new approach
  - extending analysis to $\Lambda_b \rightarrow \Lambda$ transitions
Backup Slides
The elephant in the room
Joint LHCb measurement of $R_K$ and $R_{K^*}$

- lepton-flavour-nonuniversality in $b \to s \ell^+ \ell^-$ is gone!
  - not the longest standing anomaly by far!
  - not the only one, either!
- I prefer to think of it as a precision measurement of $\mathcal{B}(B \to K^{(*)} e^+ e^-)$
  - gives rise to a new anomaly
  - $\mathcal{B}(B \to Ke^+ e^-)$ deviates from SM prediction by roughly the same amount as $\mathcal{B}(B \to K \mu^+ \mu^-)$!