Year-end Special Theory Group contest

Hints on puzzles

Problem 1

(a) How is the network of points constructed?

For instance, let us consider M points.

Here each line connecting the two points is assumed to have resistance R. The networks are plotted for M points (M=2, 3, 4, 5, 6, 7, 8, 9, and 10)

In[17]:=
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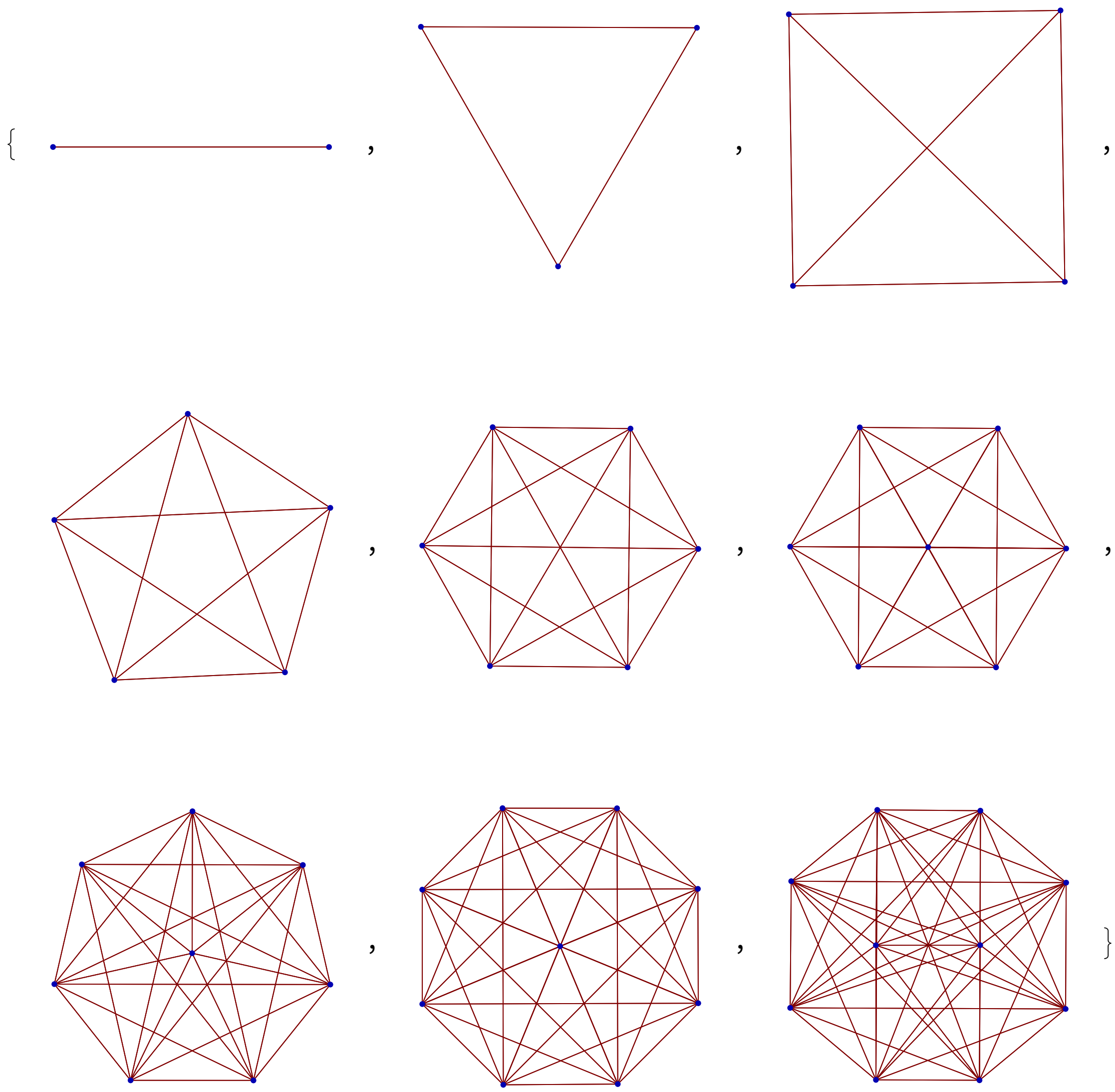
Table[GraphPlot[Table[If[i ≠ j, i → j], {j, 1, M, 1}, {i, 1, M, 1}]], {M, 2, 10, 1}]
```

Out[17]=



- Now consider the case of M=4 from the above set of networks. Now choose any two points from the available 4. There is an effective resistance r between the two points, irrespective of which pair has been chosen (i.e., same resistance r between any pair). Try finding the resistance r between the two points for M=4. (Also, it is obvious in the cases of M=2,3). Does this give you any ideas on how any network of M points can be handled?

(b and c) Choosing two points, let us assume you remove a resistance R in the network. Now let the effective resistance between the chosen two points be r'. If we choose any other pair, is the effective resistance between that pair r'?

Problem 2

How do you think the ring of digits (in the illustration in Puzzles.pdf) is constructed for n=3?

Let us consider the set of binary digits between 0 and 2^n-1 for n=3.

In[22]:=
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n = 3;
Table[IntegerString[i, 2, 3], {i, 0, 2^n - 1}]
```

Out[23]=
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{000, 001, 010, 011, 100, 101, 110, 111}
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Let us choose the largest first, i.e., 111, with the register starting at the leftmost 1 as shown in the figure in the question (in Puzzles.pdf). Now when the register moves to the middle 1, we have "11x_1" where x_1 can be 0 or 1. Now x_1 cannot be 1 as 111 is already the number in the initial register position. Therefore x_1=0. So we now have 1110 as the string. The register moves to the third position in this string. We now have the number "10x_2". Now x_2 can be both 0 or 1. Here if you choose 0, you will run into a case where you cannot avoid repeating numbers. So you chose x_2 to be 1. If you truly understand why choosing x_2=0 leads to trouble, the problem can be easily extended and solved in n=5 case too.

Problem 3

Let us play Twenty Questions with numbers first, in the range 1 to 2^{20}. Please think of any number in this range.

In the first question, I will ask whether the number is less than or equal to 2^{19}. You say either yes or no (here we assume that you say the truth). Either way, my range is now halved. Let us say you said yes, my next question would be whether the number is less than or equal to 2^{18}. This proceeds till the twentieth question, when I will know the number that you have thought of.

In the case of Twenty Questions with numbers, why does splitting the set in equal parts minimize the number of question you probably will have to ask?

Because it is equally probable for a person to think of any number in the given range. So you are splitting the probabilities too equally. So now in the case of the 45 cards in the problem, is there a way by which the cards can be combined into sets of cards, such as 4s and 5s (or perhaps it is 3s and 4s :P), that will split the probabilities of choosing the cards as equally as you possibly can (at least somewhat equally).

All the best! Hope you have fun!