Start your summer with puzzles

Theory group seminar

Problem 1

Consider 2023 points with each point connected to every other point by a resistor of resistance R.

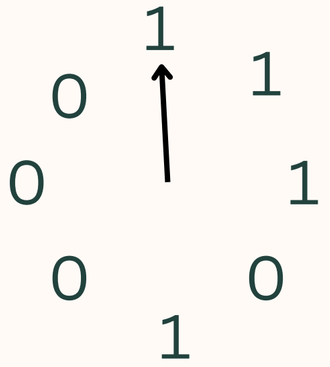
(a) What is the effective resistance between any two points?

(b) Choosing two points, can you remove a single resistor such that the resistance between the chosen two points becomes the same as that when there are 2021 points with resistance R between any two points?

(c) You are given two networks, (i) a network with 2021 points with R between any two points of this network, (ii) a network built with 2023 points as described in (b) of this question. Can you differentiate between the two without counting the points?

PROBLEM 2

We can arrange 2^n binary digits in a ring such that every n-digit pattern taken in the clockwise (respectively, anticlockwise) sense is unique in that sense. To illustrate, consider the ring given below corresponding to n=3 case.



3 digit sets correspond to 7,6,5,2,4,0,1,3 (clockwise, starting from the marked digit) and similarly in the anticlockwise sense too. Can you build such a ring for n=5?

PROBLEM 3

In the well-known game, Twenty Questions one person thinks of an object, such as the He-Ne Laser, and another person tries to guess the object by asking no more than twenty questions, each answerable by yes or no. For instance, if a person has chosen as his object to be a number, in twenty questions one can guess any number from 1 through 2^{20} (or 1,048,576).

Now, assume that a deck of cards consists of 1 ace of spades, 2 deuces of spades, 3 threes, and so on to 9 nines, 45 spade cards in all. The deck is shuffled and a card is drawn. How can you minimize the number of questions that you will probably have to ask?

Hint: Think probabilities!