Neutrinos and the Flavour Problem

Part I. The Flavour Problem

Part II. Neutrino Phenomenology

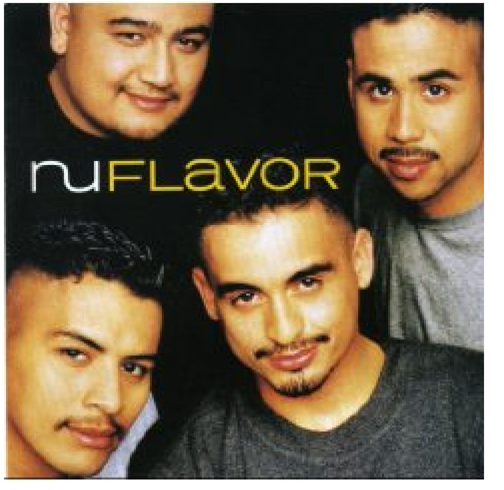
Part III. Flavour in the Standard Model

Part IV. Origin of Neutrino Mass

Part V. Flavour Models

Part VI. Discrete Family Symmetry

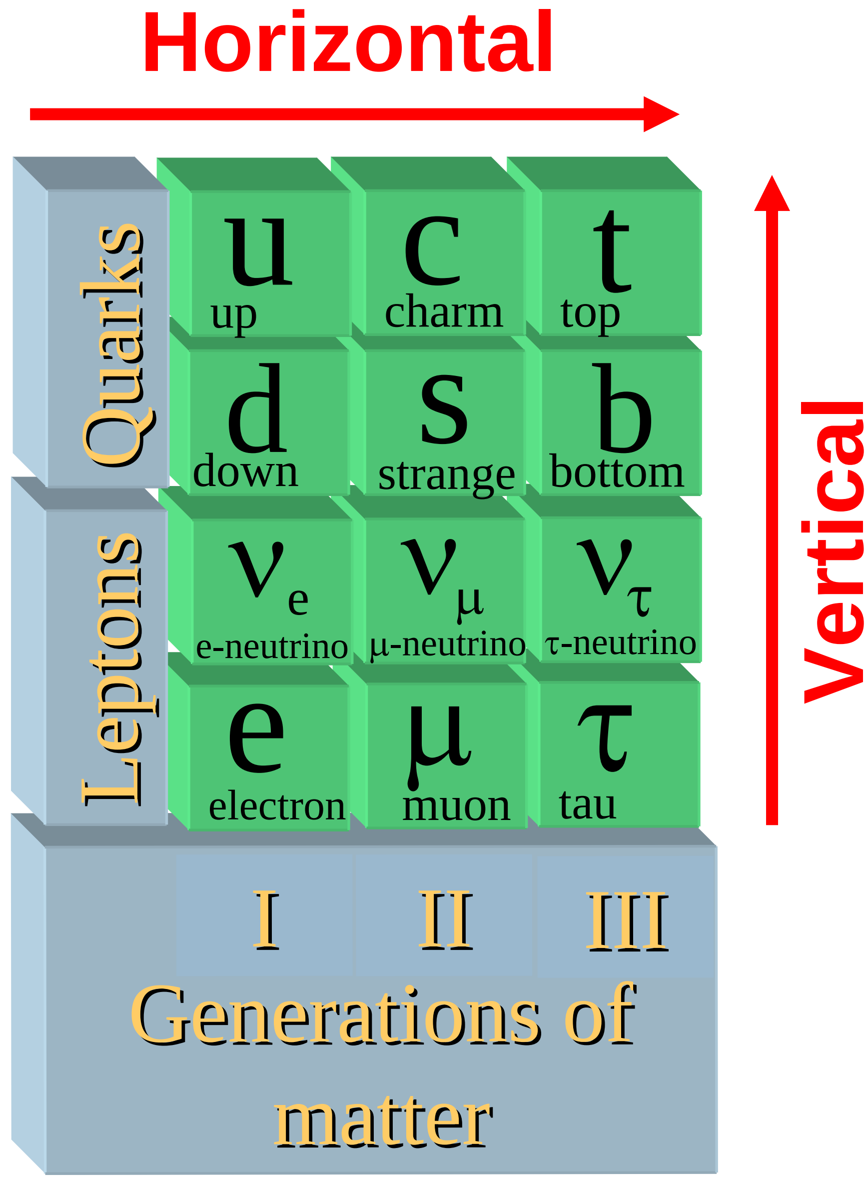
Part VII. GUT Models



Part I.
The Flavour Problem

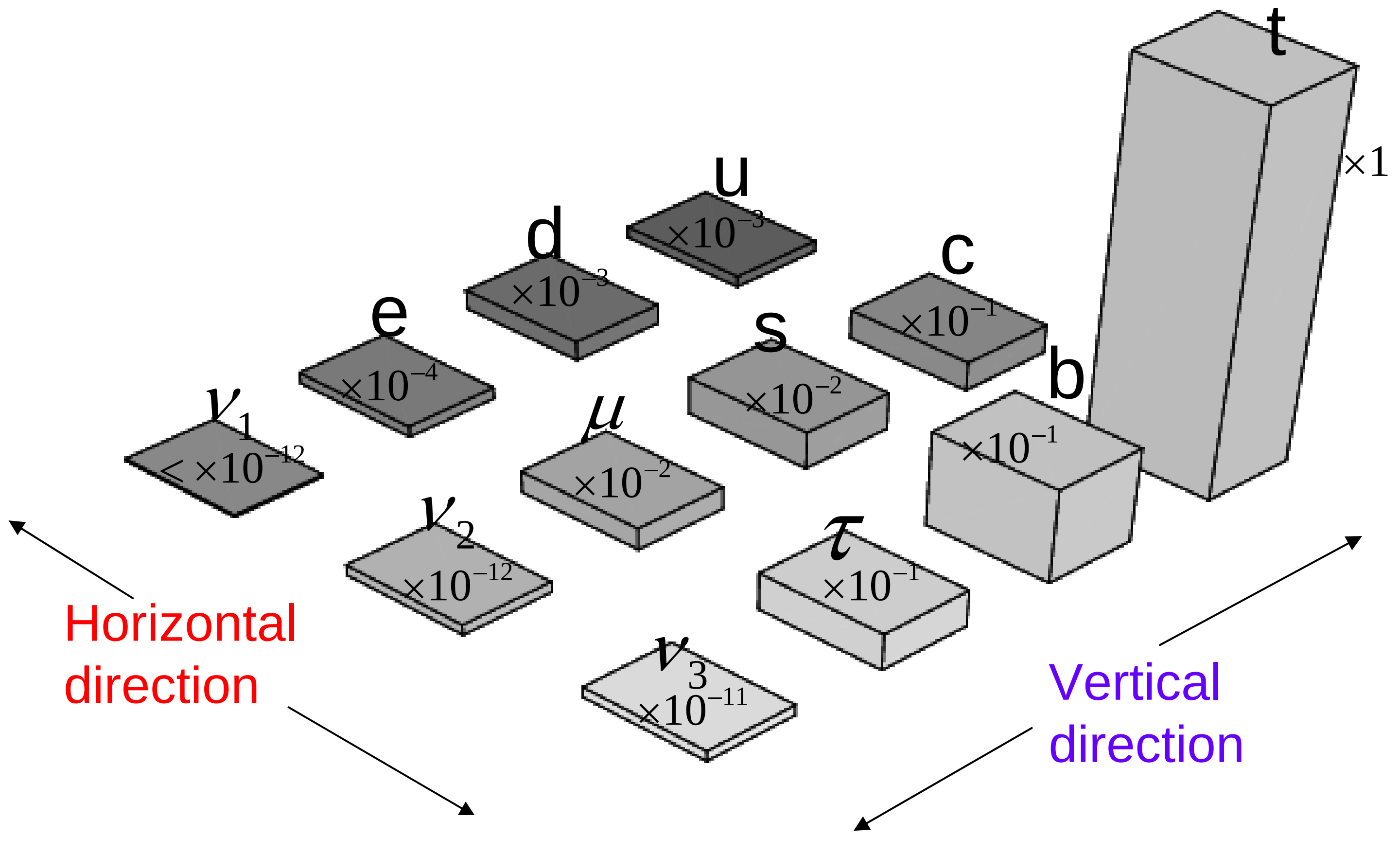
The Flavour Problem

1. Why are there three families of quarks and leptons?

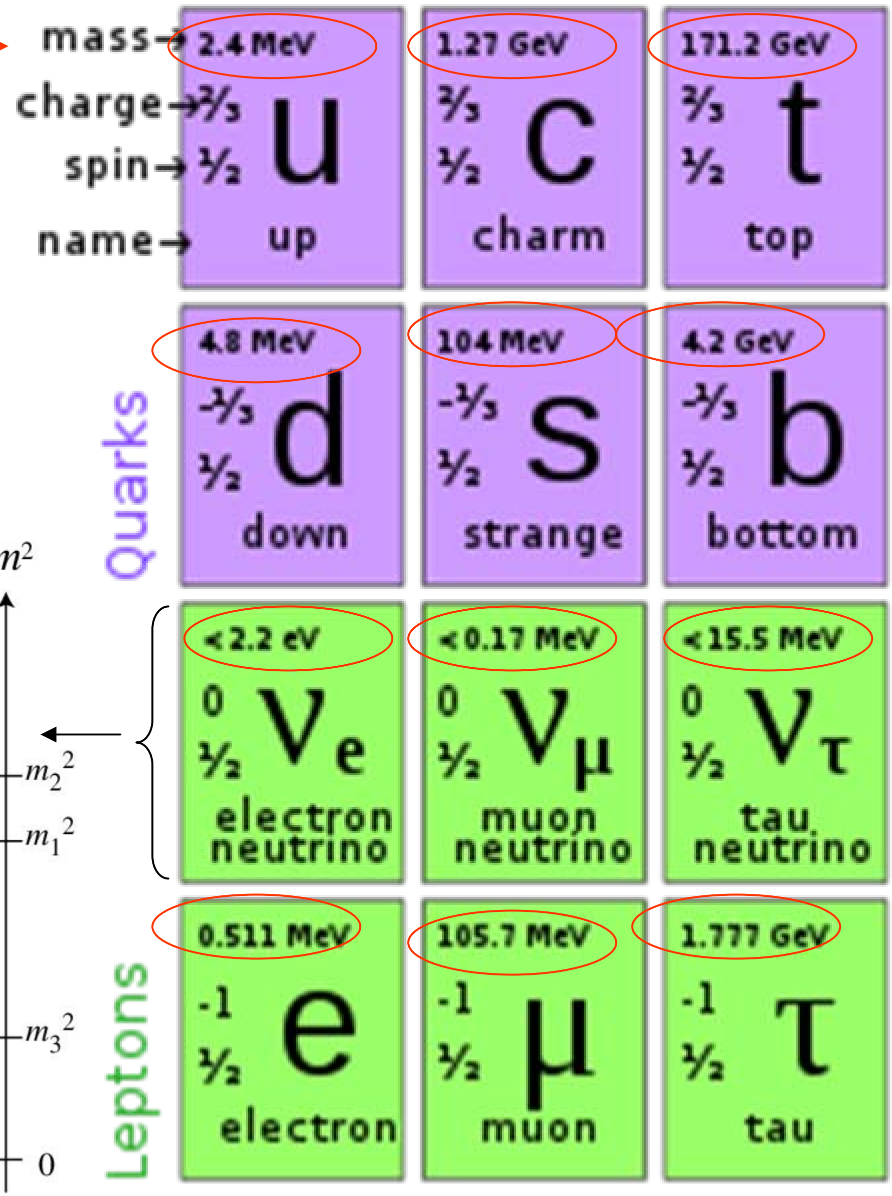


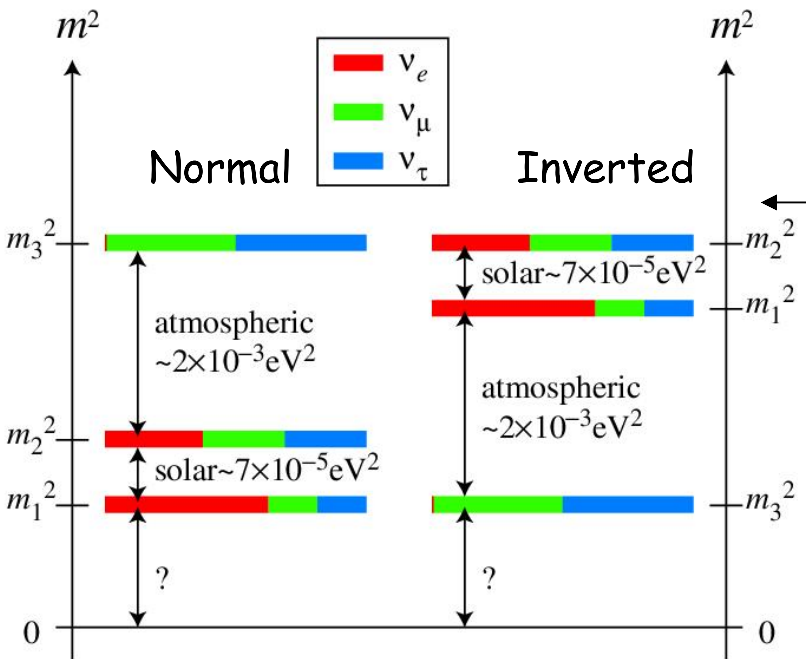
The Flavour Problem

2. What is the origin of quark and lepton masses?



quark and lepton masses

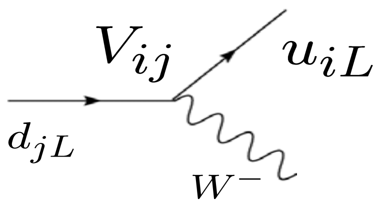




Absolute neutrino mass scale?

The Flavour Problem
3. Why is quark mixing so small?

Cabibbo
Kobayashi
Maskawa



$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{aligned} \theta_{12} &= 13^\circ \pm 0.1^\circ \\ \theta_{23} &= 2.4^\circ \pm 0.1^\circ \\ \theta_{13} &= 0.20^\circ \pm 0.05^\circ \end{aligned}\right\}$$

All these angles are pretty small – why?

While the CP phase is quite large

$\delta_{CP} \approx 70^o \pm 5^o$

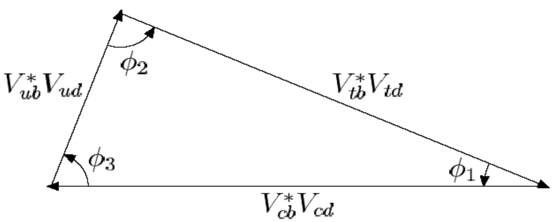
The Flavour Problem
4. What is origin of quark CP violation?

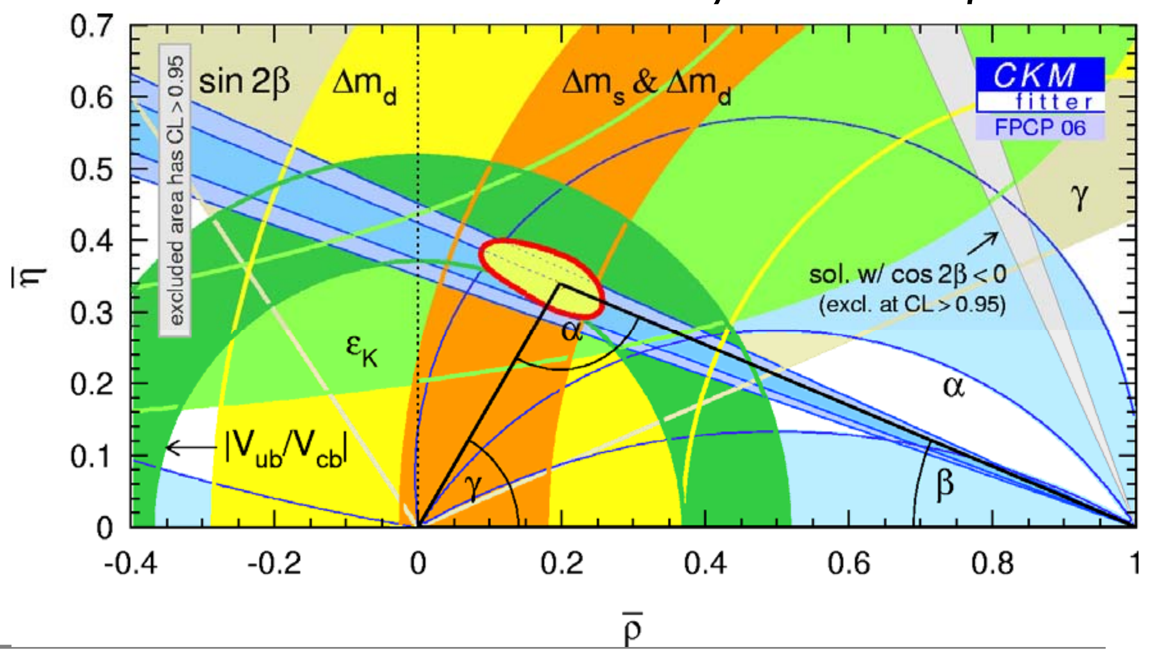
Wolfenstein

$$\begin{pmatrix} \boxed{\begin{matrix} V_{ud} \\ V_{cd} \\ V_{td} \end{matrix}} & \begin{matrix} V_{us} \\ V_{cs} \\ V_{ts} \end{matrix} & \boxed{\begin{matrix} V_{ub} \\ V_{cb} \\ V_{tb} \end{matrix}} \end{pmatrix} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$

$$\lambda \approx 0.226 \quad A \approx 0.81 \quad \rho \approx 0.13 \quad \eta \approx 0.35$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$





$\alpha \approx 90^o \pm 4^o$

$\delta_{CP} \approx \gamma \approx 70^o \pm 5^o$

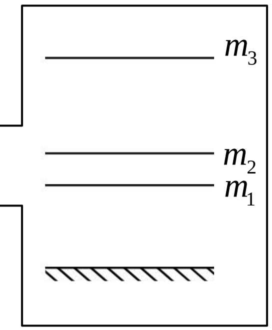
The Flavour Problem

5. Why is lepton mixing so large?

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Standard Model states

Neutrino mass states



Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric | Reactor | Solar | Majorana

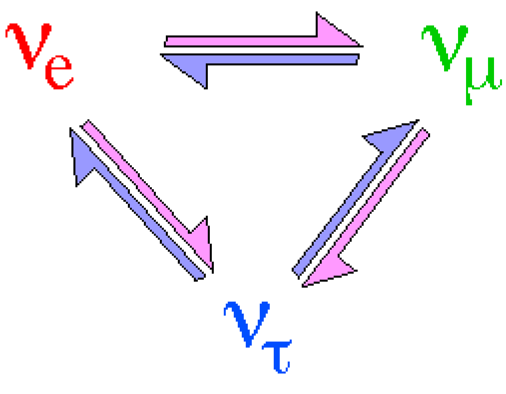
$$s_{ij} = \sin\theta_{ij}$$
$$c_{ij} = \cos\theta_{ij}$$

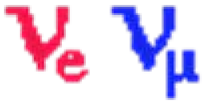
Oscillation phase δ

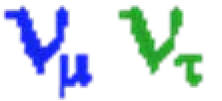
Majorana phases α_1, α_2

3 masses + 3 angles + 1(3) phase(s) = 7(9) new parameters for SM

Part II. Neutrino Phenomenology

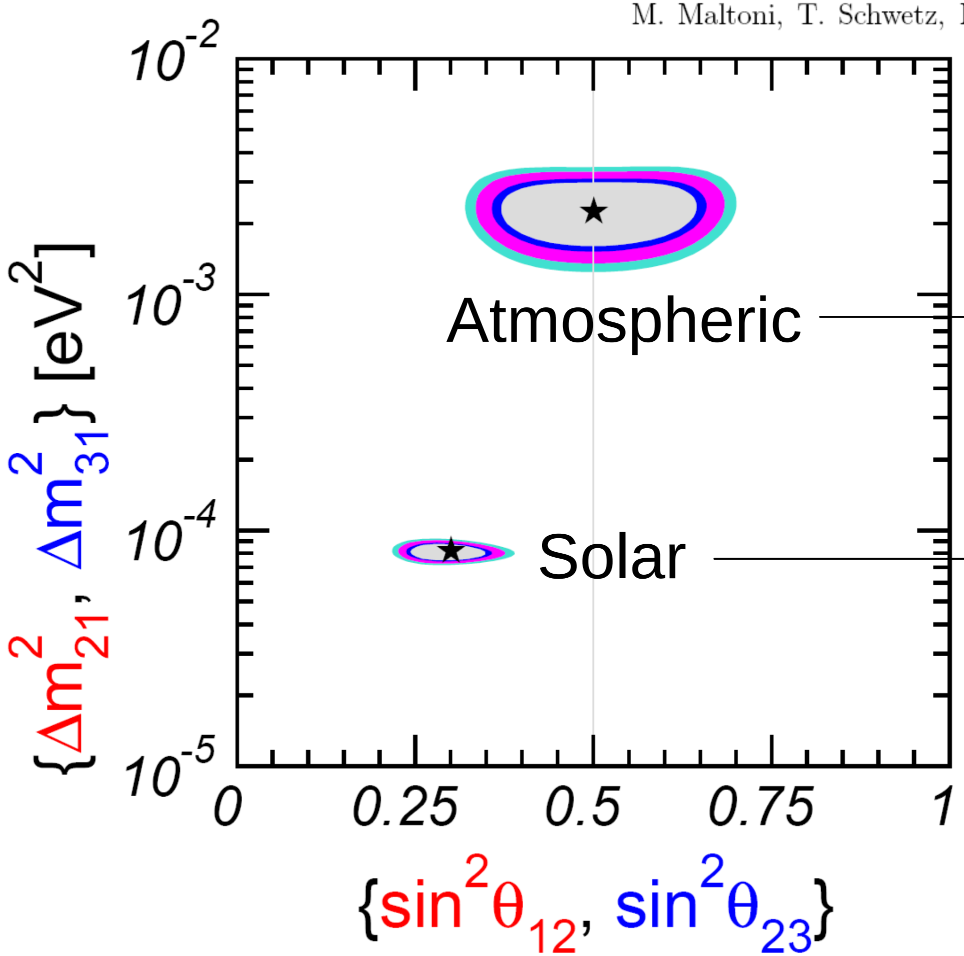






Global Fit to Atmospheric and Solar Data

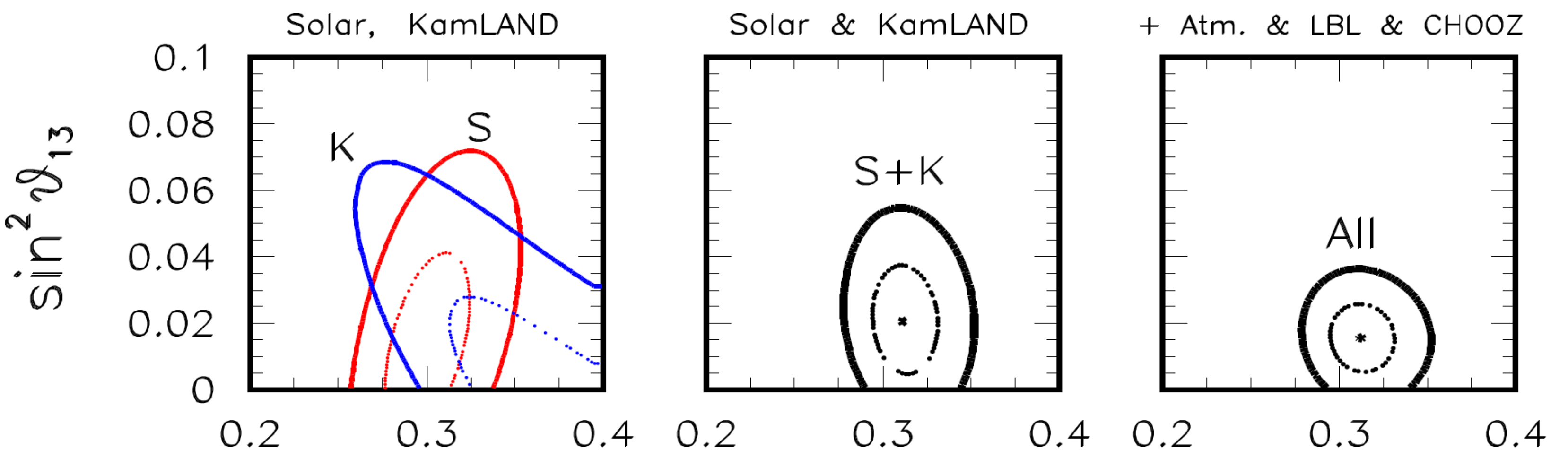
M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle,



$$\theta_{23} = 43.1^{o}{}^{+4.4^o}_{-3.5^o}$$

$$\theta_{12} = 34.5^o \pm 1.4^o$$

There is a 2σ hint for θ_{13} being non-zero



$$\sin^2\theta_{13} = 0.016 \pm 0.010$$

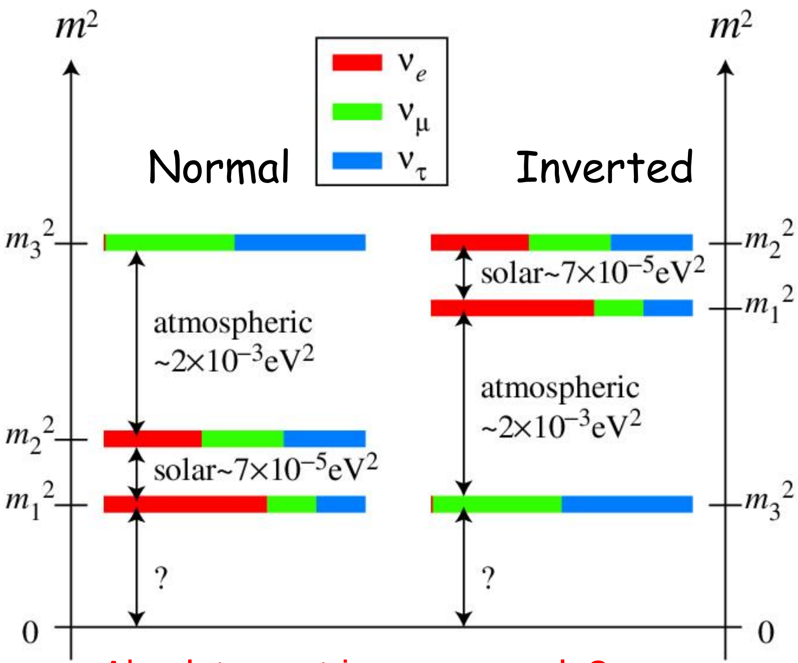
Fogli et al '08

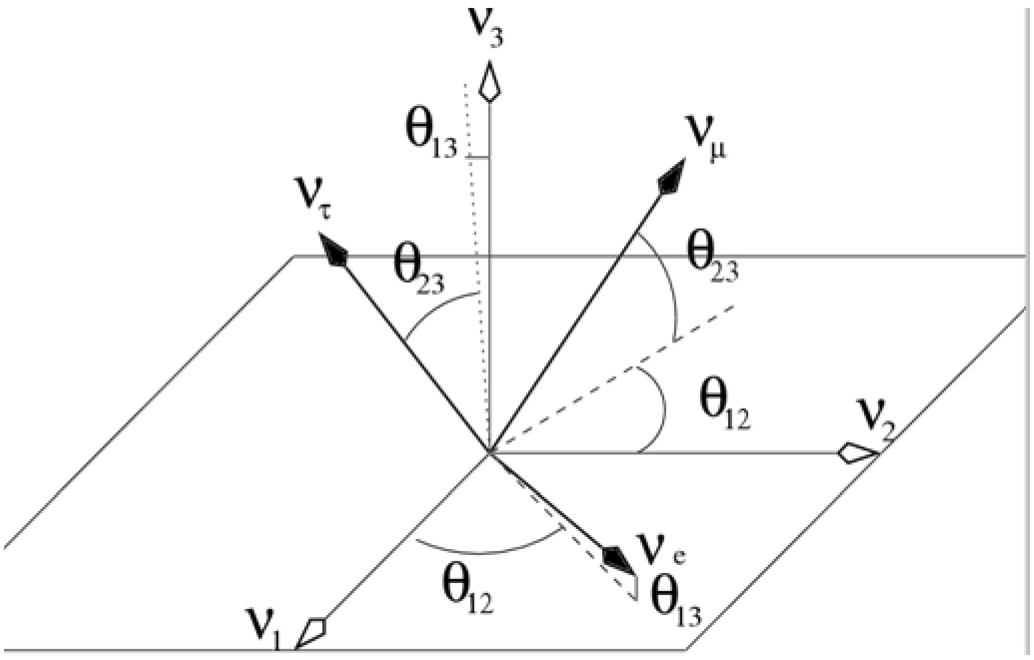
$$\sin^2\theta_{13} = 0.02 \pm 0.01$$

Fogli et al '09

The 2009 estimate includes the MINOS results which show a 1.5σ excess of events in the electron appearance channel

Neutrino mass squared splittings and angles





Absolute neutrino mass scale?

Why are neutrino masses so small ?

$$\left.\begin{aligned}\theta_{12} &= 34.5^\circ \pm 1.4^\circ \\ \theta_{23} &= 43^\circ \pm 4^\circ \\ \theta_{13} &\le 10^\circ\end{aligned}\right\}$$

Two of these angles are pretty large – why?

Tri-bimaximal mixing matrix U_{TB}

Harrison, Perkins, Scott

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

TB angles $\theta_{12} = 35^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ.$

c.f. data $\theta_{12} = 34.5^\circ \pm 1.4^\circ, \theta_{23} = 43.1^\circ \pm 4^o, \ \theta_{13} = 8^\circ \pm 2^o$

Current data is consistent with TB mixing
(ignoring the 2σ hint for θ_{13})

Useful to parametrize the PMNS mixing matrix in terms of deviations from TBM

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

SFK '07

$$0.14 < r < 0.24, \ -0.05 < s < 0.02, \ -0.04 < a < 0.10$$

r = reactor

s = solar

a = atmospheric

e.g. **r≠ 0, s=0, a=0** gives Tri-bimaximal-reactor (TBR) mixing

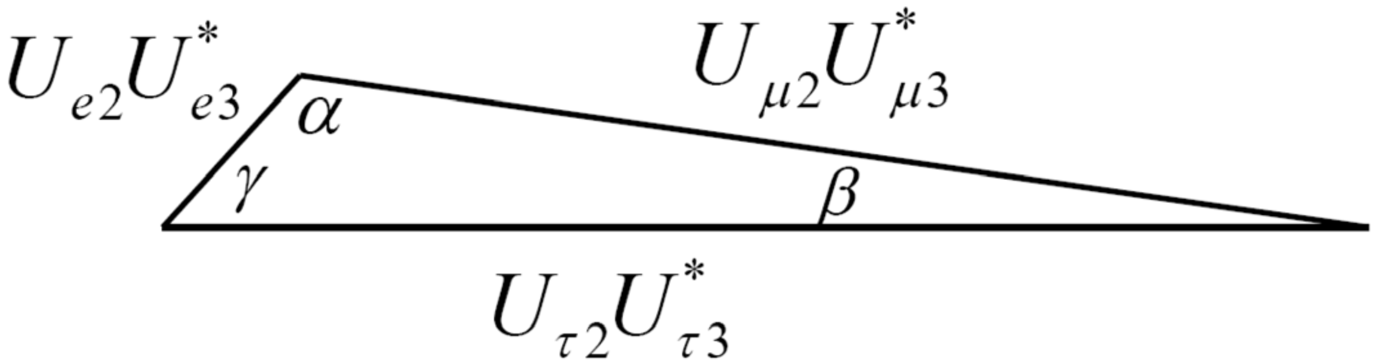
$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + r e^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1 - r e^{i\delta}) & -\frac{1}{\sqrt{3}}(1 + \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix}$$

SFK '09

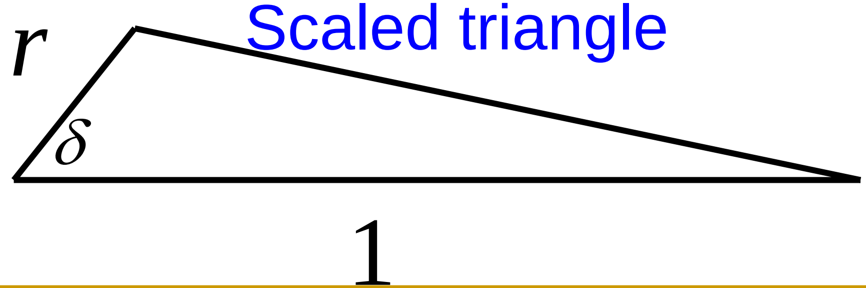
TBR not as simple as TB but is required if $\theta_{13} \neq 0$

Leptonic CP violation is unknown

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$



$\nu_2.\nu_3$



Scaled triangle

Neither r nor δ is measured – UT could be a straight line!

Neutrinos can have Dirac and/or Majorana Mass

CP conjugate

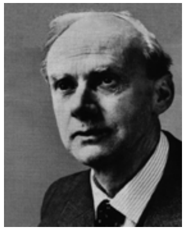
Majorana masses

$$\left.\begin{array}{l} \boxed{m_{LL}\overline{\nu}_L\nu_L^c} \\ \boxed{M_{RR}\overline{\nu}_R\nu_R^c} \end{array}\right\}$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino



$$\boxed{m_{LR}\overline{\nu}_L\nu_R}$$

Conserves L

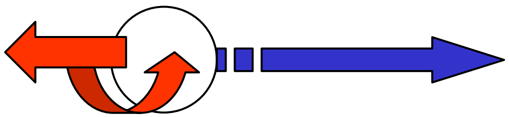
Violates L_e, L_μ, L_τ

Neutrino $\neq$ antineutrino

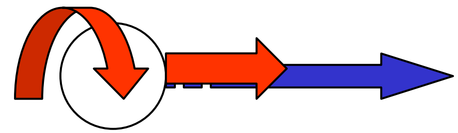
Dirac mass

In general a mass term can be thought of as an interaction between left and right-handed chiral fields

e^-_L e^-_R



$$m_e \overline{e^-_L}\, e^-_R$$

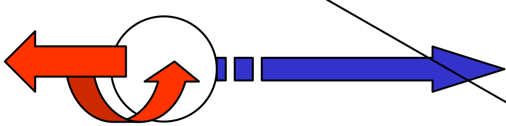


Left-handed neutrinos ν_L can form masses with either right-handed neutrinos ν_R or with their own CP conjugates ν^c_L

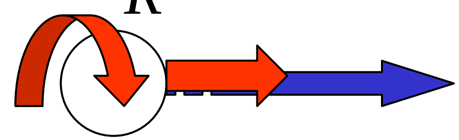
ν_L

Dirac

ν_R



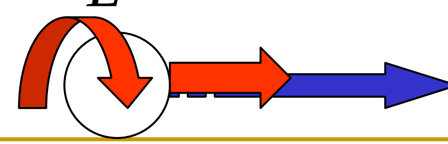
$$m^\nu_{LR}\,\overline{\nu}_L\,\nu_R$$



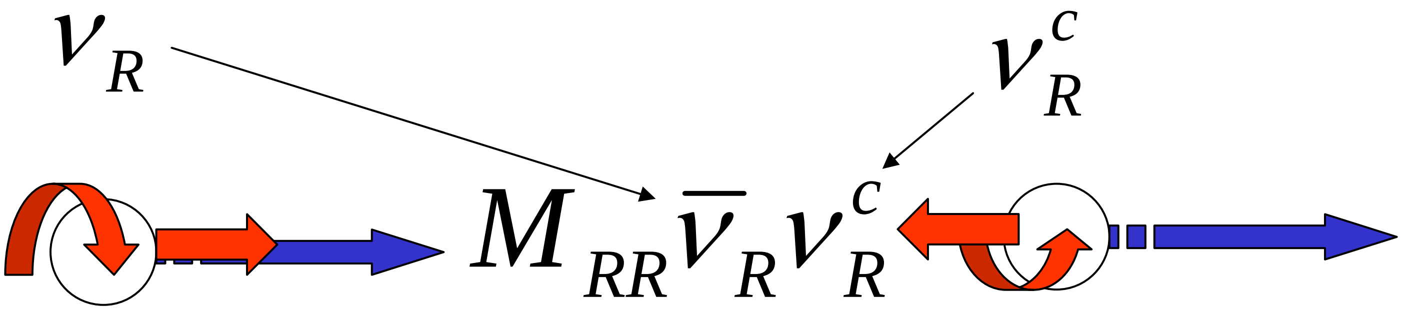
ν^c_L

Majorana

$$m^\nu_{LL}\,\overline{\nu}_L\,\nu^c_L$$



Right-handed neutrinos ν_R can also form masses with their own CP conjugates $\nu_R{}^c$

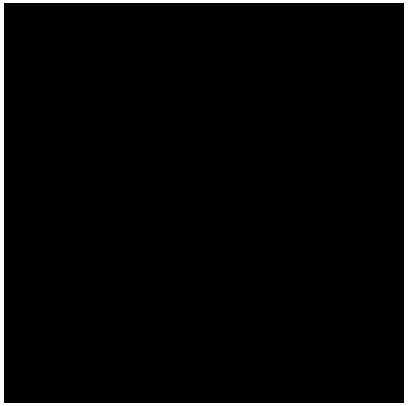


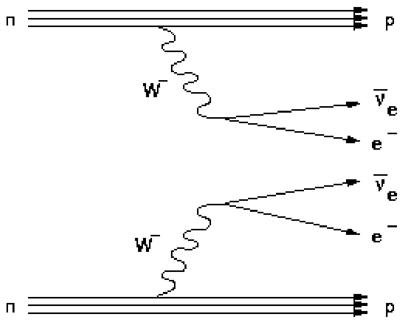
Majorana

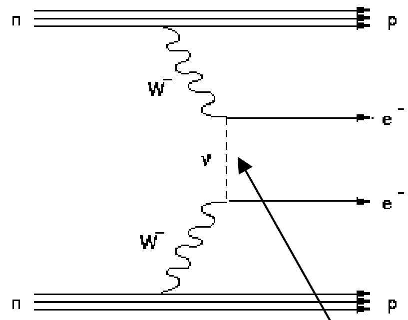
In principle there is nothing to prevent the right-handed Majorana mass M_{RR} from being arbitrarily large since ν_R is a gauge singlet e.g. $M_{RR}=M_{GUT}$

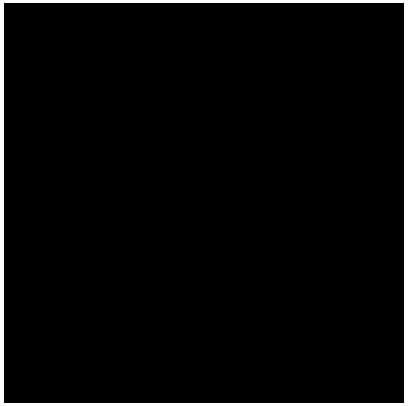
On the other hand it is possible that $M_{RR}=0$ which could be enforced by lepton number L conservation

Absolute ν mass scale and the nature of ν mass









Majorana (no signal if Dirac)

Neutrinoless double beta decay

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$$

Tritium beta decay

$$|m_{\nu_e}| = \left|\sum_i |U_{ei}|^2 m_i\right|$$

Present Mainz	< 2.2 eV	
KATRIN	~0.35eV	

Klapdor-	^{76} Ge	~0.4 eV (signal)
Majorana	^{76} Ge	~0.05 eV
GERDA	^{76} Ge	~0.1 eV (phase II)
		~0.01 eV (phase III)

Non- ^{76} Ge:

CUORE, NEMO3
SuperNEMO →50meV=0.05 eV
COBRA Cd116 →??
EXO Xe136 1 ton 5y 50-70meV=0.05-0.07eV
SNO+ Nd150 start 2011? →50meV=0.05 eV

The Future

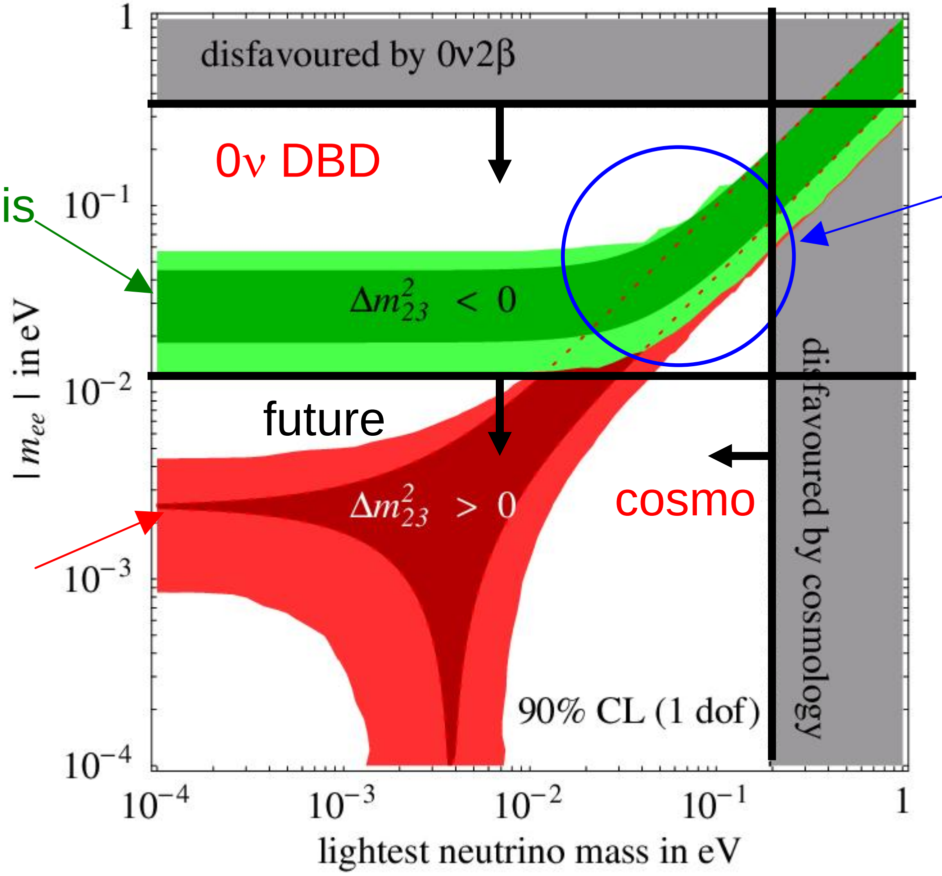
I 100-500meV SNemo CUORE GERDA EXO SNO+
II 15-50meV 1 ton 10y?
III 2-5meV 100 tons 20y??

$$U_{e1} = c_{12}c_{13}e^{i\alpha_1/2}, \quad U_{e2} = s_{12}c_{13}e^{i\alpha_2/2}, \quad U_{e3} = s_{13}e^{-i\delta}$$

Cosmology vs Neutrinoless DBD

Inverted hierarchy is TESTABLE

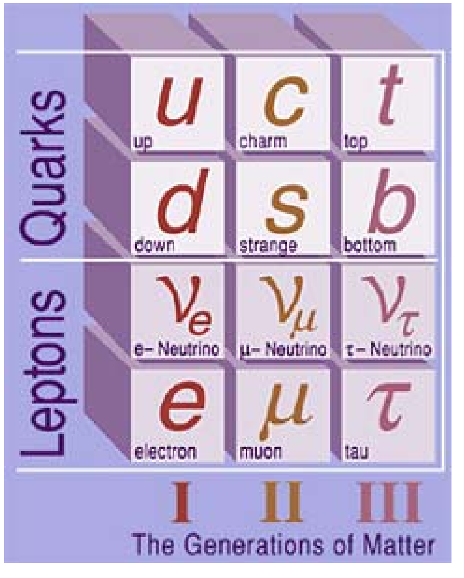
Normal hierarchy is NOT TESTABLE



Approx. degeneracy is TESTABLE

from: F. Feruglio, A. Strumia, F. Vissani ('02)

Part III. Flavour in the Standard Model



Standard Model

$$\underbrace{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

	Interactions			
	strong	electro-weak	gravitational	unified ?
Theory	QCD	GSW	quantum gravity ?	SUGRA ?
Symmetry	$SU(3)$	$SU(2)\times U(1)$	?	SU(5)?
Gauge bosons	$g_1 \cdots g_8$ gluons	photon $W^{\pm}$,Z^0 bosons	G graviton	X,Y ? GUT bosons?
charge	colour	weak isospin weak hypercharge	mass	?

$$\boxed{Q = T_3 + \frac{1}{2} Y_W}$$

Yukawa matrices

$$H \psi_L^i Y_{ij} \psi_R^j$$

$$Y_{ij} \rightarrow Y_{ij}^U, Y_{ij}^D, Y_{ij}^E, Y_{ij}^N$$

$$e.g. \quad Y_{ij}^E (\bar{\nu}_e \quad \bar{e})_L^i \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R^j$$

helicity	Generations 1.	2.	3.	Quantum Numbers Q	T_3	Y_W
ψ_L	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 −1	1/2 −1/2	−1 −1
	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	2/3 −1/3	1/2 −1/2	1/3 1/3
ψ_R	e_R	μ_R	τ_R	-1	0	-2
	u_R d_R	c_R s_R	t_R b_R	2/3 −1/3	0 0	4/3 −2/3

Quark mixing matrix V_{CKM}

$$V^{U_L} Y_{LR}^{U} V^{U_R \dagger} v = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad V^{D_L} Y_{LR}^{D} V^{D_R \dagger} v = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Defined as $\quad V_{CKM} = V^{U_L} V^{D_L \dagger} \quad$ 5 phases removed

Lepton mixing matrix U_{PMNS}

Light neutrino Majorana mass matrix

$$V^{E_L} Y_{LR}^{E} V^{E_R \dagger} v = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad V^{\nu_L} m_{LL}^{\nu} V^{\nu_L T} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Defined as $\quad U_{PMNS} = V^{E_L} V^{\nu_L \dagger} \quad$ 3 phases removed

Recall the origin of the electron mass in the SM are the Yukawa couplings:

$$\begin{pmatrix} <H^+> \\ <H^0> \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$y^e \begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R \longrightarrow y^e \begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} 0 \\ v \end{pmatrix} e_R = \underbrace{y^e v}_{m_e} \bar{e}_L e_R$$

Yukawa coupling y_e must be small since $<H^0>=v=175$ GeV

$$m_e = y_e \langle H^0 \rangle \approx 0.5\, MeV \Leftrightarrow y_e \approx 3.10^{-6}$$

Unsatisfactory

Introduce right-handed neutrino ν_{eR} with zero Majorana mass

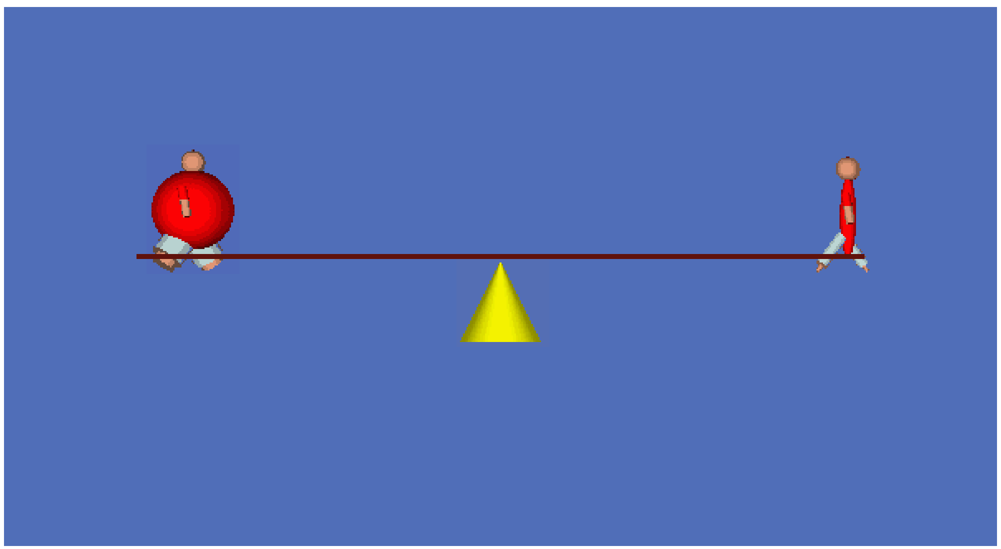
$$y_\nu \bar{L} H^c \nu_{eR} = y_\nu \langle H^0 \rangle \bar{\nu}_{eL} \nu_{eR}$$

then Yukawa coupling generates a Dirac neutrino mass

$$m_{LR}^\nu = y_\nu \langle H^0 \rangle \approx 0.2\, eV \Leftrightarrow y_\nu \approx 10^{-12}$$

Even more unsatisfactory

Part IV. The Origin of Neutrino Mass



Three important features of the SM

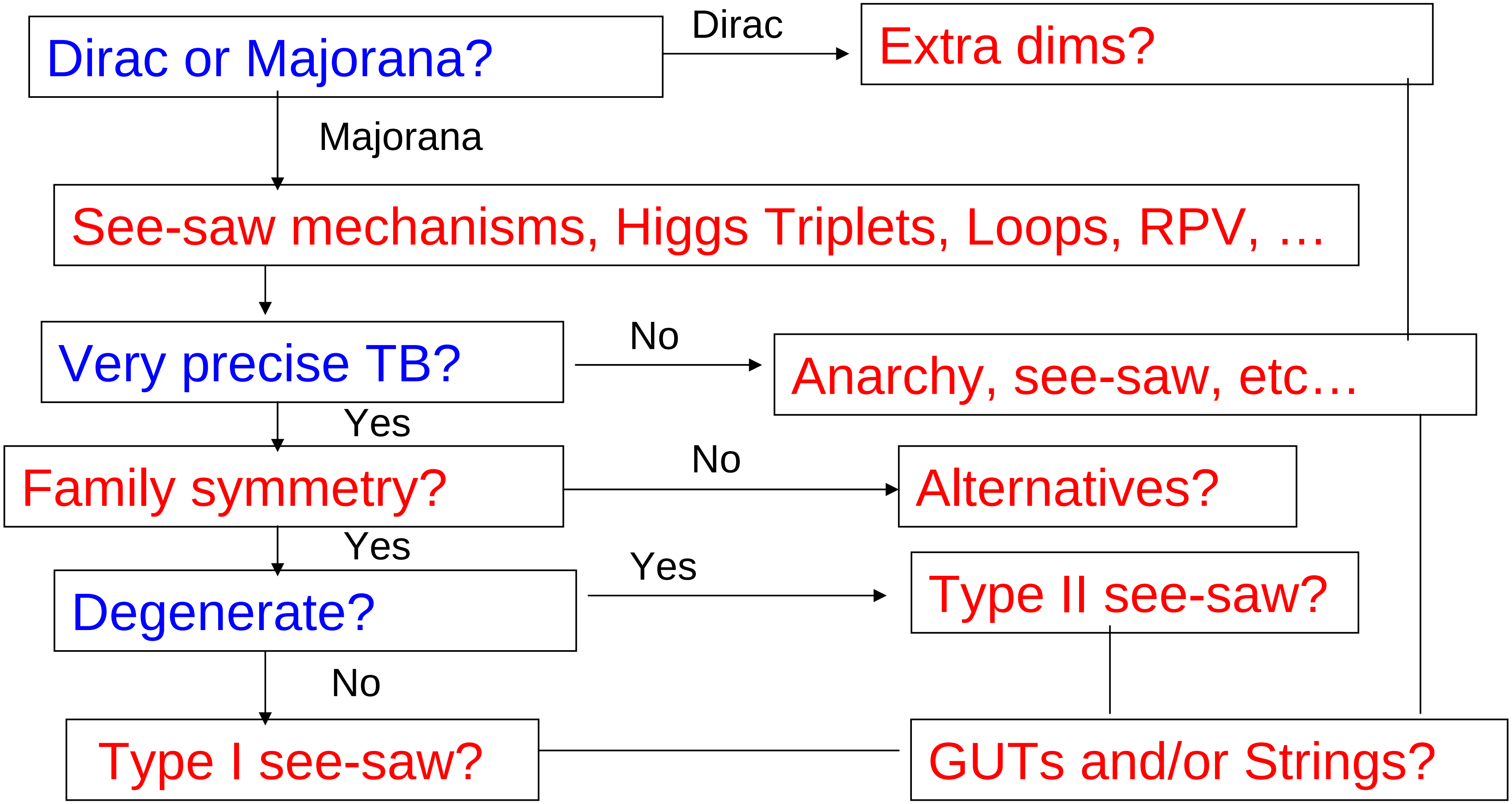
1. There are no right-handed neutrinos ν_R
2. There are only Higgs doublets of $SU(2)_L$
3. There are only renormalizable terms

In the Standard Model neutrinos are massless, with ν_e , ν_μ , ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ

Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L=L_e+L_\mu+L_\tau$

To generate neutrino mass we must relax 1 and/or 2 and/or 3

Neutrino Mass Models Road Map



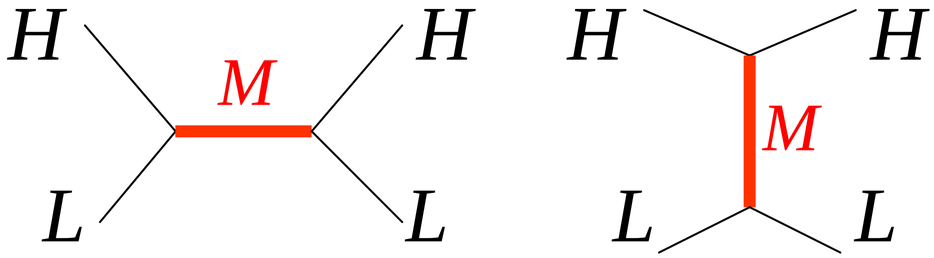
Origin of Majorana Neutrino Mass

Renormalisable ΔL =2 operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable ΔL =2 operator $\frac{\lambda_\nu}{M} LLHH = \frac{\lambda_\nu}{M} \langle H^0 \rangle^2 \, \overline{\nu}_{eL} \nu^c_{eL}$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL} \sim \langle H^0 \rangle^2 / M$ where M is some high energy scale

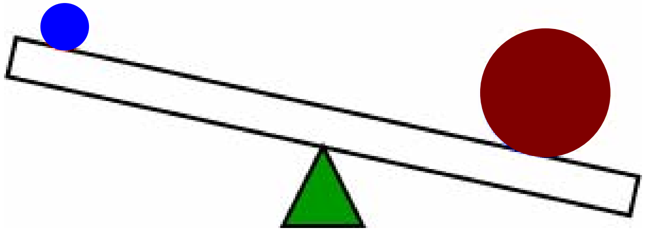
The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)



e.g. see-saw mechanism

The See-Saw Mechanism

Light neutrinos

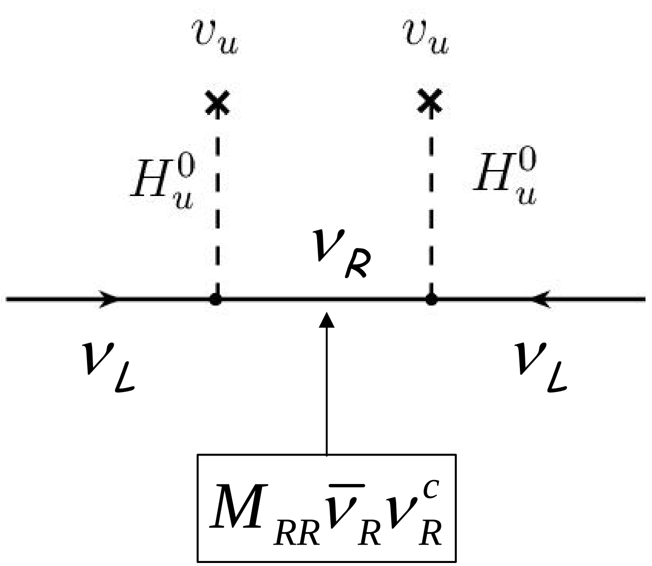


Heavy particles

The see-saw mechanism

Type I see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)

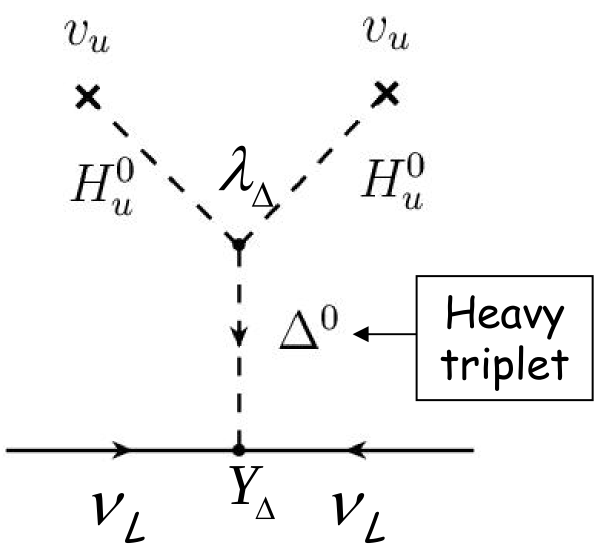


$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type I

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Type II

The See-Saw Matrix

Type II contribution

Dirac matrix

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} m_{LL}^{II} & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

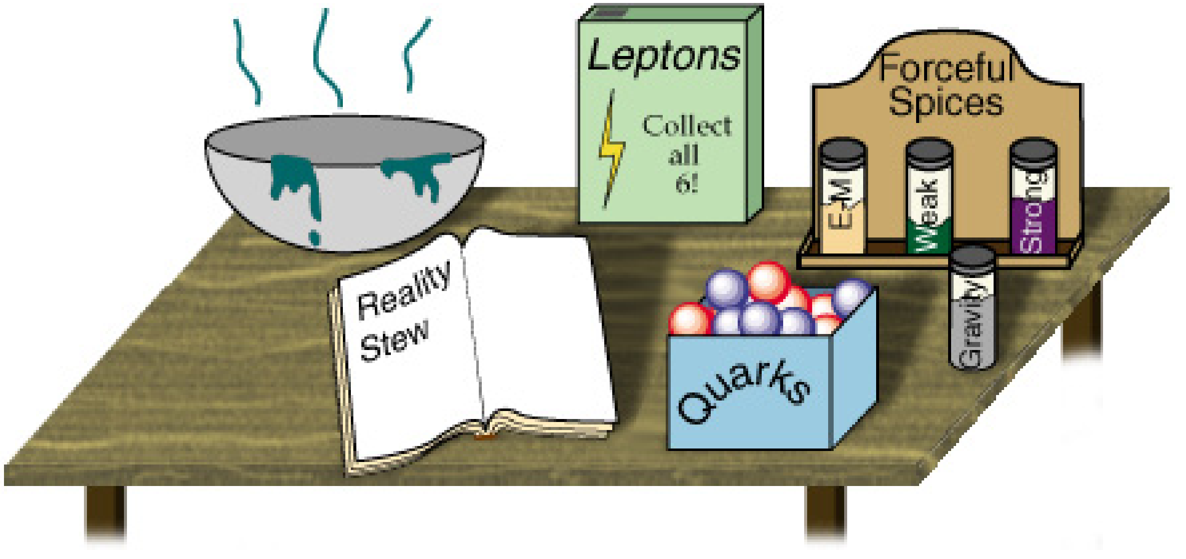
Heavy Majorana matrix

Diagonalise to give effective mass $\rightarrow m_{LL}\overline{\nu}_L \nu_L^c$

Light Majorana matrix $\longrightarrow$ $\boxed{m_{LL}^{\nu} \approx m_{LL}^{II} - m_{LR} M_{RR}^{-1} m_{LR}^T}$

$m_{LL}^{\nu} \sim m_{LR}^2 / M_{RR}$ suggests new high energy mass scale(s) → radiative corrections

Part V.
Flavour Models



Hierarchical Symmetric Textures

Consider the following ansatz for the upper 2x2 block of a hierarchical Y^d

$$\left[Y_{ij}^{d}\right]_{1-2} = \begin{pmatrix} 0 & y_{12} \\ y_{12} & y_{22} \end{pmatrix} \longrightarrow |V_{us}| \approx \left|\sqrt{\frac{m_d}{m_s}}\right| \approx \lambda$$

Gatto et al

successful prediction

This motivates having a symmetric down quark Yukawa matrix with a 1-1 "texture zero" and a hierarchical form

$$Y^d \sim \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} y_b$$

$$|V_{cb}| \approx \left|\frac{(m_{LR}^D)_{23}}{m_b}\right| \approx \lambda^2 \qquad |V_{ub}| \approx \left|\frac{(m_{LR}^D)_{13}}{m_b}\right| \approx \lambda^3$$

successful predictions

$\lambda \approx 0.2$ is the Wolfenstein Parameter

G.Ross et al

Up quarks are more hierarchical than down quarks

This suggests different expansion parameters for up and down

$$Y_{LR}^{D} \sim \begin{pmatrix} 0 & \varepsilon_d^3 & \varepsilon_d^3 \\ \varepsilon_d^3 & \varepsilon_d^2 & \varepsilon_d^2 \\ \varepsilon_d^3 & \varepsilon_d^2 & 1 \end{pmatrix} y_b \qquad \varepsilon_d \sim 0.15$$

$$Y_{LR}^{U} \sim \begin{pmatrix} 0 & \varepsilon_u^3 & \varepsilon_u^3 \\ \varepsilon_u^3 & \varepsilon_u^2 & \varepsilon_u^2 \\ \varepsilon_u^3 & \varepsilon_u^2 & 1 \end{pmatrix} y_t \qquad \varepsilon_u \sim 0.05$$

$$m_d : m_s : m_b = \varepsilon_d^4 : \varepsilon_d^2 : 1$$

$$m_u : m_c : m_t = \varepsilon_u^4 : \varepsilon_u^2 : 1$$

Charged leptons are well described by similar matrix to the downs but with a numerical factor of about 3 in the 2-2 entry (Georgi-Jarlskog)

$$Y_{LR}^{E} \sim \begin{pmatrix} 0 & \varepsilon_d^3 & \varepsilon_d^3 \\ \varepsilon_d^3 & 3\varepsilon_d^2 & \varepsilon_d^2 \\ \varepsilon_d^3 & \varepsilon_d^2 & 1 \end{pmatrix} y_b \qquad \varepsilon_d \sim 0.15$$

$$m_e : m_\mu : m_\tau = \frac{m_d}{3} : 3m_s : m_b \qquad \text{at } M_U$$

$$RGE \to m_e \approx \frac{m_d}{9}, \quad m_\mu \approx m_s, \quad m_\tau \approx \frac{m_b}{3} \qquad \text{at } m_b$$

N.B. Electron mass is governed by an expansion parameter $\varepsilon_d \sim 0.15$ which is not unnaturally small – providing we can generate these textures from a theory

Textures from U(1) Family Symmetry

Consider a U(1) family symmetry spontaneously broken by a flavon vev $\langle \phi \rangle \neq 0$

For D-flatness we use a pair of flavons with opposite U(1) charges $Q(\phi) = -Q(\bar{\phi})$

Example: U(1) charges as Q (ψ_3)=0, Q (ψ_2)=1, Q (ψ_1)=3, Q(H)=0, Q(ϕ)=-1,Q($\bar{\phi}$)=1

Then at tree level the only allowed Yukawa coupling is H ψ_3 ψ_3 $\rightarrow Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The other Yukawa couplings are generated from higher order operators which respect U(1) family symmetry due to flavon ϕ insertions:

$$\overset{\boxed{-1+0+1+0=0}}{\frac{\phi}{M} H\psi_2\psi_3} + \left(\frac{\phi}{M}\right)^2 H\psi_2\psi_2 + \left(\frac{\phi}{M}\right)^3 H\psi_1\psi_3 + \left(\frac{\phi}{M}\right)^4 H\psi_1\psi_2 + \left(\frac{\phi}{M}\right)^6 H\psi_1\psi_1$$

When the flavon gets its VEV it generates small effective Yukawa couplings in terms of the expansion parameter $\varepsilon = \frac{\langle \phi \rangle}{M}$

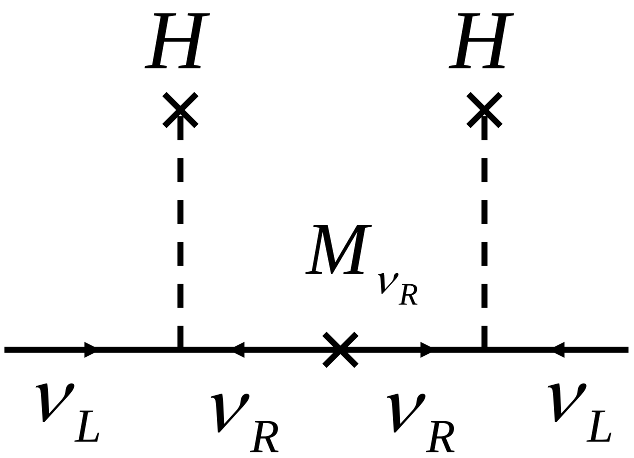
$$\rightarrow Y = \begin{pmatrix} \varepsilon^6 & \varepsilon^4 & \varepsilon^3 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon \\ \varepsilon^3 & \varepsilon & 1 \end{pmatrix}$$

Not quite of desired form
→ non-Abelian

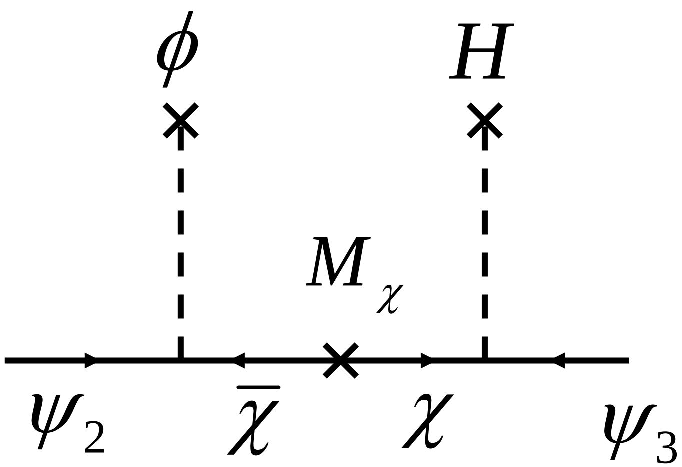
Froggatt-Nielsen Mechanism

What is the origin of the higher order operators?

Froggat and Nielsen took their inspiration from the see-saw mechanism



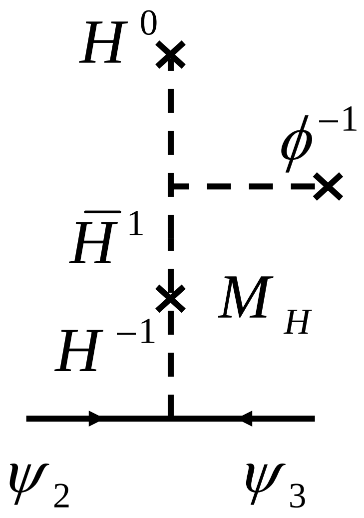
$$\rightarrow \frac{H^2}{M_{\nu_R}}\nu_L\nu_L$$

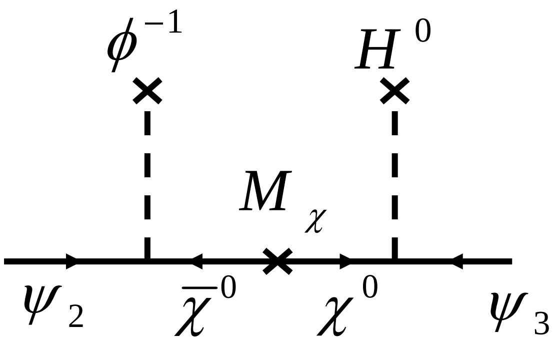


$$\rightarrow \frac{\phi}{M_\chi}H\psi_2\psi_3$$

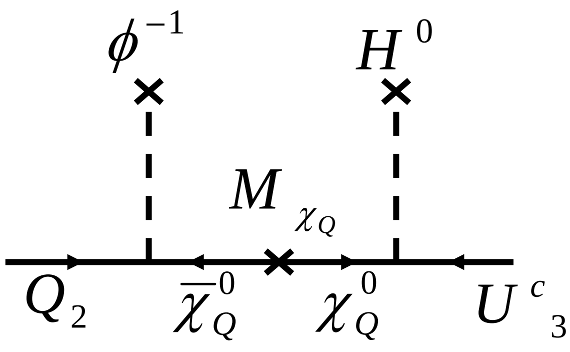
Where χ are heavy fermion messengers
c.f. heavy RH neutrinos

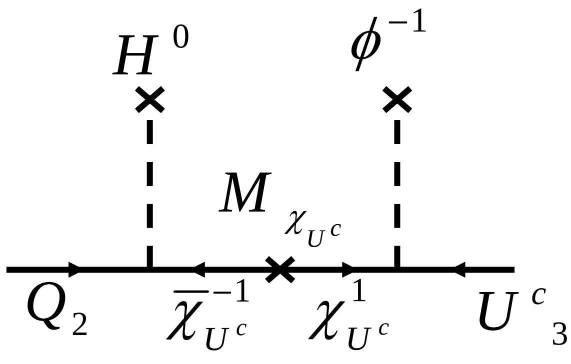
There may be Higgs messengers or fermion messengers





Fermion messengers may be $SU(2)_L$ doublets or singlets





SFK, Ross,Varzielas

Textures from SU(3) Family Symmetry

In SU(3) with ψ_i=3 and H=1 all tree-level Yukawa couplings $H\psi_i\ \psi_j$ are forbidden.

$$Y^{SU(3)}_{tree-level} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In SU(3) with flavons $\phi^i = \overline{3}$ the lowest order Yukawa operators allowed are:

$$\frac{1}{M^2}\phi^i\phi^j H\psi_i\psi_j$$

For example suppose we consider a flavon $\phi_3{}^i$ with VEV $\langle \phi_3{}^i \rangle = (0,0,1)u_3$ then this generates a (3,3) Yukawa coupling

$$\frac{1}{M^2}\phi_3{}^i\phi_3{}^j H\psi_i\psi_j \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{u_3{}^2}{M^2}$$

Note that we label the flavon $\phi_3{}^i$ with a subscript 3 which denotes the direction of its VEV in the i=3 direction.

Next suppose we consider a flavon $\phi_{23}{}^{i}$ with VEV $\langle \phi_{23}{}^{i} \rangle = (0,1,1)u_2$ then this generates (2,3) block Yukawa couplings

$$\frac{1}{M^2}\phi_{23}{}^{i}\phi_{23}{}^{j}H\psi_i\psi_j \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\frac{u_2{}^2}{M^2}$$

To complete the model we use a flavon $\phi_{123}{}^{i}$ with VEV $\langle \phi_{123}{}^{i} \rangle = (1,1,1)u_1$ then this generates Yukawa couplings in the first row and column

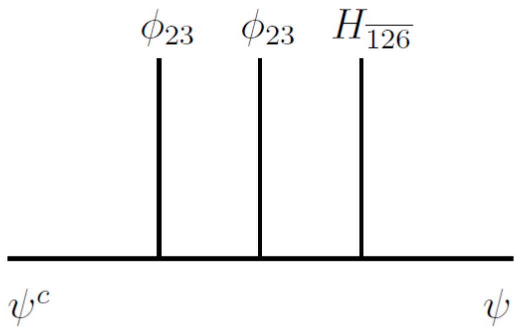
$$\frac{1}{M^2}\phi_{123}{}^{i}\phi_{23}{}^{j}H\psi_i\psi_j \rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & - & - \\ 1 & - & - \end{pmatrix}\frac{u_1u_2}{M^2}$$

Taking $1 \sim \frac{u_3{}^2}{M^2}$ $\varepsilon^2 \sim \frac{u_2{}^2}{M^2}$ $\varepsilon^3 \sim \frac{u_1u_2}{M^2}$ we generate desired structures

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\langle \phi_3 \rangle \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\langle \phi_{23} \rangle \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon^2 & \varepsilon^2 \\ 0 & \varepsilon^2 & 1 \end{pmatrix} \xrightarrow{\langle \phi_{123} \rangle \neq 0} \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

Froggatt-Nielsen diagrams

Right-handed fermion messengers dominate with $M^u \sim 3\ M^d$



$$\frac{1}{M_{u,d}^2}\phi_{23}{}^i\phi_{23}{}^j H\psi_i\psi_j \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_{u,d}^2 & \varepsilon_{u,d}^2 \\ 0 & \varepsilon_{u,d}^2 & \varepsilon_{u,d}^2 \end{pmatrix}$$

$\varepsilon_d = \langle\phi_{23}\rangle/M^d \sim 0.15$ and $\varepsilon_u = \langle\phi_{23}\rangle/M^u \sim 0.05$

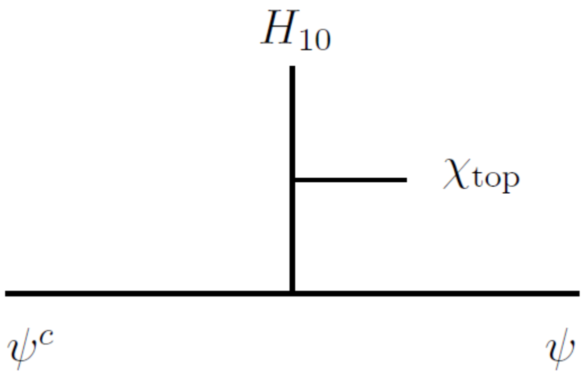
Unsatisfactory features:

1. Suggests $y_b > y_t$!
2. First row/column is only quadratic in the messenger mass.

To improve model we introduce sextet and singlet flavons

Flavon sextets for the third family

Idea is to use flavon sextets χ = 6 and Higgs messengers to generate the third family Yukawa couplings



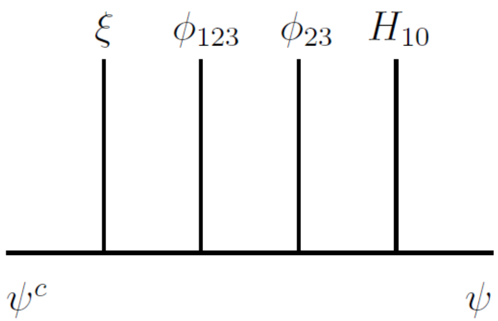
$$\frac{1}{M_H} H\psi_i \chi^{ij} \psi_j \xrightarrow{\langle \chi^{33} \rangle = V} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{V}{M_H}$$

Third family Yukawas $\quad y_t \sim y_b \sim V / M_H$

SFK,Luhn'09

Flavon singlet for first row/column

Flavon singlet $\xi = 1$ leads to first row and column Yukawa couplings involving a cubic messenger mass



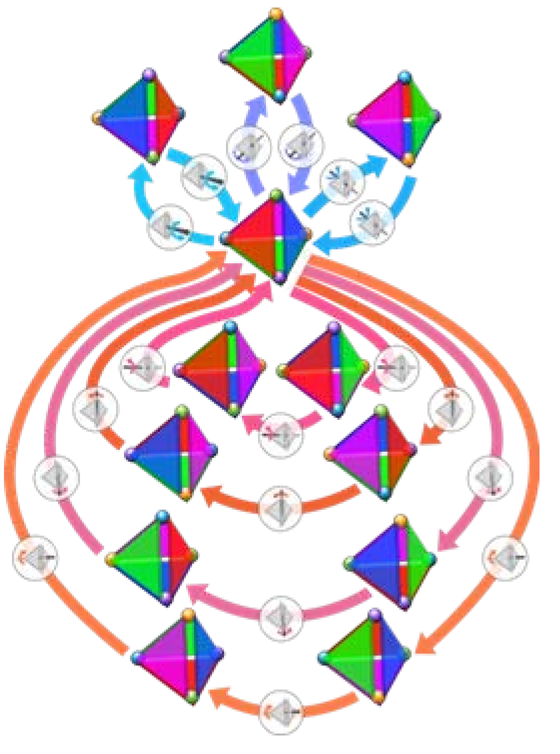
$$\frac{1}{M_{u,d}^3}\xi\phi_{123}{}^i\phi_{23}{}^j H\psi_i\psi_j \to \begin{pmatrix} 0 & \varepsilon_{u,d}^3 & \varepsilon_{u,d}^3 \\ \varepsilon_{u,d}^3 & - & - \\ \varepsilon_{u,d}^3 & - & - \end{pmatrix}$$

So finally we arrive at desired quark textures (with $y_t \sim y_b$)

$$Y_{LR}^D \sim \begin{pmatrix} 0 & \varepsilon_d^3 & \varepsilon_d^3 \\ \varepsilon_d^3 & \varepsilon_d^2 & \varepsilon_d^2 \\ \varepsilon_d^3 & \varepsilon_d^2 & 1 \end{pmatrix} y_b \qquad \varepsilon_d \sim 0.15$$

$$Y_{LR}^U \sim \begin{pmatrix} 0 & \varepsilon_u^3 & \varepsilon_u^3 \\ \varepsilon_u^3 & \varepsilon_u^2 & \varepsilon_u^2 \\ \varepsilon_u^3 & \varepsilon_u^2 & 1 \end{pmatrix} y_t \qquad \varepsilon_u \sim 0.05$$

Part VI. Discrete Family Symmetry



Discrete neutrino flavour symmetry

Consider the TB Neutrino Mass Matrix

$$M^{\nu}_{TB} = U_{TB} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T_{TB}$$

$$M^{\nu}_{TB} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

$$\Phi_1 \Phi_1^T = \frac{1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \quad \Phi_2 \Phi_2^T = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \Phi_3 \Phi_3^T = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

TB Neutrino Mass Matrix is invariant under a discrete $Z_2 \times Z_2$ group generated by S,U

$$M^{\nu}_{TB} = S M^{\nu}_{TB} S^T \qquad M^{\nu}_{TB} = U M^{\nu}_{TB} U^T$$

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

S_4 family symmetry

Lam

In this basis the charged lepton matrix is invariant under a diagonal phase symmetry T

$$M^E = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} = TM^E T^\dagger \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

S,T,U → generate the discrete group S_4

Suggests using a discrete family symmetry S_4 broken by three types of flavons ϕ_S , ϕ_T, ϕ_U which each preserve a particular generator

$$S\langle\phi_S\rangle = +1\langle\phi_S\rangle, \quad U\langle\phi_U\rangle = +1\langle\phi_U\rangle, \quad T\langle\phi_T\rangle = +1\langle\phi_T\rangle$$

$$\mathcal{L}^{Yuk} \sim \psi(\phi_T + \phi_I)\psi^c H \ ,$$

Charged leptons preserve T

$$\mathcal{L}^{Maj} \sim \psi(\phi_S + \phi_U + \phi_I)\psi H H$$

Neutrinos preserve S,U

Indirect models

SFK, Luhn

Alternatively it is possible to realise the neutrino flavour symmetry indirectly as an accidental symmetry

Introduce three triplet flavons ϕ_1 , ϕ_2 , ϕ_3 with VEVs along columns of U_{TB}

$$\langle\phi_1\rangle = v_1\Phi_1, \quad \langle\phi_2\rangle = v_2\Phi_2, \quad \langle\phi_3\rangle = v_3\Phi_3$$

$$\Phi_1 = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

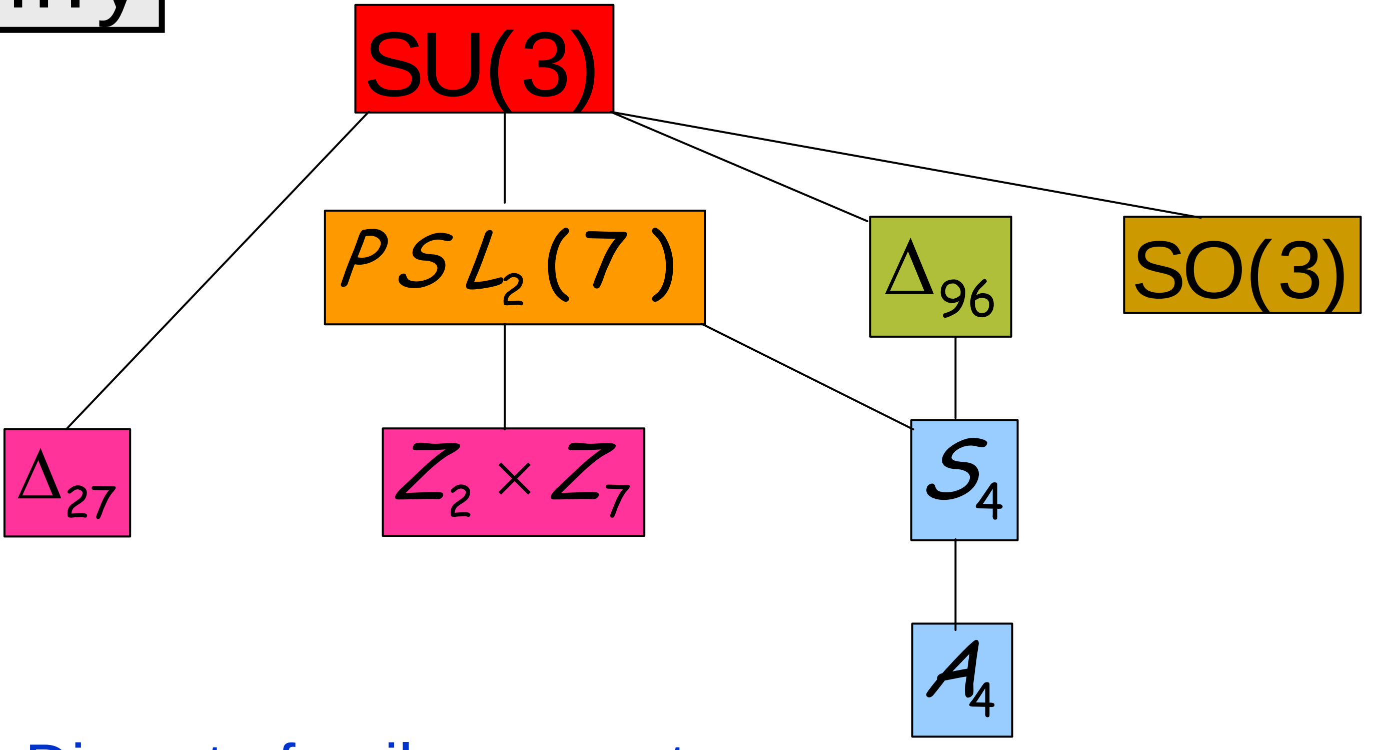
These flavons break S,U

The following Majorana Lagrangian preserves S,U accidentally

$$\mathcal{L}^{Maj} \sim \psi(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\psi HH$$

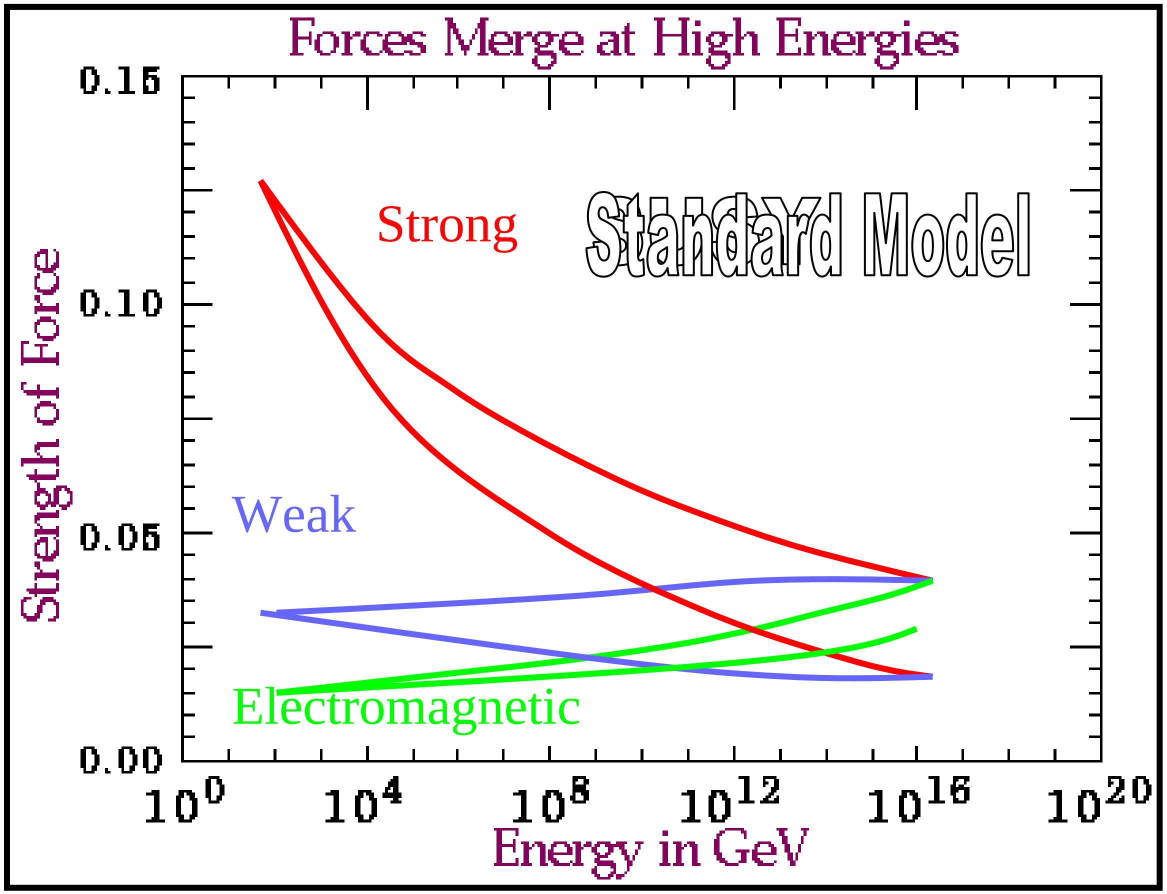
$$\longrightarrow M^{\nu}_{TB} = m_1\Phi_1\Phi_1^T + m_2\Phi_2\Phi_2^T + m_3\Phi_3\Phi_3^T$$

G_{Family}



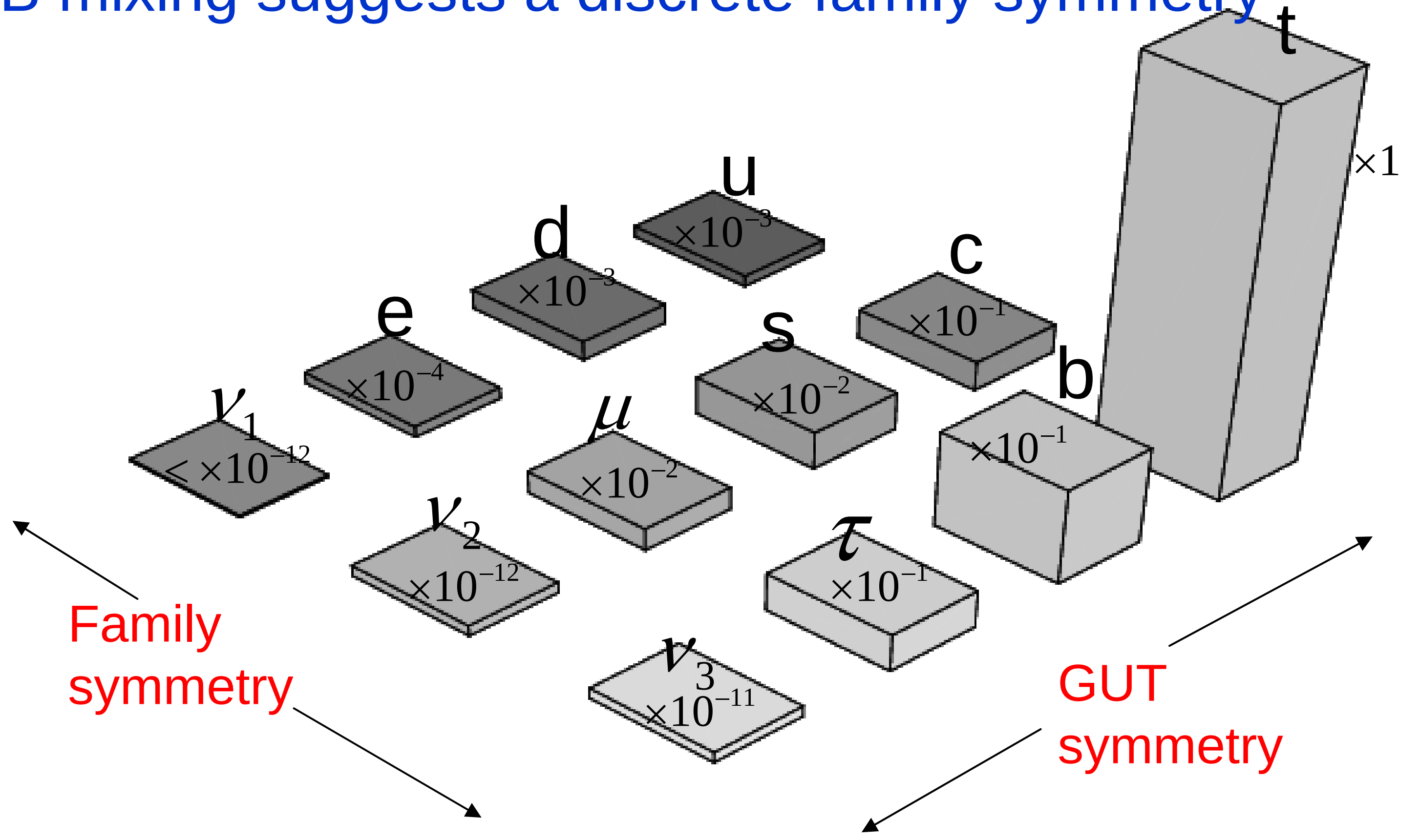
Discrete family symmetry preferred by:
- Vacuum alignment
- String theory

Part VII. GUT Models

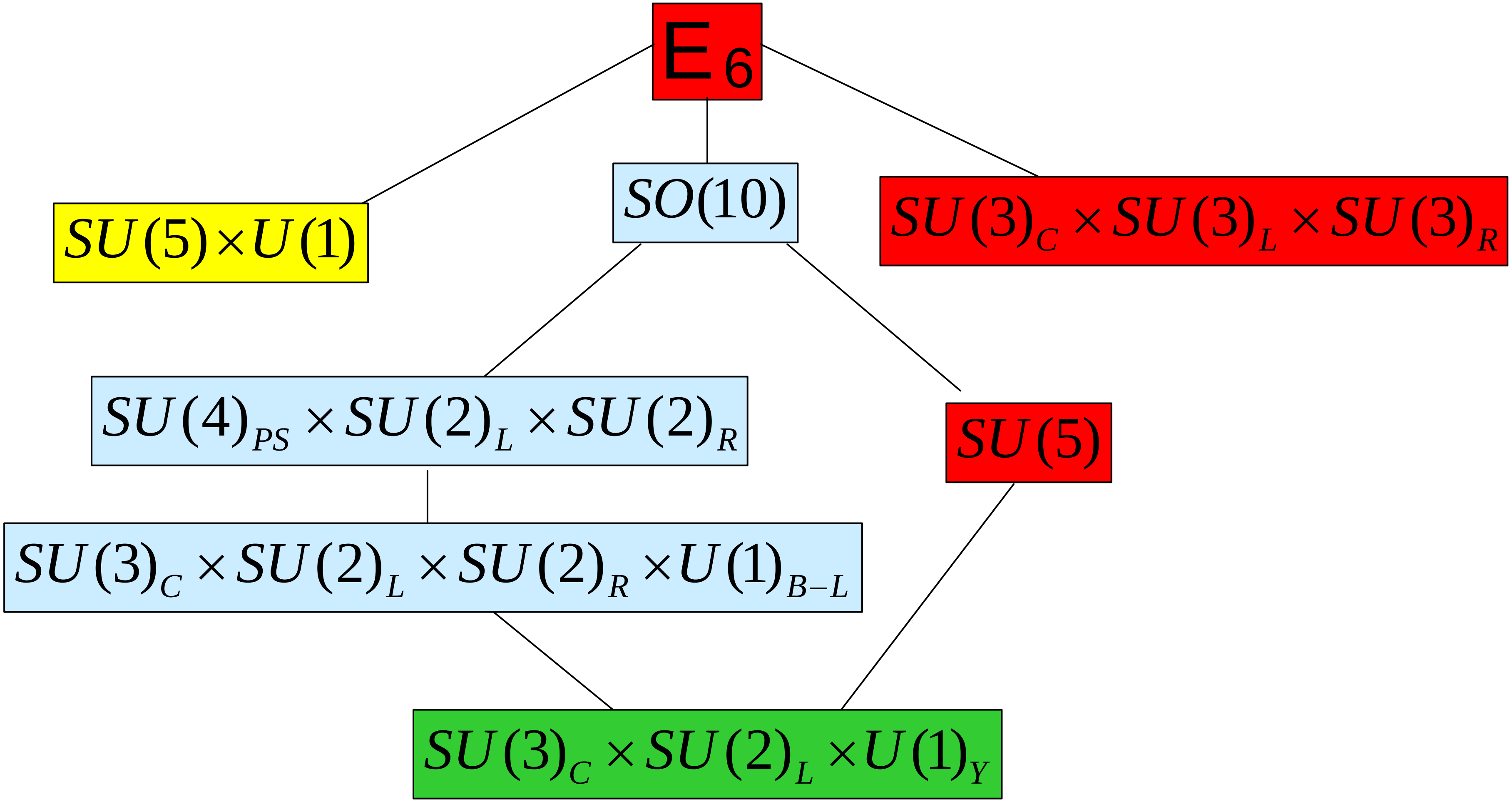


GUTs and Family Symmetry

- Gauge coupling unification suggests GUTs
- TB mixing suggests a discrete family symmetry



G_{GUT}



E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

GUT relations and sum rule

$Y_u \xleftrightarrow{V_{CKM}} Y_d$

Class of models: $\theta_{12}^d \gg \theta_{12}^u$

$\theta_{12}^d \approx \theta_C$

Georgi-Jarlskog

GUT relation

See-saw ⇒

$m_\nu = v_{EW}^2 Y_\nu M_R^{-1} Y_\nu^T$

$m_\nu \xleftrightarrow{V_{MNS}} Y_e$

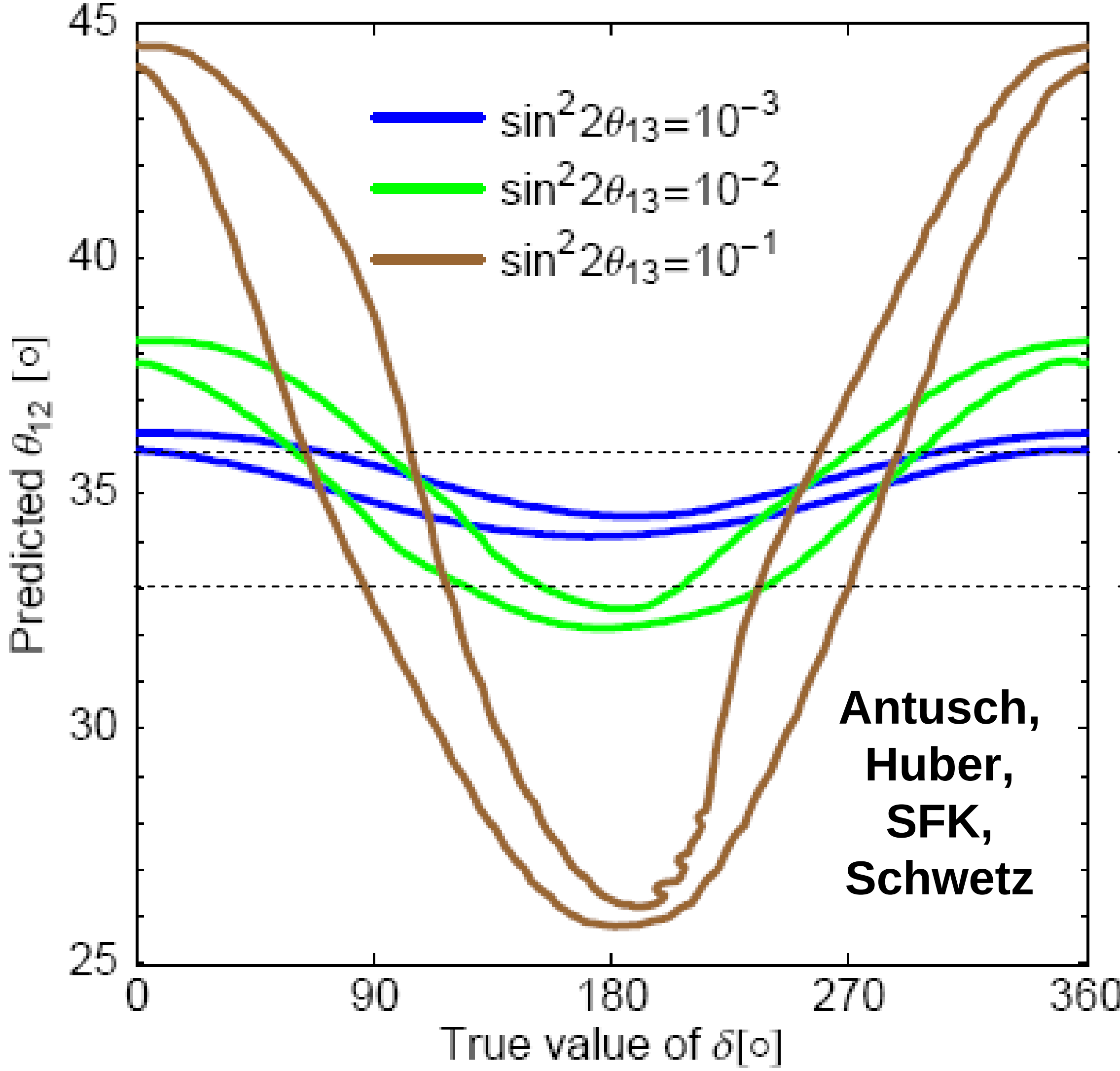
$$\theta_{12}^e \approx \frac{\theta_{12}^d}{3} \approx \frac{\theta_C}{3}$$

TB + charged lepton corrections + GUTs leads to predictions: $\theta_{13} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^\circ,$ $\theta_{12} \approx 35.3^o + \theta_{13}\cos\delta$ Sum Rule

Bjorken; Ferrandis, Pakvasa; SFK

SFK, Antusch, Masina, Malinsky, Boudjemaa

Testing the sum rule $\theta_{12} \approx 35.3^o + \theta_{13}\cos\delta$



Bands show 3σ error for optimized neutrino factory determination of $\theta_{13}\cos\delta$

$\theta_{12}=34.5^o \pm 1.4^o$

(current value)

Conclusions

- **Neutrino mass and mixing is first new physics BSM and adds impetus to solving the flavour problem**

- **Many possible origins of neutrino mass, but focus on ideas which may lead to a theory of flavour: see-saw mechanism and family symmetry broken by flavons**

- **If TB mixing is accurately realised this may imply discrete family symmetry**

- **GUTs $\times$ discrete family symmetry with see-saw is very attractive framework for TB mixing**

SFK,Luhn'09

$PSL_2(7)$ x SO(10) Model

Attractive features of $PSL_2(7)$:

- The smallest simple finite group containing complex triplets and sextets

- Contains S_4 as subgroup (i.e. contains the generators S,T,U)

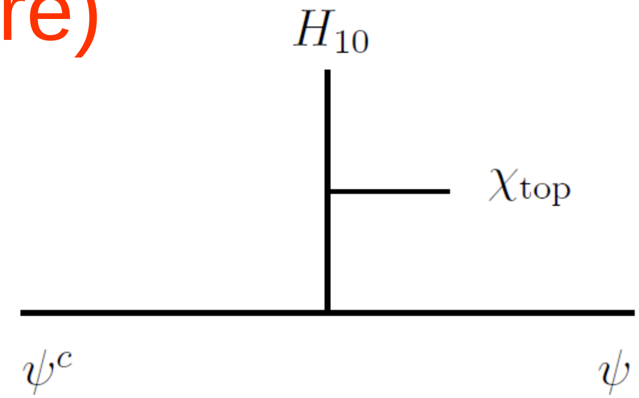
- Contains sextet reps → allows flavon sextets χ = 6 used both for third family Yukawas and for TB mixing where $\langle\chi_{TB}\rangle$ preserve S,U

- Flavon triplets ϕ_{23}, ϕ_{123} and flavon singlet ξ as before

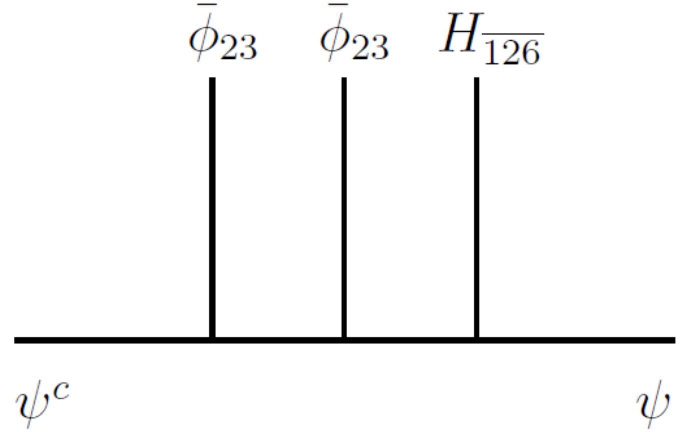
field	ψ	H_{10}	$H_{\overline{126}}$	$\Delta_{\overline{126}}$	χ_{top}	$\chi_{TB}^{[p]}$	$\bar{\phi}_{23}$	$\bar{\phi}_{123}$	ξ
$SO(10)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{126}}$	$\overline{\mathbf{126}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$PSL_2(7)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}$	$\mathbf{6}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$
$U(1)$	0	1	4	2	-1	-2	-2	4	-3

Yukawa Sector (as before)

$$\frac{1}{M_H}\psi_i\hat{\chi}^{ij}_{\rm top}\psi^c_j H_{10}$$

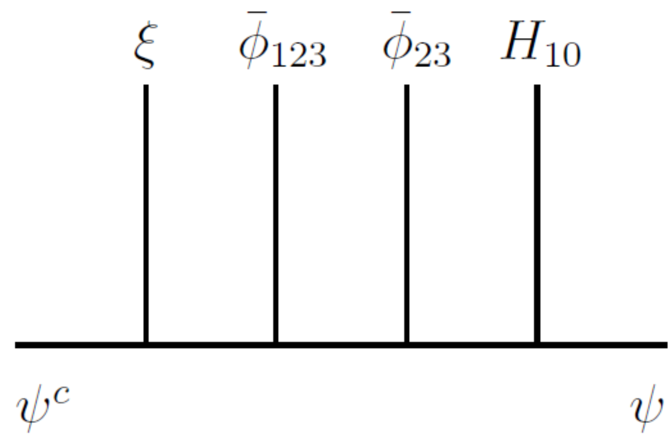


$$\frac{1}{M^2}\psi_i\bar{\phi}^i_{23}\bar{\phi}^j_{23}\psi^c_j H_{\overline{126}}$$



$$\frac{1}{M^3}\psi_i(\bar{\phi}^i_{23}\bar{\phi}^j_{123}$$

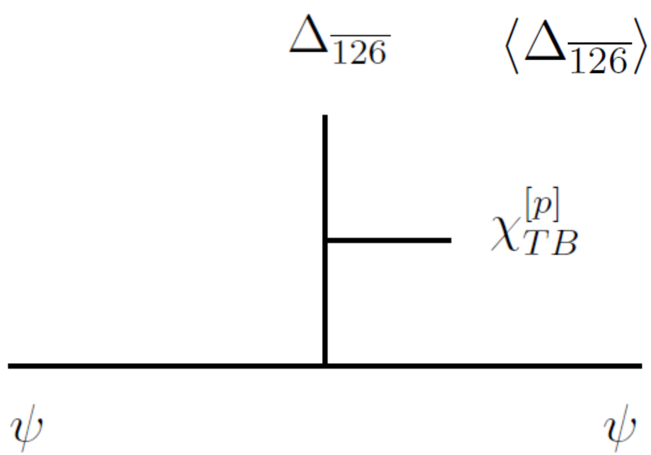
$$+\ \bar{\phi}^j_{23}\bar{\phi}^i_{123})\psi^c_j\,\xi H_{10}$$



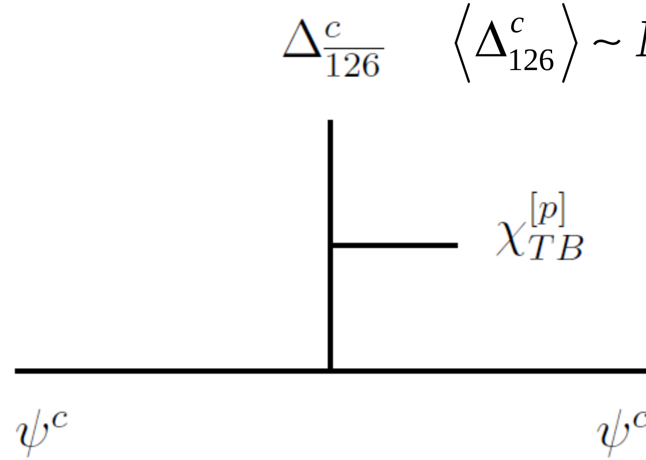
Majorana Sector (new)

$$\mathcal{L}_{\text{Maj}} \sim \frac{1}{M}\sum_{p=0}^{2}\left(\psi_i \hat{\chi}_{TB}^{[p]\,ij}\psi_j \Delta_{\overline{126}} + \psi_i^c \hat{\chi}_{TB}^{[p]\,ij}\psi_j^c \Delta_{\overline{126}}^c\right)$$

$$\langle \Delta_{\overline{126}} \rangle \sim \frac{v_u^2}{M}$$



$$\left\langle \Delta_{126}^c \right\rangle \sim M_{GUT}$$



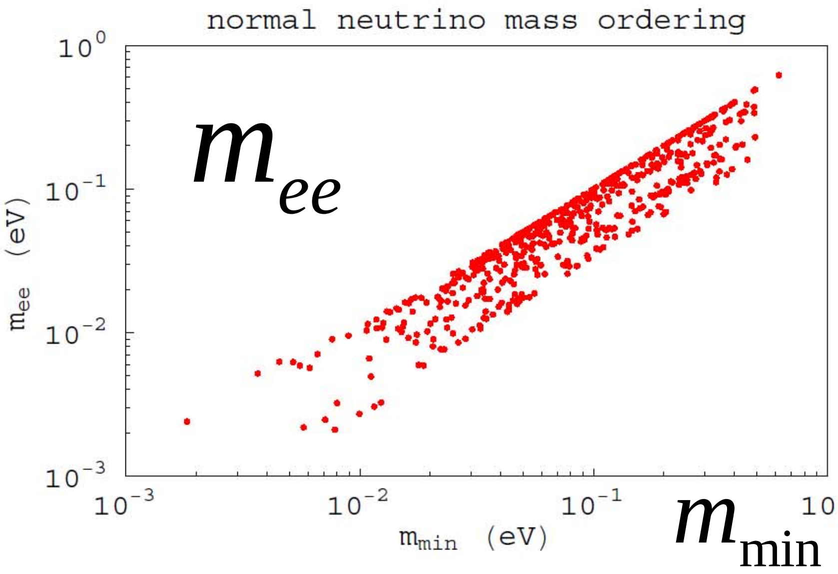
$$m^{\nu}_{\text{type II}} \sim \sum_{p=0}^{2} \langle \hat{\chi}_{TB}^{[p]} \rangle \cdot \frac{v_u^2}{M^2}$$

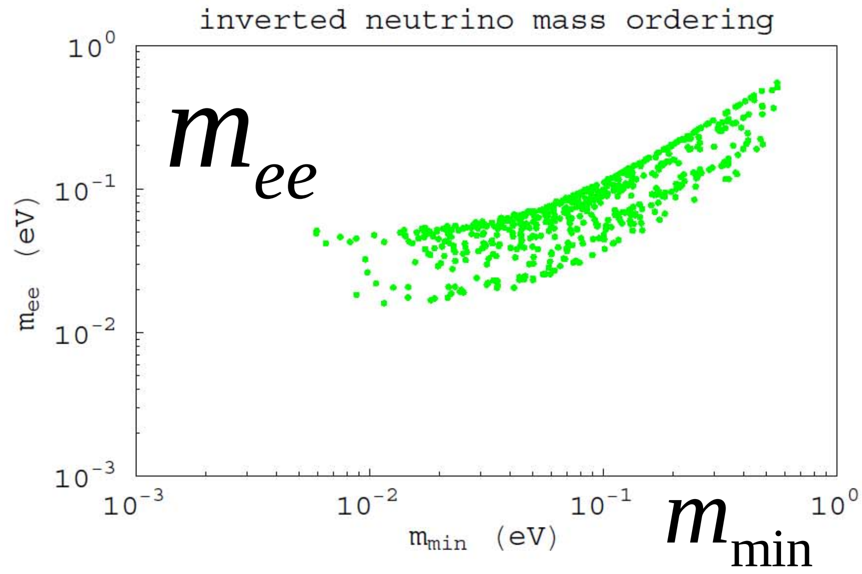
Type II see-saw is dominant

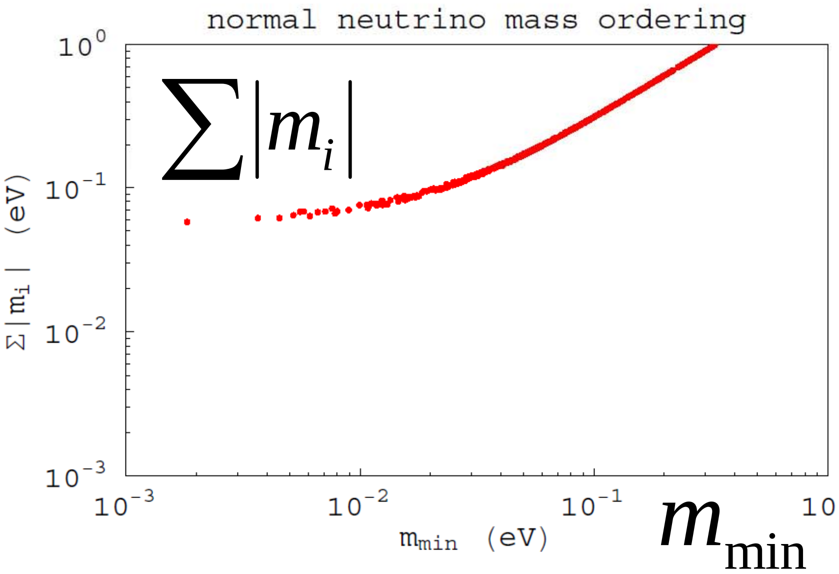
$$M_{RR} \sim \sum_{p=0}^{2} \frac{\langle \hat{\chi}_{TB}^{[p]} \rangle \langle \Delta_{\overline{126}}^c \rangle}{M^2}$$

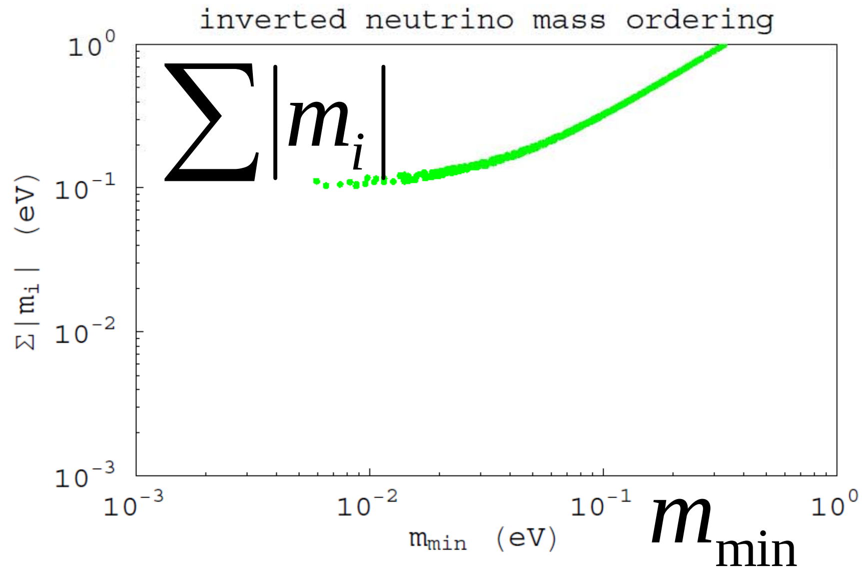
Very heavy → type I see-saw is subdominant

Type II neutrino phenomenology









Indirect type I see-saw models

SFK

Consider $$\mathcal{L}_N^{Yuk} \sim L_i(\phi_1^i N_1^c + \phi_2^i N_2^c + \phi_3^i N_3^c)H \ ,$$

$$\mathcal{L}_N^{Maj} \sim M_1 N_1^c N_1^c + M_2 N_2^c N_2^c + M_3 N_3^c N_3^c$$

$$\longrightarrow \quad \mathcal{L}^{Maj} \sim L\left(\frac{\phi_1\phi_1^T}{M_1} + \frac{\phi_2\phi_2^T}{M_2} + \frac{\phi_3\phi_3^T}{M_3}\right) LHH$$

$$\longrightarrow \quad M_{TB}^{\nu} = m_1\Phi_1\Phi_1^T + m_2\Phi_2\Phi_2^T + m_3\Phi_3\Phi_3^T$$

$$m_1 = a_1^2/M_1,\ m_2 = a_2^2/M_2,\ m_3 = a_3^2/M_3$$

Example of Form dominance → TB mixing independently of masses

Constrained sequential dominance corresponds to $m_1 \to 0$