## Puzzle set 2

## Convoluted ways to get $\pi$

## Puzzle 1: Colliding blocks

Consider two sliding blocks in a perfectly idealized world where there's no friction, and all collisions are perfectly elastic, meaning no energy is lost. One block of mass $M$ (with $M$ ) is sent towards the other of mass $m=1 \mathrm{~kg}$. The mass $m$ starts off stationary with a wall behind it so that this block bounces back and forth between $M$ and the wall. This happens until $m$ redirects the $M$ 's momentum enough so that $M$ goes away faster than $m$ from the wall never to collide again with $m$. Now it is seen that if $M=100$ kg , the total number of collisions suffered by $m$ (both with $M$ and the wall) is 31 . If $M=100^{2} \mathrm{~kg}$, there are 314 collisions, and if $M=100^{3} \mathrm{~kg}$, there are 3141 collisions. If $M=100^{6} \mathrm{~kg}$, there are 3141592 collisions. Can you explain why $\pi$ shows up here?

## Puzzle 2: Ships that pass in the night

Schwinger and Tomonaga are standing on two corners of an $n \times n$ lattice. Schwinger is in bottom left corner and Tomonaga is on the top right. At every time step, Schwinger can move either one step right or up with equal probability. Similarly Tomonaga can move only one step to the left or down, again with equal probability. They both start moving from their respective corners at the same time. Let the probability they meet be P. Can you show that $\left(P^{2} n\right)^{-1}$ is $\pi$ ?

## Puzzle 3: Unscramble to decrypt puzzle solutions


(_) (_) ( ) (repeated twice) will show you the solution.

