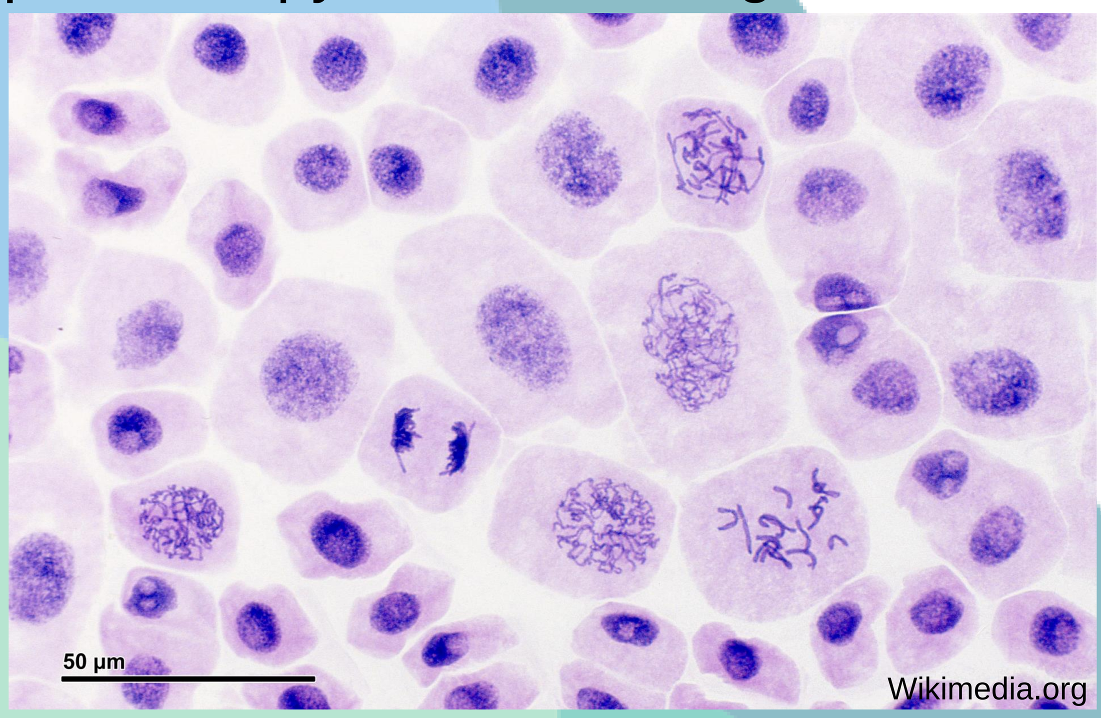
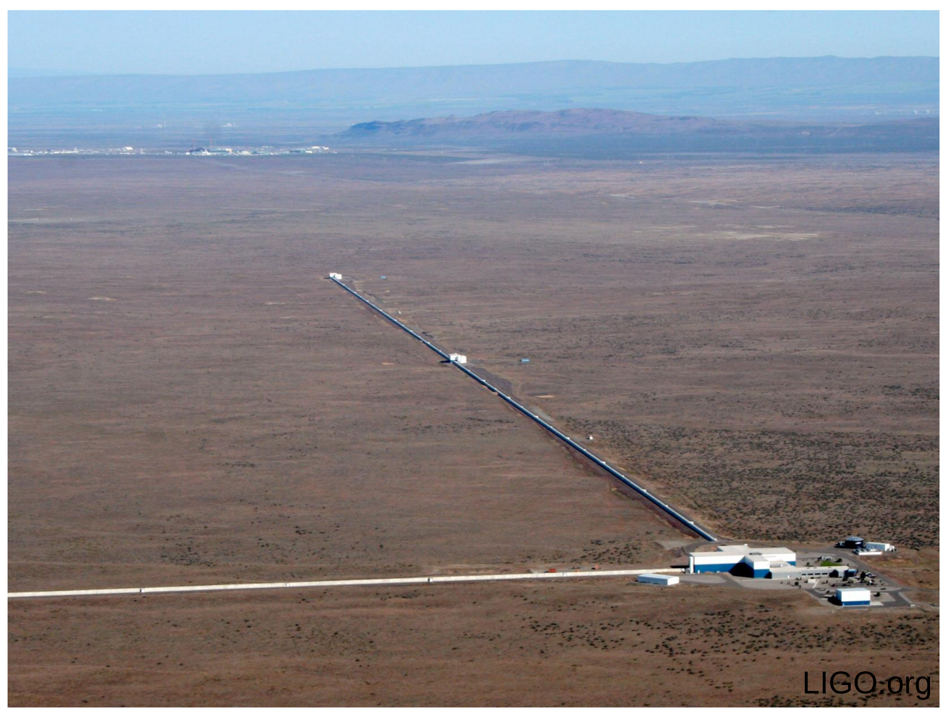
Gaussian systems for quantum-enhanced imaging

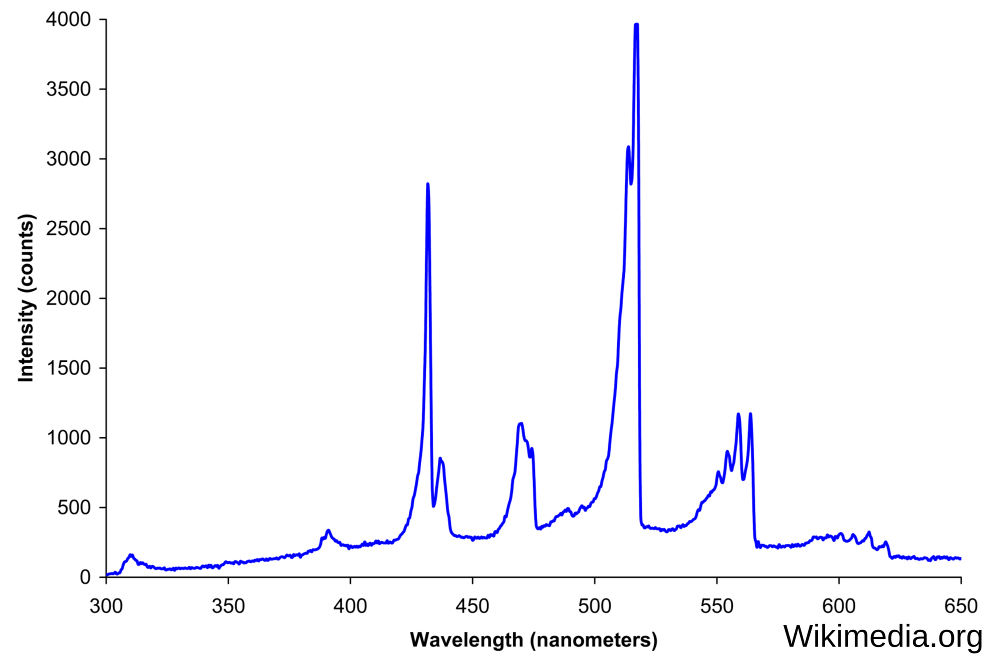
Dominic Branford, Christos N. Gagatsos, Animesh Datta
Department of Physics, University of Warwick, Coventry, CV4 7AL

Applications

Performing measurements with high precision has ramifications across a range of technologies, including imaging, spectroscopy, manufacturing and scientific research







Estimation and resources

Parameter estimation schemes can be judged by the quantum Cramér-Rao bound which gives a bound to attainable precision. From the quantum Fisher information H for μ repeated measurements the uncertainty in the parameters is

$$\mathrm{Cov}(\{\varphi_1, \cdots, \varphi_d\}) \geq \frac{1}{\mu} H^{-1}.$$

In metrology experiments there are a finite number of resources---typically energy (or equivalently photon number) of the probe state $E_{Tot.}$---available to estimate d parameters. A scheme's performance can be judged by the scaling of $\mathrm{Tr}(H^{-1}) = \sum_{i=1}^{d} (\Delta\varphi_i)^2$ with number N or average energy/number $E_{Tot.}$ as well as parameter number d.

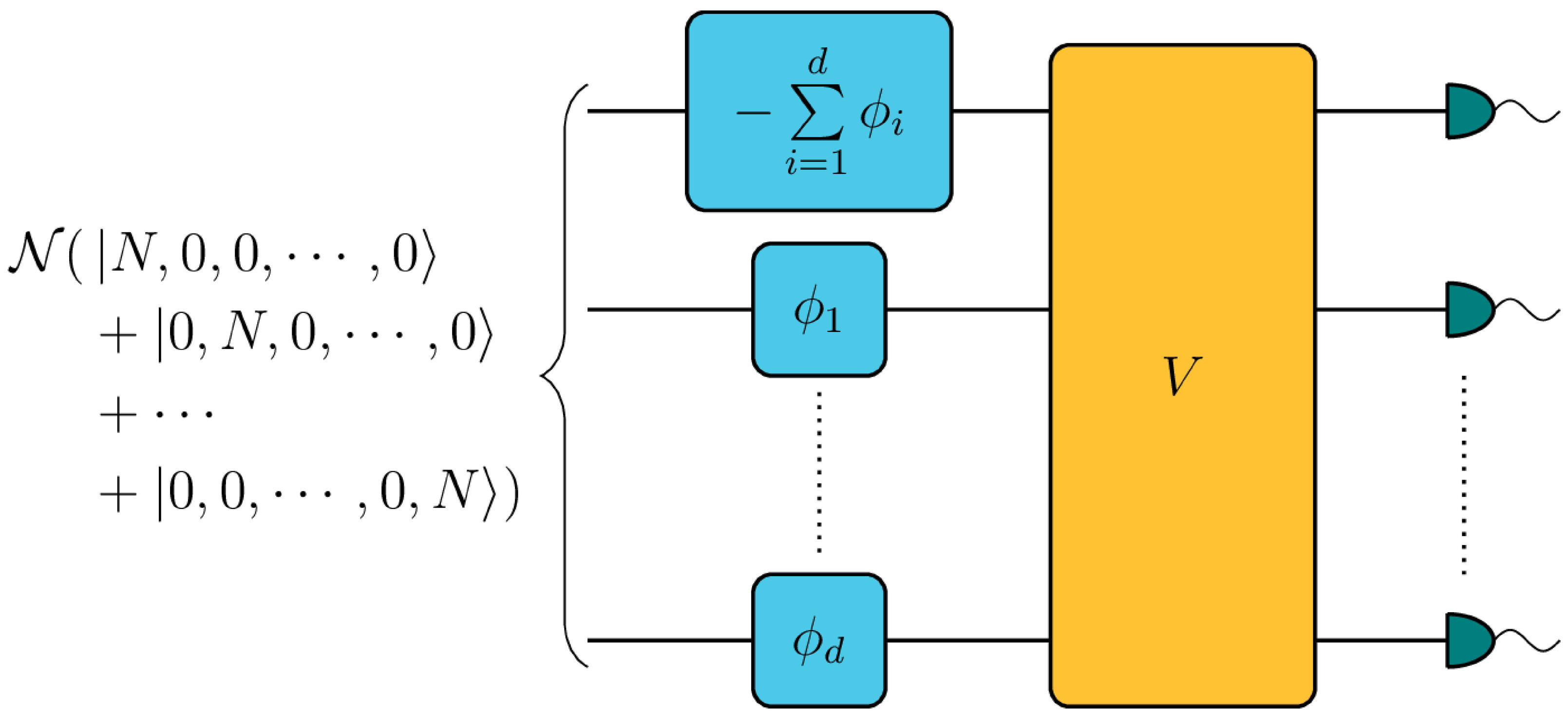
Non-Gaussian states (peak-power constraint)

Precision can be improved by a factor of $E_{Tot.}$ through the use of quantum-mechanical states, such as N00N and squeezed vacuum states [Lang & Caves. PRA **90**, 025802 (2014)].

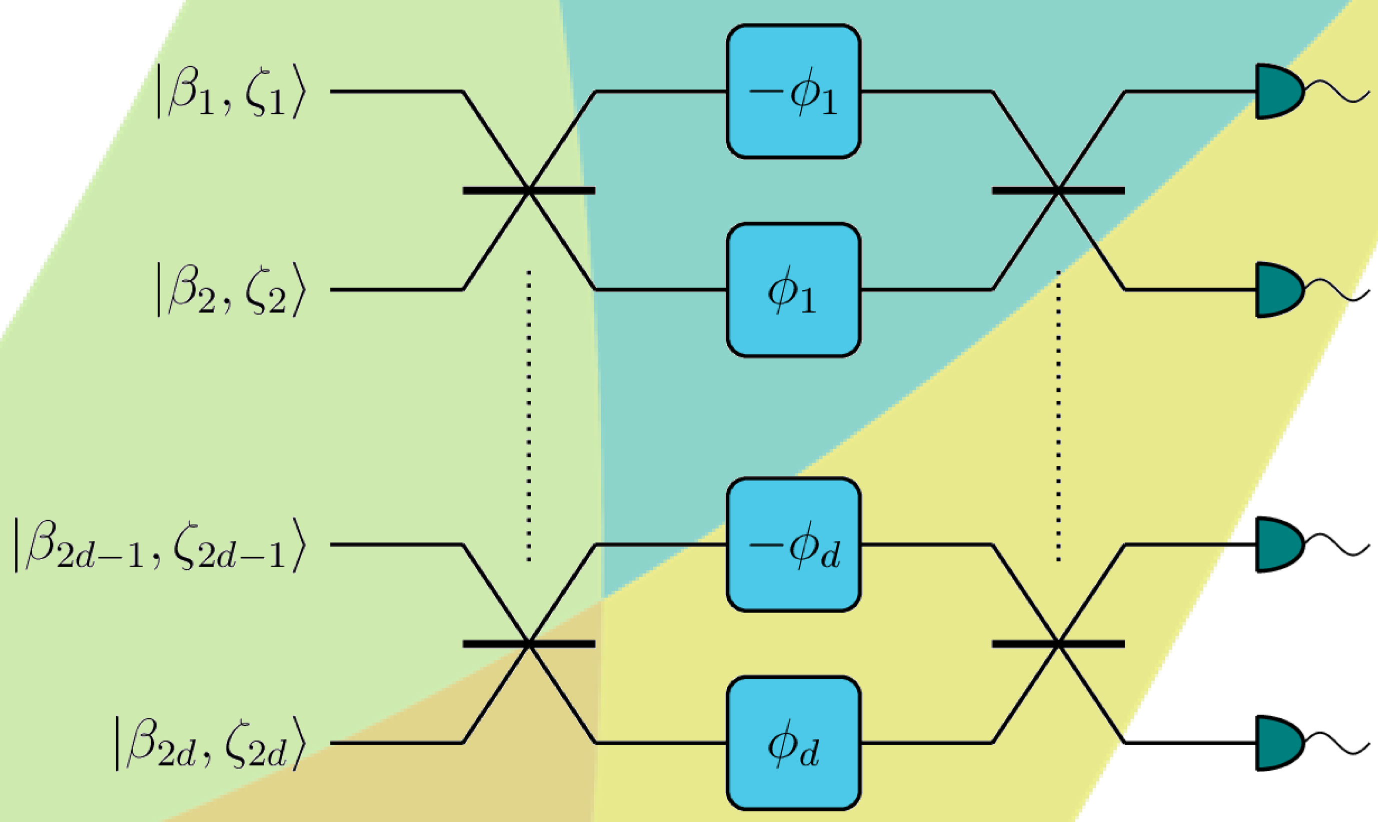
Humphreys et al. [PRL **116**, 030801 (2013)] show fixed number states attain an improved scaling with the number, of phases d, and a total N photons,

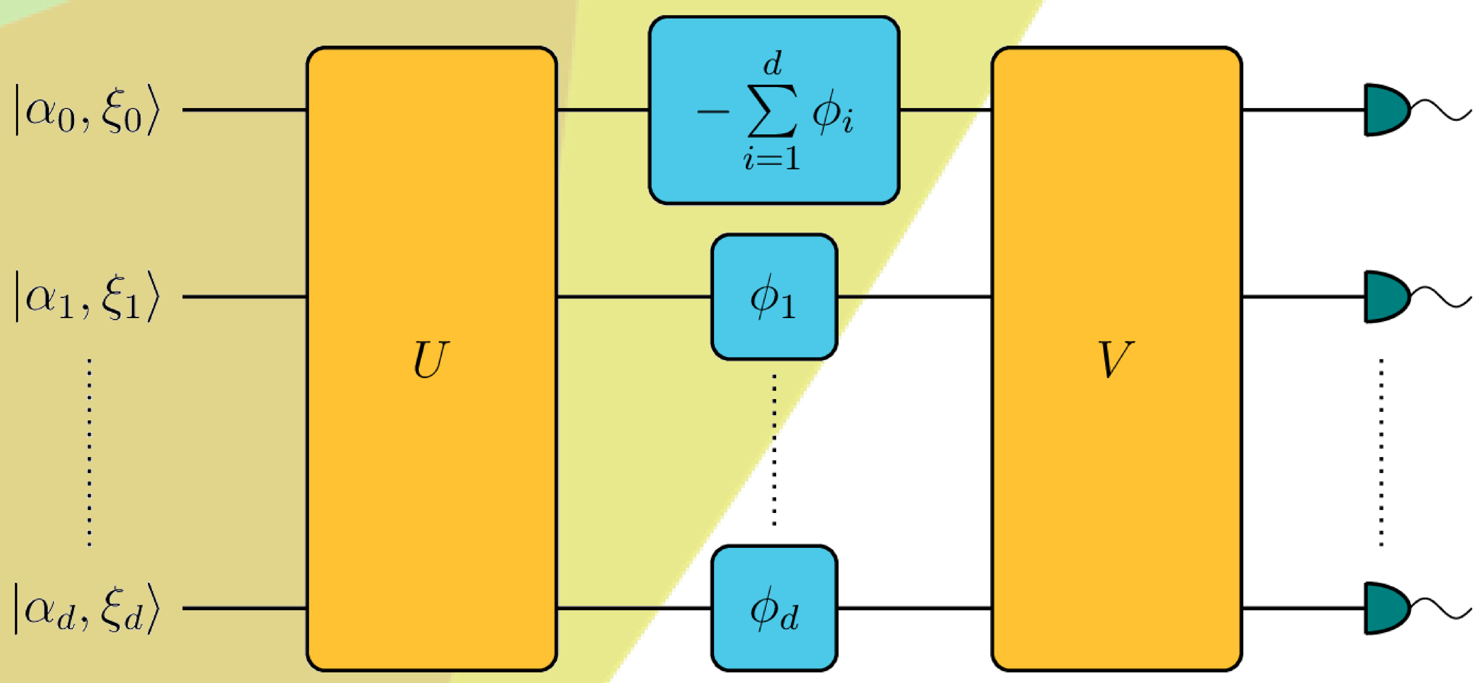
$$\mathrm{Tr}\left(H_{Ind.}^{-1}\right) = \frac{d^3}{N^2}, \qquad \mathrm{Tr}\left(H_{Sim.}^{-1}\right) = \frac{\left(1+\sqrt{d}\right)^2 d}{4N^2} \sim \frac{d^2}{N^2}.$$

Simultaneous estimation methods with these fixed number states attain an improvement of $R = \frac{\mathrm{Tr}(H_{Sim.}^{-1})}{\mathrm{Tr}(H_{Ind.}^{-1})} \sim \frac{1}{d}$.



Gaussian states (average-power constraint)





Gaussian states are an exciting class of quantum states which exhibit non-classical phenomena yet can be practically produced with current resources. Lasers, amplifiers, squeezers, and linear optics all produce Gaussian states and operations.

In Gagatsos et al. [PRA **94** 042342 (2016)] we find bounds for the simultaneous and individual cases, subject to a constrained average energy.

Under assumptions of an orthogonal U and equal magnitude squeezings ($\xi_i = \xi,\ \forall i$), the optimal state is parallel squeezed vacuum $|\xi, \xi, \cdots, \xi\rangle$,

$$\mathrm{Tr}(H_{Ind.}^{-1}) = \frac{d^3}{4E_{Tot.}(E_{Tot.} + 2d)},$$

$$\mathrm{Tr}\left(H_{Sim.}^{-1}\right) = \frac{d^2(d+1)}{8E_{Tot.}(E_{Tot.} + d + 1)},$$

$$R = \frac{\mathrm{Tr}(H_{Sim.}^{-1})}{\mathrm{Tr}(H_{Ind.}^{-1})} = \frac{(d+1)(E_{Tot.} + 2d^2)}{2d(E_{Tot.} + d + 1)} \leq \frac{d+1}{2d}.$$

For single phase estimation Gaussian and fixed number states perform comparably, yet at the multiple parameter level Gaussian states perform worse---instead optimum sensitivity seems to necessitate some non-Gaussianity.

WARWICK
THE UNIVERSITY OF WARWICK

UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

QUANTIC