Gaussian systems for quantum enhanced multiple phase estimation

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Introduction

Phase estimation is a well studied problem in quantum metrology, with implications for gravitational wave detection, imaging and manufacturing. Recent works have looked at simultaneous estimation strategies with fixed number states and found significant improvements [2]. Do the same advantages exist for the more practical class of Gaussian states?

Phase definitions

Phases of the form $e^{i\bar{\varphi}\hat{n}_{\text{Tot}}}$ have no measurable effect in a linear optics setup [3]. The phases on each mode $\{\varphi_j\}$ cannot all be resolved without the addition of reference states. Rewrite the Hamiltonian, isolating the unmeasurable phase $\bar{\varphi}$:

$$\exp\left(i\sum_{j=0}^{d}\varphi_j\hat{n}_j\right) = \exp\left(i\bar{\varphi}\hat{n}_{\text{Tot}} + i\sum_{j=1}^{d}\phi_j(\hat{n}_j - \hat{n}_0)\right). \tag{1}$$

Parameter estimation

The quantum Cramér-Rao bound (QCRB) provides a measure to assess the effectiveness of an estimation scheme. The QCRB is

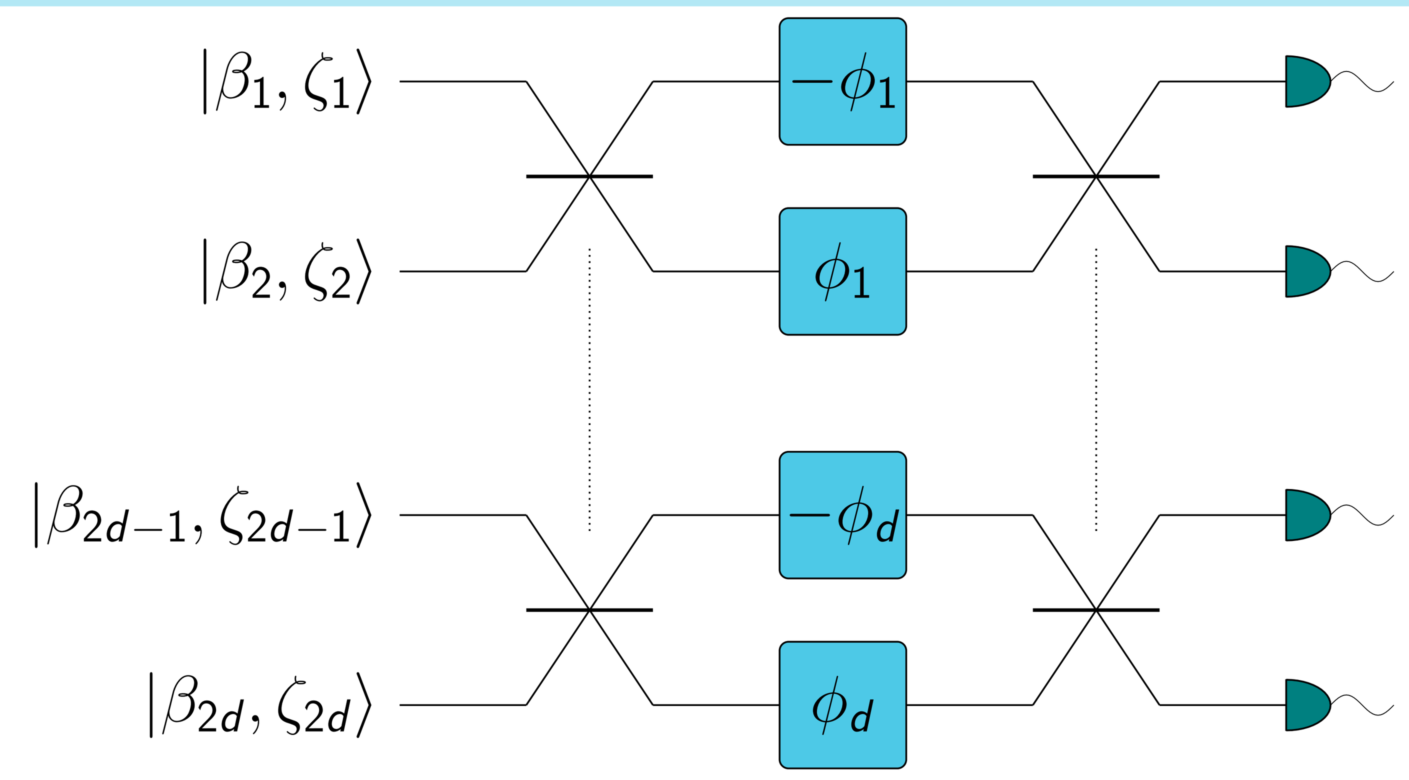
$$\text{Cov}(\{\phi_1, \cdots, \phi_d\}) \geq \frac{1}{\nu}F^{-1} \geq \frac{1}{\nu}H^{-1}, \tag{2}$$

where the Fisher information and quantum Fisher information (QFI) provide lower bounds to the mean square error. With a probe state $|\psi\rangle$ the QFI, H is given by the variance of the phase generators,

$$\begin{aligned} H_{j,k} =& 4\left(\langle\partial_j\psi|\partial_k\psi\rangle + \langle\partial_j\psi|\psi\rangle\langle\partial_k\psi|\psi\rangle\right) \\ =& 4\text{Cov}(\hat{n}_j - \hat{n}_0, \hat{n}_k - \hat{n}_0). \end{aligned} \tag{3}$$

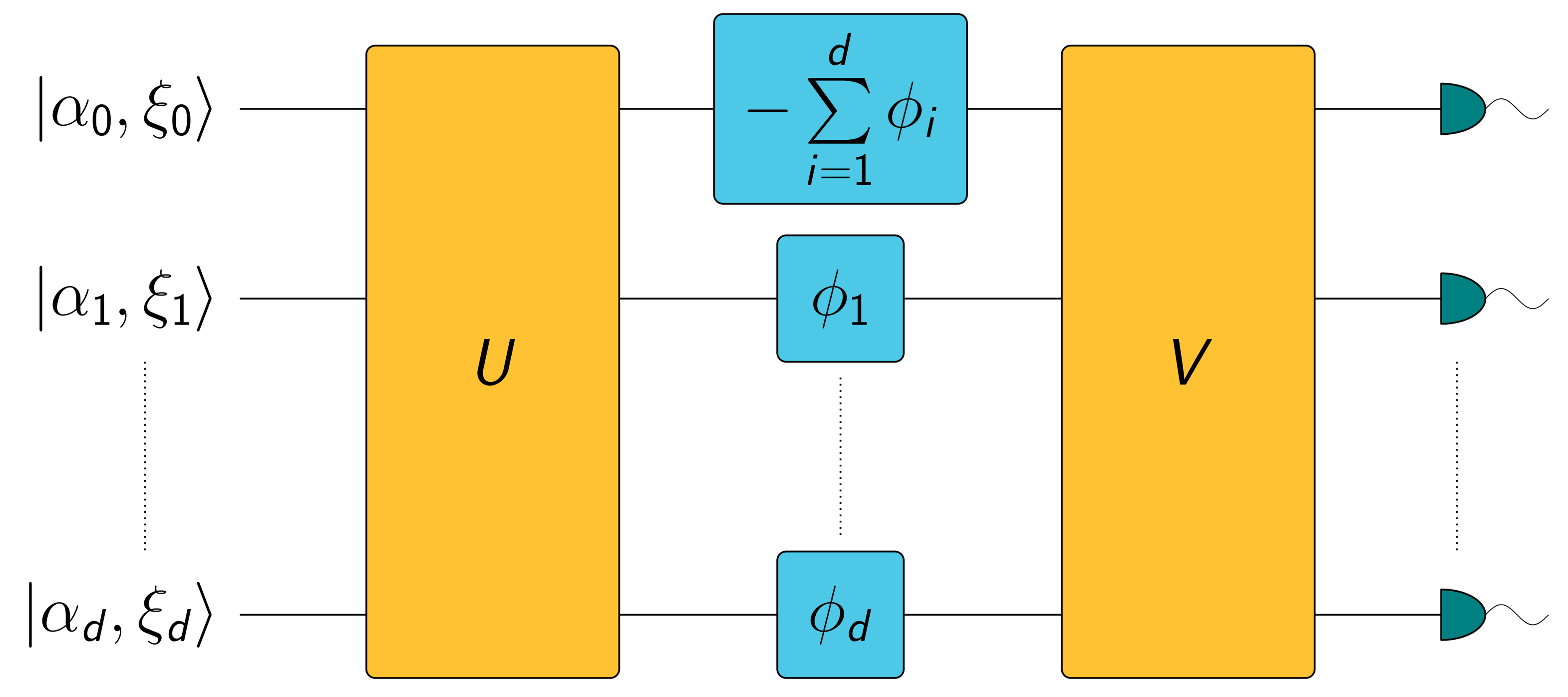
For a QFI, H, the figure of merit used is $\sum_{i=1}^{d}(\Delta\phi_i)^2 \geq \text{Tr}(H^{-1})$.

Individual estimation strategies



Squeezed vacuum $|\zeta,\zeta\rangle$ is the optimal Gaussian state and has a QFI of $H = 2d\sinh^2(2\zeta)$ which obtains the Heisenberg scaling [4].

Simultaneous estimation strategies



The state $U|\alpha_0,\xi_0,\cdots,\alpha_d,\xi_d\rangle$ corresponds to a general pure Gaussian state which then approaches the phases. The unitary V followed by the detectors illustrates an ideal detection scheme.

Calculating the QFI

The covariance matrix of the number operators $\text{Cov}(\{\hat{n}_0,\cdots,\hat{n}_d\})$ can be constructed and then input to Eq. (3) to form the QFI. Under assumptions of $U \in O(d+1)$ and $\xi_j = \xi, \forall j$ to simplify the expressions the mathematics reduces to [1]

$$\text{Tr}(H^{-1}) = \sum_{i=1}^{d}\frac{1}{h_j} - \left(\sum_{j=0}^{d}\frac{1}{h_j}\right)^{-1}\sum_{j=0}^{d}\frac{1}{h_j^2}, \qquad h_j = 2\sinh^2(2\xi) + 4\left(e^{2\xi}\left[\text{Re}(U\vec{\alpha})\right]_j + e^{-2\xi}\left[\text{Im}(U\vec{\alpha})\right]_j\right). \tag{4}$$

Optimising the QFI

We find that $\text{Tr}(H^{-1})$ is minimised by $\vec{\alpha} \to 0$ which gives the bound

$$\text{Tr}(H^{-1}) = \frac{d^2(d+1)}{8E_{\text{Tot.}}(E_{\text{Tot.}} + d + 1)}. \tag{5}$$

The optimal state is thus squeezed vacuum, as is the case for single phase estimation [4].

Comparisons

Versus repeated individual estimation the improvement at fixed $E_{\text{Tot.}}$ is

$$\frac{\text{Tr}(H_{\text{Sim.}}^{-1})}{\text{Tr}(H_{\text{Ind.}}^{-1})} = \frac{(d+1)(E_{\text{Tot.}} + 2d^2)}{2d(E_{\text{Tot.}} + d + 1)}, \tag{6}$$

in the high energy regime this is bound by

$$\lim_{E_{\text{Tot.}}\to\infty}\frac{\text{Tr}(H_{\text{Sim.}}^{-1})}{\text{Tr}(H_{\text{Ind.}}^{-1})} = \frac{d+1}{2d}. \tag{7}$$

Conclusions

For single phase estimation Gaussian and fixed number states perform comparably, yet a limitation of Gaussian states appears to emerge at the multiple parameter level. This provides a first step towards understanding multiple phase estimation with Gaussian states in realistic scenarios with noise and practical measurement schemes.

References

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