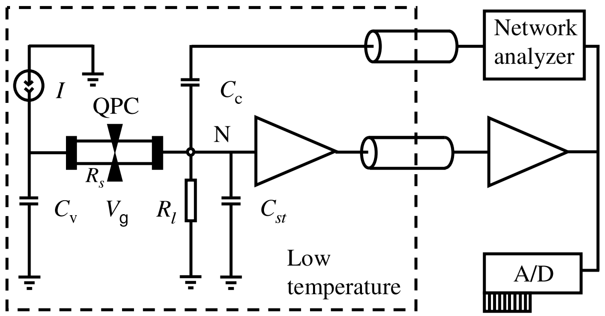
# *Full Counting Statistics as Geometry*

N. d'Ambrumenil, Warwick

June 2008

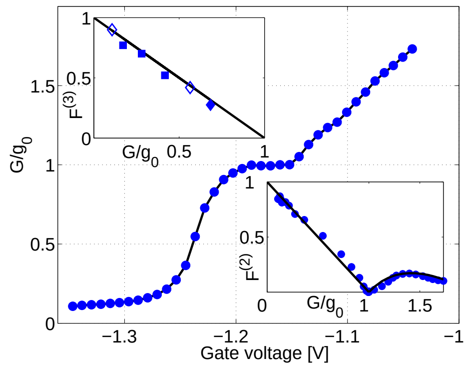
Boris Muzykantskii, Amitesh Pratap and Yury Sherkunov

# Fluctuations & Entropy




Gershon et al, 0710.1852

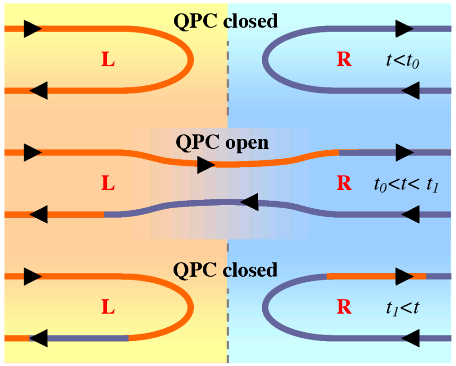
T ≈ 1K, ≈ 5K, $\nu_{>} = \frac{1}{R_{\|} C_{\|}} \sim$ 7MHz




Transmission:

$$\frac{G}{g_0} = T = |t|^2$$

$$S^{(k)} = eIF^{(k)} = \frac{\langle\langle q^k \rangle\rangle}{t_f} \qquad (= e^{k-1} I \quad \text{classical Poisson process})$$




“Entanglement
Entrometer”

Klich & Levitov, 0804.1377

# Quantum Noise & FCS

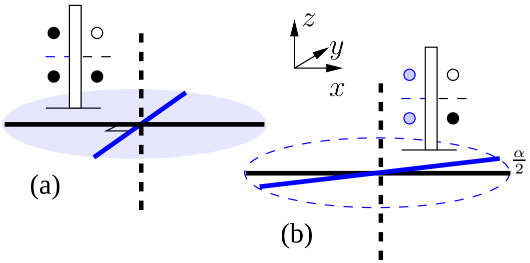
$$\langle\langle q^2(t_f)\rangle\rangle = e(1-T)It_f$$ Noise, DC (Lesovik 1989), MES

$$\chi(\lambda, t_f) = \sum_{n=-\infty}^{\infty} P_n(t_f)e^{in\lambda}$$ FCS, Levitov & Lesovik 1996

$$= \det|1-\hat{n}+\hat{n}\hat{G}|, \qquad \hat{G} = \hat{U}^\dagger(t_f)e^{i\lambda\hat{Q}_L}\hat{U}(t_f)e^{-i\lambda\hat{Q}_L}$$

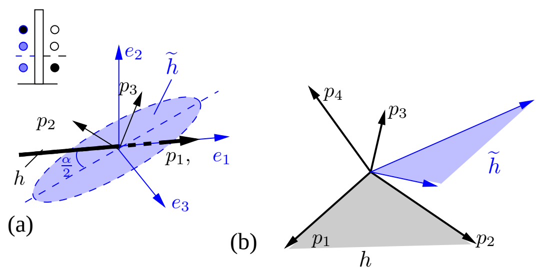
Uni- and bi-directional events Vanevic Nazarov & Belzig 07, AP 2008

$$\chi(\lambda) = (R+Te^{i\kappa_\lambda\lambda})^{N_u}\prod_{i=1}^{N_b}\left(1+RT\sin^2\frac{\alpha_j}{2}(e^{i\lambda}+e^{-i\lambda}-2)\right) \qquad \kappa_\lambda = \pm 1$$

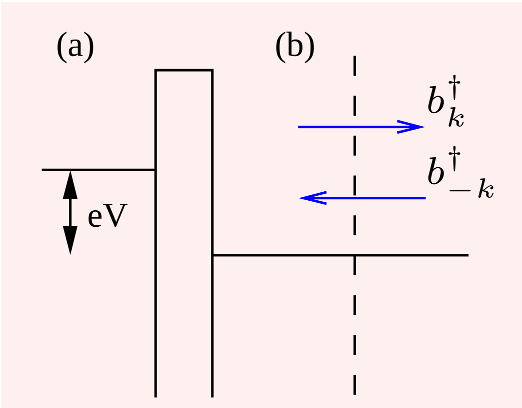



YS, AP, BM, NdA

PRL 100, 196601



# Quantum Noise




$$\overbrace{\begin{pmatrix} \hat{a}^\dagger_{-\mathbf{k}} \\ \hat{b}^\dagger_{\mathbf{k}} \end{pmatrix}}^{\text{Out}} = \overbrace{\begin{pmatrix} r & -t \\ t^* & r^* \end{pmatrix}}^{s\text{ matrix}} \overbrace{\begin{pmatrix} \hat{a}^\dagger_{\mathbf{k}} \\ \hat{b}^\dagger_{-\mathbf{k}} \end{pmatrix}}^{\text{In}}$$

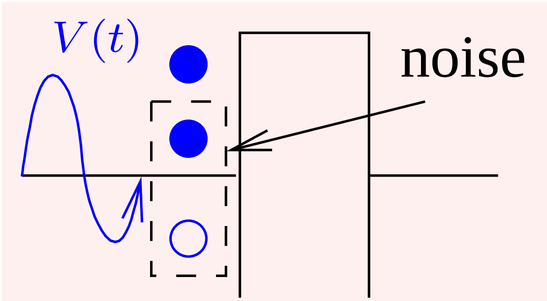
$$\begin{aligned} I^{dc} &\sim e\sum_k v_k (t^* a_k^\dagger + r^* b_{-k}^\dagger)(t a_k + r b_{-k}) - b_{-k}^\dagger b_{-k} \\ \langle I \rangle &= \frac{e}{h}\int d\epsilon |t|^2 (\langle n^a \rangle - \langle n^b \rangle) = |t|^2 \frac{e^2}{h} V \end{aligned}$$

Noise power spectrum $S^{(2)} \sim (\langle I^2 \rangle - \langle I \rangle^2)$

$$\begin{aligned} S^{(2)} &\sim e^2 \sum_{k,k'} v_k v_{k'} |t|^2 |r|^2 (\overbrace{\langle a_k^\dagger b_{-k} b_{-k'}^\dagger a_{k'} \rangle}^{n^a(1-n^b)} + \overbrace{\langle b_{-k}^\dagger a_k a_{k'}^\dagger b_{-k} \rangle}^{n^b(1-n^a)}) \\ &= e(1 - |t|^2) I \end{aligned}$$

Lesovik 1989

## CSAC or MES




$$\langle\langle Q^2\rangle\rangle \sim n^a(1-n^b)+n^b(1-n^a) = \int_0^\infty \left(|f_+(\omega)|^2+|f_-(\omega)|^2\right)\frac{\omega d\omega}{2\pi}$$

$$\langle\langle Q\rangle\rangle \sim n^a(1-n^b)-n^b(1-n^a) = \int_0^\infty \left(|f_+(\omega)|^2-|f_-(\omega)|^2\right)\frac{\omega d\omega}{2\pi}$$

$$e^{i\phi(t)} = f_+(t) + f_-(t)$$

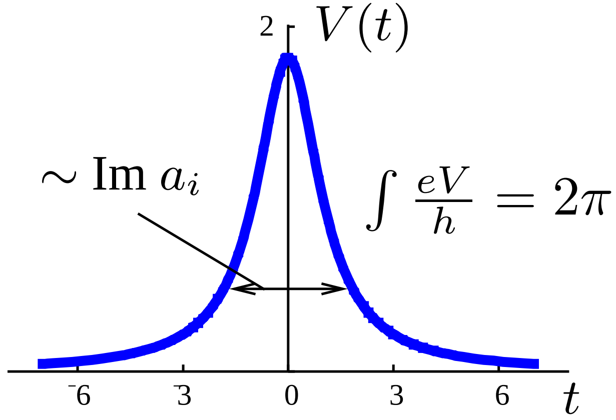
$f_+$ analytic Im $t > 0$

Minimal noise when $f_\pm = 0$

Ivanov et al 96, Keeling et al 06

Lorentzian pulses with quantized area

$$e^{i\phi(t)} = \prod_{i=1}^{N}\left(\frac{t-a_i^*}{t-a_i}\right), \qquad \mathrm{Im}\, a_i < 0$$



# Full Counting Statistics

$$\chi(\lambda) = \sum_{n=-\infty}^{\infty} P_n e^{in\lambda} = \langle 0| \underbrace{\hat{U}^\dagger(t_f) e^{i\lambda \hat{Q}_L} \hat{U}(t_f) e^{-i\lambda \hat{Q}_L}}_{\hat{G}} |0\rangle$$

$$= \det|1 - n + nG| \qquad G_{li,l'i'} = \langle li|\hat{G}|l'i'\rangle$$

RH problem, $(1 - n + nG)^{-1}$, AP+NdA 07
Keldysh, VNB 07

Why? Moments of charge transfer, MES

IK+LL 08

$z = (1 - e^{i\lambda})^{-1}$

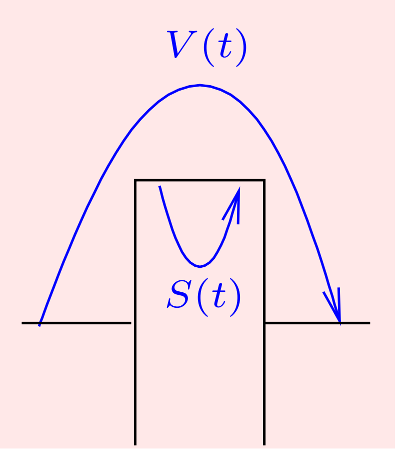
$\vdots$

$$\chi(z) = \det\left|(z - n_L) e^{-i(nP)_U \lambda(z)} (1 - e^{i\lambda(z)})\right|$$

Entanglement entropy $\mathrm{Im}\, \partial_z \log(\chi(z - i0)) \rightarrow$ spectral density of $n_L$

# Time Domain—Slow Variations

Voltage pulse or vary scattering matrix $0 < t < t_f$:




$$\overbrace{\langle|\hat{a}_{l\epsilon}\hat{U}(t_f)\hat{a}^\dagger_{l'\epsilon'}|\rangle}^{\sigma_{ll'}(\epsilon-\epsilon',E)} = \int dt\,\sigma(t,E)e^{i(\epsilon-\epsilon')t}$$

$$\approx \int dt\,S_{ll'}(t,0)e^{i(\epsilon-\epsilon')t}$$

$\sigma(t,0) \approx S(t)$ so that if $\hbar\frac{\partial S^\dagger}{\partial t}\frac{\partial S}{\partial E} \ll 1$

$$\psi(t) \to \tilde{\psi}(t) \approx S(t)\psi(t)$$

$(\hbar S^\dagger \frac{\partial S}{\partial E}$ is Wigner delay time)

$n$ not diagonal in $t$:

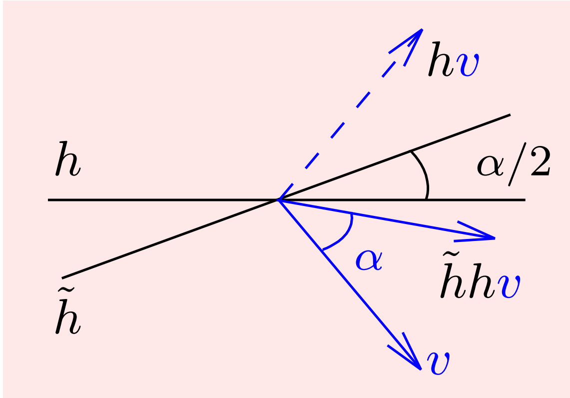
$$n(t,t') = \frac{i}{2\pi}\frac{1}{t-t'+i0}$$

$\psi(t')$ analytic for Re $t' > 0$:

$$n\psi = \int n(t,t')\psi(t')dt' = \psi(t)$$

# Geometry

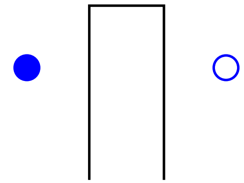
$$\left.\begin{array}{rcccl} h & = & & & (2n-1) \\ \tilde{h} & = & UhU^\dagger & = & (2\tilde{n}-1) \end{array}\right\} \text{e-values } \pm 1$$




$h$, $\tilde{h}$ define mirror planes (single particle states)

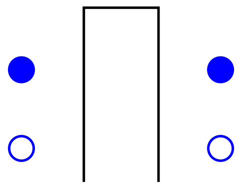
$h\tilde{h}$ unitary, e-values $e^{\pm i\alpha}$, e-vectors $v_\alpha$, $hv_\alpha$.

## Unidirectional

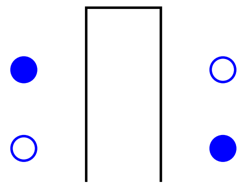


$$\begin{aligned} \dim \tilde{h} &= \dim h + 1 \\ \chi(\lambda) &= R + Te^{i\lambda} \end{aligned}$$

## Bidirectional



$\chi(\lambda) = 1$

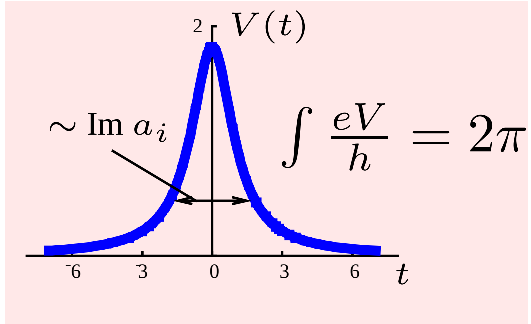


$\chi(\lambda) = (R + Te^{i\lambda})(R + Te^{-i\lambda})$

$$\chi(\lambda) = \cos^2\tfrac{\alpha}{2} + \sin^2\tfrac{\alpha}{2}\left(R + Te^{i\lambda}\right)\left(R + Te^{-i\lambda}\right)$$

# Lorentzian Pulses

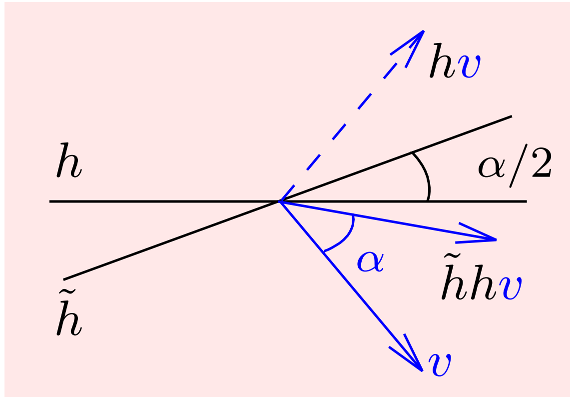
$$e^{i\phi(t)} = \prod_{i=1}^{N} \frac{t - a_m^*}{t - a_m} = 1 + \sum_m \frac{A_m}{t - a_m}$$




Cont'n to $\chi(\lambda)$ is 1 if $\qquad (|\psi_m\rangle \sim \frac{1}{t-a_m})$

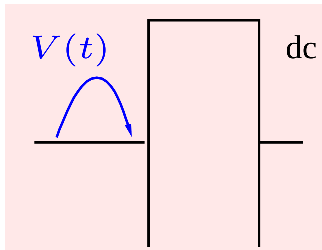
$$0 = (h\tilde{h} - 1)|\psi\rangle$$

$$= \underbrace{\sum_{m,m'} \text{sign}(\text{Im}a_m) \frac{2A_m A_{m'}^*}{a_m^* - a_{m'}} |\psi_m\rangle\langle\psi_m|\,\psi\rangle}_{\text{Projector onto N states}}$$




## Examples

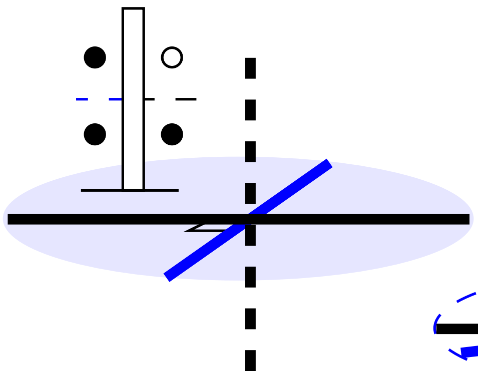
i Im $a_m$ all same sign, $h\tilde{h}|\psi_m\rangle = -|\psi_m\rangle$, $\alpha_i = \pi$. $N$ unidirectional events.



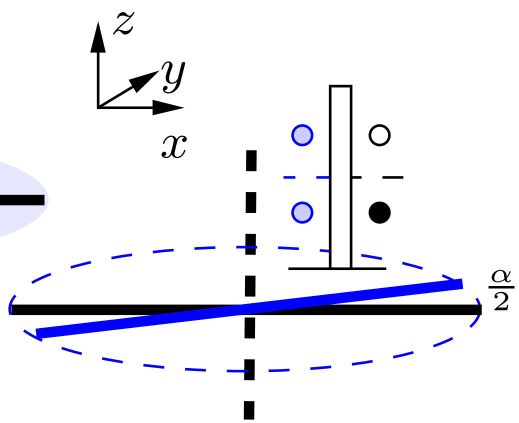

ii sgn(Im $a_1$) sgn(Im $a_2$) $= -1$

$$e^{i\phi(t)}|\psi_1^*\rangle \rightarrow \cos\frac{\alpha}{2}e^{i\phi_1}|\psi_1\rangle + \sin\frac{\alpha}{2}e^{i\phi_2}|\psi_2\rangle$$

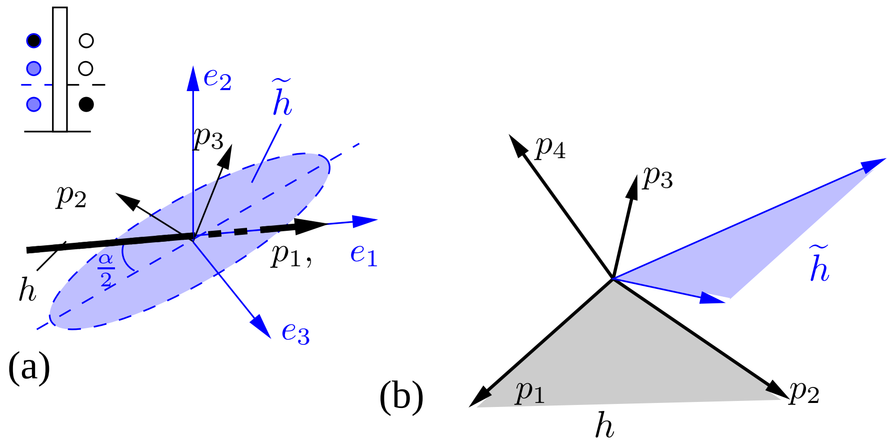
# Pictures



(a) $N_u = 1$, $N_b = 0$




(b) $N_u = 0$, $N_b = 1$

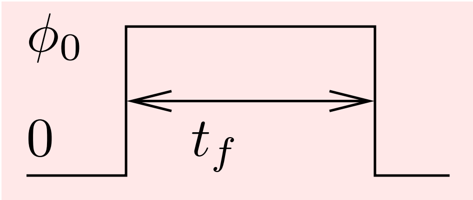



(a) $N_u = 1$, $N_b = 1$, (b) $N_u = 0$, $N_b = 2$
$p_i$ space with $h\tilde{h} \neq 1$; $e_i$ basis for $h\tilde{h}$

# Other Cases

FES (1 ch), Equilibrium Noise (2)

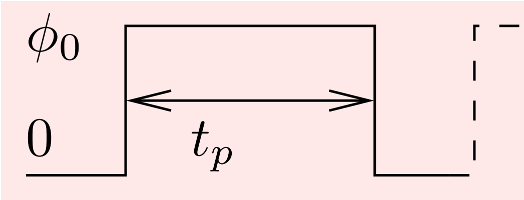
$$\langle 0|\hat{U}(t_f)|0\rangle = \prod_k \cos\frac{\alpha_k}{2} \approx \left(\frac{1}{\xi t_f}\right)^{\phi_0^2/\pi^2}$$




(see also Abanov & Ivanov 08)

Pulse Train

$$\chi(\lambda) = 2N\frac{RT}{\pi^2}\sin^2\phi_0 \log\left(\frac{t_p}{\tau}\right)$$




$\xi$, $\tau$ short time cutoffs

# Quantum Noise & FCS

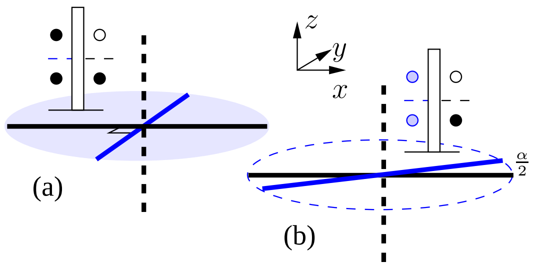
$$\langle\langle q^2(t_f)\rangle\rangle = e(1-T)It_f$$ Noise, DC (Lesovik 1989), MES

$$\chi(\lambda) = \sum_{n=-\infty}^{\infty} P_n e^{in\lambda}$$ FCS, Levitov & Lesovik 1996

$$= \det|1-\hat{n}+\hat{n}\hat{G}|, \qquad \hat{G} = \hat{U}^\dagger(t)e^{i\lambda\hat{Q}_L}\hat{U}(t)e^{-i\lambda\hat{Q}_L}$$

Uni- and bi-directional events Vanevic Nazarov & Belzig 07, AP 2008

$$\chi(\lambda) = (R+Te^{i\kappa_\lambda\lambda})^{N_u}\prod_{i=1}^{N_b}\left(1+RT\sin^2\frac{\alpha_j}{2}(e^{i\lambda}+e^{-i\lambda}-2)\right) \qquad \kappa_\lambda = \pm 1$$




YS, AP, BM, NdA

PRL 100, 196601