Relic neutrinos at accelerator experiments

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Contents

- What is the $C_{\nu B}$?
- PTOLEMY(-on-a-beam)
- Resonant neutrino capture
- Experimental challenges
Contents

● What is the $C\nu B$?

● PTOLEMY(-on-a-beam)

● Resonant neutrino capture

● Experimental challenges
What is the CνB?
What is the CvB?
What is the CvB?

- Electrons and photons are kept in equilibrium through EM interactions:

\[ e + \gamma \rightarrow e + \gamma \quad \text{and} \quad e^+ + e^- \leftrightarrow \gamma \]
What is the CνB?

- Electrons and photons are kept in equilibrium through EM interactions:

\[ e + \gamma \rightarrow e + \gamma \quad \text{and} \quad e^+ + e^- \leftrightarrow \gamma \]

- Neutrinos and electrons are kept in equilibrium through weak interactions:

\[ \nu + e \rightarrow \nu + e \quad \text{and} \quad \nu + \bar{\nu} \leftrightarrow e^+ + e^- \]
What is the CνB?
What is the CvB?

Neutrino Decoupling

$\left( e, \gamma, \nu \right)$

$T_{SM}$

1s
What is the CvB?

- Freeze-out happens when:

\[ \Gamma_\nu = H \]
What is the CvB?

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  \[ \Gamma_{\nu} = H \]

- Neutrino interaction rate is \( \Gamma_{\nu} \propto \sigma_{\nu} n_{\nu} \)
What is the CvB?

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\[ \sigma_\nu \propto G_F^2 T_{SM}^2 \]
What is the CvB?

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\[ \Gamma_\nu = H \]

- Neutrino interaction rate is \( \Gamma_\nu \propto \sigma_\nu n_\nu \)

\[ \sigma_\nu \propto G_F^2 T_{SM}^2 \]

\[ n_\nu \propto \int \frac{d^3 p_\nu}{e^T_{\nu} + 1} \propto T_{SM}^3 \]
What is the CvB?

- Freeze-out happens when:
  \[ \Gamma_\nu = H \]

- Neutrino interaction rate is
  \[ \Gamma_\nu \propto G_F^2 T_{SM}^5 \]
What is the CvB?

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- Hubble parameter scales as

\[ H^2 \propto G_N \rho \]
What is the CνB?

- Freeze-out happens when:

\[ \Gamma_\nu = H \]

- Neutrino interaction rate is \( \Gamma_\nu \propto G_F^2 T_{SM}^5 \)

- Hubble parameter scales as \( H^2 \propto G_N \rho \)

\[
\rho \propto \sum_{e,\gamma,\nu} \int \frac{p_i \, d^3 p_i}{e^{\frac{p_i}{T_i}} \pm 1} \propto T_{SM}^A
\]
What is the CvB?

- Freeze-out happens when:
  \[ \Gamma_\nu = H \]

- Neutrino interaction rate is \( \Gamma_\nu \propto G_F^2 T_{SM}^5 \)

- Hubble parameter scales as \( H^2 \propto G_N T_{SM}^4 \)
What is the CνB?

- Freeze-out happens when:

\[ G_F^2 T_{\text{dec}}^5 \approx \sqrt{G_N} T_{\text{dec}}^2 \]
What is the CvB?

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\[ \implies T_{\text{dec}} \sim \left( \frac{\sqrt{G_N}}{G_F^2} \right)^{\frac{1}{3}} \sim 1 \text{ MeV} \]
What is the CvB?

- Freeze-out happens when:

\[ G_F^2 T_{\text{dec}}^5 \simeq \sqrt{G_N} T_{\text{dec}}^2 \]

\[ \implies T_{\text{dec}} \sim \left( \frac{\sqrt{G_N}}{G_F^2} \right)^{\frac{1}{3}} \sim 1 \text{ MeV} \]

\[ t_{\text{dec}} = \frac{1}{2H} \sim 1 \text{ s} \]
What is the CvB?

Neutrino Decoupling

$T_{SM}$

$(e, \gamma, \nu)$

1s
What is the CνB?
What is the CvB?

Neutrino Decoupling

$(e, \gamma, \nu)$

$T_{SM}$

$\nu$

$T_{SM}$

$e^+ e^- \rightarrow \gamma$

Inflation, EW, QCD

$1_s$
What is the CvB?

- $E_{\gamma} \geq 1.02\text{ MeV}:

$$e^+ + e^- \leftrightarrow \gamma$$
What is the CvB?

- $E_\gamma \geq 1.02$ MeV:
  
  \[ e^+ + e^- \leftrightarrow \gamma \]

- $E_\gamma < 1.02$ MeV:
  
  \[ e^+ + e^- \rightarrow \gamma \]
What is the $C\nu B$?

- $E_\gamma \geq 1.02$ MeV:
  \[ e^+ + e^- \leftrightarrow \gamma \]

- $E_\gamma < 1.02$ MeV:
  \[ e^+ + e^- \rightarrow \gamma \]

- This process changes the photon temperature!
What is the CνB?

- In a comoving volume, total entropy is conserved:

\[
\frac{dS}{dt} = 0
\]
What is the CvB?

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• Entropy before and after annihilation needs to be the same:
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- In a comoving volume, total entropy is conserved:
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- Entropy before and after annihilation needs to be the same:
  \[
  g_s^*(T_{SM}) T_{SM}^3 = g_s^*(T_\gamma) T_\gamma^3
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- In a comoving volume, total entropy is conserved:
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- Entropy before and after annihilation needs to be the same:
  \[
  g_s^*(T_{SM}) T_{SM}^3 = g_s^*(T_\gamma) T_\gamma^3
  \]

- In general:
  \[
  g_s^*(T) = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i
  \]
What is the CvB?

- Before annihilation:

\[ g_s^* (T_{SM}) = 2 \gamma + \frac{7}{8} \left( 2 \times 2 \right) \]
What is the C\nuB?

- Before annihilation:

\[
g_s^*(T_{SM}) = \frac{11}{2}
\]
What is the CνB?

- Before annihilation:
  \[ g_s^*(T_{SM}) = \frac{11}{2} \]

- After annihilation:
  \[ g_s^*(T_\gamma) = 2 \]
What is the CvB?

- Photon temperature satisfies:

\[ \frac{11}{2} T_{SM}^3 = 2 T_{\gamma}^3 \]
What is the CvB?

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- Recalling that the neutrinos are still at \( T_{SM} \):
What is the CvB?

- Photon temperature satisfies:
  \[ \frac{11}{2} T_{SM}^{3} = 2 T_{\gamma}^{3} \]

- Recalling that the neutrinos are still at \( T_{SM} \):

\[
T_{\nu} = \left( \frac{4}{11} \right)^{\frac{1}{3}} T_{\gamma}
\]
What is the CvB?

Neutrino Decoupling

$T_{SM}$

$(e, \gamma, \nu)$

$e^+e^- \rightarrow \gamma$

$T_{SM}$

$(e, \gamma)$

$1_s$

$t$
What is the CvB?
What is the CvB?
What is the CνB?

Neutrino Decoupling

$(e, \gamma, \nu)$

$T_{SM}$

$e^+ e^- \rightarrow \gamma$

$T_{SM}$

$T_{\gamma}$

1 s 1 m $10^5$ y

Inflation, EW, QCD
What is the CvB?
The CMB today

- Redshifted to temperature:

\[ T_{\nu,0} = \left( \frac{4}{11} \right)^{\frac{1}{3}} T_{\text{CMB}} \]
The CνB today

- Redshifted to temperature:

\[ T_{\nu,0} = 0.168 \text{ meV} \]
The CνB today

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- At least two neutrino states are non-relativistic!
The CνB today

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- At least two neutrinos states are non-relativistic!

- Exist today as freely propagating mass eigenstates
The CvB today

- Expect these to follow a Fermi-Dirac distribution with:

\[ n_\nu = 56 \text{ cm}^{-3} \]
The CvB today

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The CvB today

- Expect these to follow a Fermi-Dirac distribution with:
  \[ n_\nu = 56 \text{ cm}^{-3} \]
- These should all be left helicity states
- ...but neutrinos have mass!
- This may lead to CDM profile, overdensities, helicity mixing etc.
Why detect the CνB?

- The CMB is the furthest we can currently look back through time
Why detect the CνB?

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- A rare source of non-relativistic neutrinos!
Why detect the CνB?

- The CMB is the furthest we can currently look back through time
- A rare source of non-relativistic neutrinos!
- Perhaps they’re not there at all
Why detect the $C\nu B$?
So...why haven’t we detected them yet?

- Neutrinos are notoriously hard to look for...

\[ \sigma_\nu \sim G_F^2 E_\nu^2 \sim 5 \cdot 10^{-50} \left( \frac{E_\nu}{1 \text{ keV}} \right)^2 \text{ cm}^2 \]
So...why haven’t we detected them yet?

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- Compare this to a typical EM process:

\[ \sigma_{e\mu} = \frac{4\pi \alpha^2}{3s} \sim 10^{-25} \left( \frac{1 \text{ MeV}}{E_e} \right)^2 \text{ cm}^2 \]
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- Existing neutrino detection experiments have thresholds:

\[ \bar{\nu}_e + p + (1.8 \text{ MeV}) \rightarrow e^- + n \]
But...there is hope!
How might we detect the CνB?

- **Threshold:**
  - Remove it completely!
  - Find some way to bridge it
How might we detect the $C\nu B$?

- **Threshold:**
  - Remove it completely!
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- **Event rate:**
  - Use a huge number of targets
  - Increase the cross section
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The PTOLEMY experiment

- Proposed by Weinberg in 1962 [1]:

\[ \nu_e + ^3\text{H} \rightarrow e^- + ^3\text{He}^+ \]

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  \[ \nu_e + ^3\text{H} \rightarrow e^- + ^3\text{He}^+ \]
- This process has no threshold
- Tritium already well understood from neutrino mass experiments

The PTOLEMY experiment

![Graph showing relative number of events vs electron kinetic energy]
The PTOLEMY experiment

- Neutrino capture cross section [2]:

\[
\langle \sigma \beta_\nu \rangle \propto G_F^2 E_e p_e \sim 10^{-45} \text{ cm}^2
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\[ R = N_T n_\nu \langle \sigma \nu_\nu \rangle \sim 4 \text{ y}^{-1} \]

The PTOLEMY experiment

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- This event rate is doubled for Majorana neutrinos

What’s the catch?

- Extreme sensitivity required to detect signal:

\[ \Delta \leq 2m_\nu \]
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- Obtaining and storing 100g of tritium
What’s the catch?

- Extreme sensitivity required to detect signal:
  \[ \Delta \leq 2m_\nu \]

- Obtaining and storing 100g of tritium

- cf. KATRIN, uses \( \sim 300\mu g \) of tritium [3]

Can we do better?

- Recall the cross section:

\[ \langle \sigma \beta_\nu \rangle \propto G_F^2 E_e p_e \]
Can we do better?

- Recall the cross section:

\[ \langle \sigma \beta_\nu \rangle \propto G_F^2 E_e p_e \]

- This scales quadratically with energy!

\[ E_e = m_e + |Q_H| + E_\nu \]

\[ p_e = \sqrt{(E_\nu + |Q_H|)(E_\nu + 2m_e + |Q_H|)} \]
Can we do better?

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- We can increase our neutrino energy by using a beam
Setup

- Accelerate (tritium) ions on a beam
Setup

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- Treat neutrinos as at rest in lab frame:

\[ E_\nu \simeq m_\nu \]
Setup

- Accelerate (tritium) ions on a beam

- Treat neutrinos as at rest in lab frame:
  \[ E_\nu \approx m_\nu \]

- Relevant beam rest frame quantities:
  \[ \tilde{E}_\nu \approx \frac{m_\nu}{M} E \quad \tilde{\phi} = \gamma \phi \quad \tilde{t} = \frac{t}{\gamma} \quad \tilde{R} = \gamma R \]
Setup

- In the beam rest frame:

\[ \langle \sigma \tilde{\beta}_\nu \rangle \propto G_F^2 \tilde{E}_e \tilde{p}_e \]

\[ \tilde{E}_e = m_e + |Q_H| + \tilde{E}_\nu \]

\[ \tilde{p}_e = \sqrt{(\tilde{E}_\nu + |Q_H|)(\tilde{E}_\nu + 2m_e + |Q_H|)} \]
In the beam rest frame:

\[
\langle \sigma \beta_\nu \rangle \propto G_F^2 \tilde{E}_e \tilde{p}_e
\]

\[
\tilde{E}_e = m_e + |Q_H| + \tilde{E}_\nu
\]

\[
\tilde{p}_e = \sqrt{(\tilde{E}_\nu + |Q_H|)(\tilde{E}_\nu + 2m_e + |Q_H|)}
\]

Quadratic enhancement begins when:

\[
\tilde{E}_\nu > 2m_e \implies E \gtrsim 3 \text{ PeV}
\]
Difficulties

- Huge energy requirements
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- Still need a large amount of tritium
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- Still need a large amount of tritium
- Almost no way to recover a signal
Difficulties

- Huge energy requirements
- Still need a large amount of tritium
- Almost no way to recover a signal
- ...but, large energy presents an opportunity!
Large energy allows us to use the inverse process:

\[ ^3\text{He}^{++} + \bar{\nu}_e \rightarrow ^3\text{H}^+ + e^+ \]
In “Inverse PTOLEMY-on-a-beam”

- Large energy allows us to use the inverse process:

\[ {^3\text{He}^{++} + \bar{\nu}_e} \rightarrow {^3\text{H}^+ + e^+} \]

- Positron energy given by:

\[ \tilde{E}_e = m_e - Q_{\text{He}} + \tilde{E}_\nu \]

\[ \tilde{p}_e = \sqrt{(\tilde{E}_\nu - Q_{\text{He}})(\tilde{E}_\nu + 2m_e - Q_{\text{He}})} \]
- Large energy allows us to use the inverse process:

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- Positron energy given by:

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\tilde{E}_e = m_e - Q_{\text{He}} + \tilde{E}_\nu
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\[
\tilde{p}_e = \sqrt{ (\tilde{E}_\nu - Q_{\text{He}})(\tilde{E}_\nu + 2m_e - Q_{\text{He}}) }
\]

- This process has a ‘unique’ signal
“Inverse PTOLEMY-on-a-beam”

- Large energy allows us to use the inverse process:
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- Positron energy given by:
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  \tilde{E}_e = m_e - Q_{\text{He}} + \tilde{E}_\nu
  \]
  \[
  \tilde{p}_e = \sqrt{(\tilde{E}_\nu - Q_{\text{He}})(\tilde{E}_\nu + 2m_e - Q_{\text{He}})}
  \]

- This process has a ‘unique’ signal

- Signal is now unstable
“Inverse PTOLEMY-on-a-beam”
Can we do better?

- Not really...
Can we do better?

- Not really...

- Cross section still tiny at huge energies:

\[ \langle \sigma \beta_\nu \rangle \propto G_F^2 E_e p_e \]
Can we do better?

- Not really...

- Cross section still tiny at huge energies:

  \[ \langle \sigma \tilde{\beta}_\nu \rangle \propto G_F^2 E_e p_e \]

- But we have learnt some lessons!
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Resonant neutrino capture

- Tiny cross sections → use a resonance!
Resonant neutrino capture

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- Tunable beam energy naturally invites resonances
Resonant neutrino capture

- Tiny cross sections → use a resonance!
- Tunable beam energy naturally invites resonances
- e.g. Z-resonance: \( \nu + \bar{\nu}_{C\nu_B} \rightarrow Z \rightarrow ? \)
Resonant neutrino capture

- Tiny cross sections → use a resonance!

- Tunable beam energy naturally invites resonances

- e.g. Z-resonance: \( \nu + \bar{\nu}_{C_{\nu B}} \rightarrow Z \rightarrow ? \)

- Vastly larger cross section: \( \sigma \propto \frac{1}{M_Z^2} \propto G_F \)
Resonant neutrino capture

- Resonant electron capture (REC):

\[
\frac{A}{Z}P + e^- (\text{bound}) + \bar{\nu}_e \rightarrow \frac{A}{Z-1}D
\]
Resonant neutrino capture

- Resonant electron capture (REC):
  \[ \frac{A}{Z}P + e^- (\text{bound}) + \bar{\nu}_e \rightarrow \frac{A}{Z-1}D \]

- Resonant bound beta decay (RB\(\beta\)):
  \[ \frac{A}{Z}P + \nu_e \rightarrow \frac{A}{Z+1}D + e^- (\text{bound}) \]
Resonant neutrino capture

- Resonant electron capture (REC):
  \[ \frac{A}{Z} P + e^- (\text{bound}) + \bar{\nu}_e \rightarrow \frac{A}{Z-1} D \]

- Resonant bound beta decay (RB\(\beta\)):
  \[ \frac{A}{Z} P + \nu_e \rightarrow \frac{A}{Z+1} D + e^- (\text{bound}) \]

- Parent ionised down to one s-shell electron (REC) or completely ionised (RB\(\beta\))
Resonant neutrino capture

- Cross section for resonant neutrino capture [4]:

\[ \sigma \propto \frac{1}{\tilde{E}_\nu^2} \left[ \frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \text{Br}(D \rightarrow P) \]

Resonant neutrino capture

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\]

1, \tilde{E}_\nu = Q

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\[
\sigma_{\text{peak}} \propto \frac{1}{Q^2} \text{Br}(D \to P)
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\[ \sigma \propto \frac{1}{\tilde{E}_\nu^2} \left[ \frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \text{Br}(D \to P) \]

\[ \sigma_{\text{peak}} \propto \frac{1}{Q^2} \text{Br}(D \to P) \]

- Peak cross section is independent of \( G_F \)!

Resonant neutrino capture

- Cross section for resonant neutrino capture [4]:

\[ \sigma \propto \frac{1}{\tilde{E}_\nu^2} \left[ \frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \text{Br}(D \to P) \]

\[ \sigma_{\text{peak}} = 2.5 \cdot 10^{-15} \left( \frac{1 \text{ keV}}{Q} \right)^2 \text{Br}(D \to P) \text{ cm}^2 \]

- Peak cross section is independent of \( G_F \)!

Resonant neutrino capture

- Capture rate per target given by:

\[
\frac{R}{N_T} = \int_Q^{\infty} d\tilde{E}_\nu \sigma(\tilde{E}_\nu) \frac{d\phi}{d\tilde{E}_\nu}
\]
Resonant neutrino capture

- Capture rate per target given by:

$$\frac{R}{N_T} = \int d\tilde{E}_\nu \sigma(\tilde{E}_\nu) \frac{d\phi}{d\tilde{E}_\nu}$$

- For narrow resonances:

$$\left[ \frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \rightarrow \pi \Gamma \delta(\tilde{E}_\nu - Q)$$
Resonant neutrino capture

- Capture rate per target given by:

\[
\frac{R}{N_T} = \frac{\pi}{2} \sigma_{\text{peak}} \Gamma \frac{d\phi}{d\tilde{E}_\nu} \bigg|_{\tilde{E}_\nu=Q}
\]
Resonant neutrino capture

- Capture rate per target given by:

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\frac{R}{N_T} = \frac{\pi}{2} \sigma_{\text{peak}} \Gamma \frac{d\phi}{d\tilde{E}_\nu} \bigg|_{\tilde{E}_\nu = Q}
\]

- Assuming Gaussian distribution:

\[
\frac{R}{N_T} \propto \frac{\Gamma}{Q^2} \frac{\phi}{\tilde{\Delta}_E} \text{Br}(D \rightarrow P)
\]
Accounting for the finite width of the CνB

\[ \text{Neutrino Flux} \left(10^{6} \text{ cm}^{-2} \text{s}^{-1} \text{meV}^{-1}\right) \]

- \[2\Delta_{\nu} \approx 0.7 \text{ meV}\]

- \[\Delta_{b}\]

\[ E - \Delta_{b} \quad E \quad E + \Delta_{b}\]
Accounting for the finite width of the CVB
Accounting for the finite width of the CvB

- Treating widths of distributions as uncertainty:

\[ \tilde{\Delta}_E = \sqrt{\left( \Delta_{\nu} \frac{\partial\tilde{E}_{\nu}}{\partial\nu} \right)^2 + \left( \Delta_{b} \frac{\partial\tilde{E}_{\nu}}{\partial p} \right)^2} \]
Treating widths of distributions as uncertainty:

\[ \tilde{\Delta}_E = \sqrt{\left( \Delta_{\nu} \frac{\partial \tilde{E}_\nu}{\partial p_{\nu}} \right)^2 + \left( \Delta_b \frac{\partial \tilde{E}_\nu}{\partial p} \right)^2} \]

For non-relativistic neutrinos, relativistic beam:

\[ \tilde{\Delta}_E = Q \sqrt{\delta_{\nu}^2 + \delta_b^2} \]
Accounting for the finite width of the CvB

- Total capture rate per target:

\[
\frac{R}{N_T} \propto \frac{\Gamma}{Q^3} \frac{\phi}{\sqrt{\delta^2_{\nu} + \delta^2_b}} \text{Br}(D \rightarrow P)
\]
Accounting for the finite width of the CvB

- Total capture rate per target:

\[ \frac{R}{N_T} \propto \frac{\Gamma}{Q^3} \frac{\phi}{\sqrt{\delta^2_{\nu} + \delta^2_b}} \text{Br}(D \rightarrow P) \]

- More convenient to introduce quality factor:

\[ R_\tau = \frac{\gamma R}{\Gamma N_T} = 1.7 \cdot 10^{-17} \frac{\text{Br}(D \rightarrow P)}{\sqrt{\delta^2_{\nu} + \delta^2_b}} \left[ \frac{0.1 \text{eV}}{m_\nu} \right] \left[ \frac{1 \text{keV}}{Q} \right]^2 \]
2-state systems

- Resonant electron capture:

\[ P \xleftarrow{\text{REC}} B_\beta \xrightarrow{\text{REC}} D \xrightarrow{C_\beta} P^+ \]
2-state systems

- Resonant electron capture:

\[ P \xrightarrow{\text{REC}} D \xrightarrow{\text{C}_{\beta}} P^+ \]

- Resonant bound beta decay:

\[ P \xrightarrow{\text{RB}_{\beta}} D \xrightarrow{\text{EC}} P \]
2-state systems

![Graph showing bound beta decay fraction vs. Q (keV) for different values of Z. The graph has logarithmic scales on both axes, with curves for Z = 1, Z = 10, and Z = 50.](image_url)
2-state systems

- Number of states on the beam:

\[
\frac{dN_P}{d\tilde{t}} = -\gamma \frac{R}{N_T} N_P(\tilde{t}) + \frac{\text{Br}(D \rightarrow P)}{\tau_D} N_D(\tilde{t})
\]
2-state systems

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\]

- Working in terms of dimensionless variables:

\[
x = \frac{t}{\gamma \tau_D} = \frac{m_\nu}{Q} \frac{t}{\tau_D} \quad R_\tau = \gamma \frac{R}{\Gamma N_T} = \gamma \tau_D \frac{R}{N_T}
\]
2-state systems

- Number of states on the beam:

\[
\frac{dN_P}{dx} = -R_\tau N_P(x) + \text{Br}(D \rightarrow P) N_D(x)
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2-state systems

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\[
\frac{dN_D}{dx} = R_\tau N_P(x) - N_D(x)
\]

- Number of daughter states reaches an equilibrium value!
2-state systems

- Number of states on the beam:

\[ N_D(x) = N_0 R_\tau (1 - e^{-x}) + \mathcal{O}(R_\tau^2) \]
2-state systems

- Number of states on the beam:

\[ N_D(x) = N_0 R_{\tau} (1 - e^{-x}) + O(R_{\tau}^2) \]

- 2-state systems are limited to converting small fraction of the beam
2-state systems

- Number of states on the beam:

\[ N_D(x) = N_0 R_\tau (1 - e^{-x}) + \mathcal{O}(R_\tau^2) \]

- 2-state systems are limited to converting small fraction of the beam

- Can we do better?
3-state systems

- Introduce a third, stable signal state:
3-state systems

- 3-state resonant electron capture:

\[ P \xrightarrow{\text{REC}} D \xrightarrow{\text{EC}} F \]

\[ P \xleftarrow{\text{BB}} D \]

\[ D \xrightarrow{\text{CB}} P^+ \]
3-state systems

- 3-state resonant electron capture:

  \[ P \xrightarrow{\text{REC}} D \xleftarrow{\text{B}\beta} P \]

  \[ D \xrightarrow{\text{C}\beta} P^+ \]

- 3-state bound beta decay:

  \[ P \xrightarrow{\text{RB}\beta} D \xleftarrow{\text{EC}} P \]

  \[ D \xrightarrow{\text{C}\beta} F^+ \]

  \[ D \xrightarrow{\text{B}\beta} F \]

  \[ F \xrightarrow{\text{EC}} F^+ \]
3-state systems

- Number of states on the beam:

\[ \frac{dN_P}{dx} = -R_\tau N_P(x) + \text{Br}(D \rightarrow P) N_D(x) \]
3-state systems

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3-state systems

- Number of states on the beam:

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\[
\frac{dN_D}{dx} = R_\tau N_P(x) - N_D(x)
\]

\[
\frac{dN_F}{dx} = \text{Br}(D \rightarrow F) N_D(x)
\]
3-state systems

- Number of states on the beam:

\[
\frac{dN_P}{dx} = -R_\tau N_P(x) + \text{Br}(D \to P)N_D(x)
\]

\[
\frac{dN_D}{dx} = R_\tau N_P(x) - N_D(x)
\]

\[
\frac{dN_F}{dx} = \text{Br}(D \to F)N_D(x)
\]

- Number of final (F) states increases monotonically!
3-state systems

- Now able to convert a significant fraction of the beam:

\[
\lim_{x \to \infty} N_F(x) = \frac{N_0 \text{Br}(D \to F)}{1 - \text{Br}(D \to P)} \gg N_0 R_\tau
\]
3-state systems

- Now able to convert a significant fraction of the beam:

\[
\lim_{x \to \infty} N_F(x) = \frac{N_0 \text{Br}(D \to F)}{1 - \text{Br}(D \to P)} \gg N_0 R_\tau
\]

- We now have a stable, clean signal with a large cross section!
3-state systems
Real world examples

- 2-state system:

\[ ^{157}\text{Gd} \xrightarrow{RB\beta} ^{157}\text{Tb} \]
Real world examples

- 2-state system:

\[ ^{157}\text{Gd} \xrightarrow{\text{RB}^\beta} ^{157}\text{Tb} \]

\[ \frac{E}{A} \approx 100 \text{ TeV} \]

\[ \frac{N_D}{N_0} \approx 10^{-24} \]
Real world examples

• 2-state system:

\[ ^{157}\text{Gd} \xrightarrow{\text{RB}_\beta} ^{157}\text{Tb} \quad \frac{E}{A} \approx 100 \text{ TeV} \quad \frac{N_D}{N_0} \approx 10^{-24} \]

• 3-state system:

\[ ^{106}\text{Cd} \xrightarrow{\text{REC}} ^{106}\text{Ag} \xrightarrow{\text{EC}} ^{106}\text{Pd} \]
Real world examples

- 2-state system:

\[ ^{157}\text{Gd} \xrightarrow{\text{RB}^\beta} ^{157}\text{Tb} \quad \frac{E}{A} \approx 100 \text{ TeV} \quad \frac{N_D}{N_0} \approx 10^{-24} \]

- 3-state system:

\[ ^{106}\text{Cd} \xrightarrow{\text{REC}} ^{106}\text{Ag} \xrightarrow{\text{EC}} ^{106}\text{Pd} \]

\[ \frac{N_D + N_F}{N_0} \approx 10^{-23} \quad \frac{E}{A} \approx 2 \text{ PeV} \]
Contents

- What is the $\text{C}_\nu\text{B}$?
- PTOLEMY(-on-a-beam)
- Resonant neutrino capture
- Experimental challenges
Experimental challenges

- Large energy requirements → appropriate choice of target, use an excited state
Experimental challenges

- Large energy requirements $\rightarrow$ appropriate choice of target, use an excited state

- Require knowledge of the neutrino mass $\rightarrow$ KATRIN, beam broadening
Experimental challenges

- Large energy requirements → appropriate choice of target, use an excited state
- Require knowledge of the neutrino mass → KATRIN, beam broadening
- Large number of targets required → reduce threshold, purpose built experiment
Strategy

- Seek processes with small threshold
  - Increased cross section
  - Shorter ‘effective’ resonance lifetime
  - Lower energy requirements
Strategy

- Seek processes with small threshold
  - Increased cross section
  - Shorter ‘effective’ resonance lifetime
  - Lower energy requirements

- Try to find a 3-state system
  - Stable, clean signal
  - Possibility to convert huge fraction of the beam
Summary

- Resonant neutrino capture has huge cross sections
Summary

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- Capture cross section is independent of $G_F$
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- Capture cross section is independent of $G_F$
- Able to perform this experiment with $\mathcal{O}(\text{TeV})$ energies!
Summary

- Resonant neutrino capture has huge cross sections
- Capture cross section is independent of $G_F$
- Able to perform this experiment with $\mathcal{O}(\text{TeV})$ energies!
- Great deal of parameter space left to be explored
Thank you! Questions?
Neutrino mass uncertainty

- Assuming wrong neutrino mass → incorrectly centred beam energy

\[ \delta_m = \frac{m_{\nu,\text{true}} - m_{\nu,\text{pred}}}{m_{\nu,\text{true}}} \]

\[ R_{\tau,\text{eff}} = R_\tau (1 - \delta_m)^2 e^{-\frac{\delta_m^2}{2(\delta_\nu^2 + \delta_b^2)}} \]

- Only capturing neutrinos from tail end of spectrum

- Partially rectifiable by appropriate choice of \( \delta_b \)
Neutrino mass uncertainty