The Nonlinear Dynamics of Flexural Ultrasonic Transducers

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Research Overview

- The flexural ultrasonic transducer (FUT) is a unimorph for operation in different fluids such as air and water
- Piezoelectric or electromagnetic
- Proximity sensing and NDE
- Development of high-frequency FUTs for hostile environments

**Objective:** Examine the nonlinear dynamics of different FUTs

<table>
<thead>
<tr>
<th>Application</th>
<th>Example Pressure (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential gas meters</td>
<td>2</td>
</tr>
<tr>
<td>Domestic water meters</td>
<td>20</td>
</tr>
<tr>
<td>Industrial gas meters</td>
<td>300</td>
</tr>
<tr>
<td>Industrial flow meters</td>
<td>300+</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Environment</th>
<th>Example Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil production</td>
<td>120</td>
</tr>
<tr>
<td>District heating</td>
<td>250</td>
</tr>
<tr>
<td>Petrochemical</td>
<td>350-450</td>
</tr>
<tr>
<td>Power plants</td>
<td>560</td>
</tr>
</tbody>
</table>

Section-view Schematic of a Classical Flexural Ultrasonic Transducer

- Piezoelectric Ceramic Disc
- Epoxy Resin Bond Layer
- Membrane
- Cap
- Silicone-type Backing Layer
Dynamic Characteristics of the FUT

(0,0) Mode

(0,1) Mode

(1,0) Mode

(1,1) Mode

Resonant ring-down

Steady-state

Edge-clamped Boundary Condition

Build-up towards steady-state

Steady-state

Resonant ring-down

Slightly off-resonance response

Time

Amplitude

FUT

F
Dynamic Nonlinearity

\[ M\ddot{x} + C\dot{x} + Kx = F(t) = B\cos\Omega t \]

- **Governing Equation**
- **Drive Frequency**
- **Resonance Frequency**
- **Amplitude**
- **Damping Term**
- **Stress Coefficients**
- **Perturbation Parameter**

- **Linear Case**

- **Nonlinear Case (Duffing-type)**

**Causes of nonlinearity**
- Geometric (boundary condition)
- Material (damping)
- Actuation mechanism
- Temperature/piezoelectric

**Why is this important?**
- Stability of dynamic response
- Reliability in industry application
- Optimised performance

Experimental Method

- Silicone backing layer
- LDV Controller
- Function Generator
- LabVIEW V.I.
- Oscilloscope
- Amplifier
- Perpendicular Focus of Laser Beam at Membrane Centre
- Polytec OFV-5000

Materials:
- Small Aluminium FUT
- Large Aluminium FUT
- Brass FUT
- Titanium FUT

Types:
- Commercial
- Custom-made
Mode Shape Measurements

Brass FUT, (0,0) Mode

Titanium FUT, (0,0) Mode

Small Aluminium FUT, (1,0) Mode

Large Aluminium FUT, (1,0) Mode

Titanium FUT, (1,0) Mode

Frequencies:
- 37.50 kHz (Brass FUT, (0,0) Mode)
- 38.76 kHz (Large Aluminium FUT, (1,0) Mode)
- 21.20 kHz (Titanium FUT, (0,0) Mode)
- 93.40 kHz (Titanium FUT, (1,0) Mode)
Primary Resonance Solution

\[ x = b \cos(\omega t - \gamma) + \frac{1}{2} \varepsilon \alpha_2 \omega_0^{-2} b^2 \left[ -1 + \frac{1}{3} \cos(2\omega t - 2\gamma) \right] + O(\varepsilon^2) \]

- Softening nonlinear response
- Asymmetry between maximum and minimum in response
- Phase shift as a function of excitation amplitude
- Linear relationship between vibration response and excitation amplitude
FUTs in Continuous Mode

\[ b = \frac{1}{\sigma} \left( \frac{9\alpha_3 \omega_0^2 - 10\alpha_2^2}{24\omega_0^3} a^3 - \frac{B}{2\omega_0} \cos \gamma \right) \]

Softening identified: \( 9\alpha_3 \omega_0^2 < 10\alpha_2^2 \)
FUTs in Continuous Mode

Small Al, (1,0) Mode

\[ \gamma = 56.6^\circ \quad \rightarrow \quad \gamma = 64.5^\circ \]

\[ \mu b = \left( \frac{B}{2\omega_0} \right) \sin \gamma \]

\[ b = \frac{1}{\sigma} \left( \frac{9\alpha_3 \omega_0^2 - 10\alpha_2^2}{24\omega_0^3} b^3 - \frac{B}{2\omega_0} \cos \gamma \right) \]

Drive Response

Asymmetry and phase shift

Log-Linear Vibration Velocity (mm/s/√V)

Peak-to-Peak Excitation Voltage (V)

\[ \frac{b}{B} = \frac{1}{2\omega_0} \]

Linearity between \( b \) and \( B \)
Nonlinearity in Resonant Decay

Aluminium FUT, Nominal 40 kHz Resonance in the (0,0) Mode

Fluctuations are likely signal processing artifacts
Analysis on a Set of FUTs

- Aluminium FUT, (0,0) mode
- Five transducers, nominally identical

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<tr>
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<th>Electrical Impedance Analysis</th>
<th>Laser Doppler Vibrometry</th>
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<tbody>
<tr>
<td></td>
<td>Coupling Coefficient $k^2$</td>
<td>Quality Factor $Q_M$</td>
</tr>
<tr>
<td>1</td>
<td>0.33</td>
<td>71.01</td>
</tr>
<tr>
<td>2</td>
<td>0.32</td>
<td>56.13</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>56.71</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>54.17</td>
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<tr>
<td>5</td>
<td>0.32</td>
<td>49.75</td>
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<tr>
<td>Mean</td>
<td>0.322</td>
<td>57.55</td>
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<tr>
<td>Standard Deviation</td>
<td>0.007</td>
<td>7.16</td>
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Summary and Future Research

- Nonlinear behaviour can manifest from boundary condition, operating mode, excitation amplitude, FUT cap material, piezoelectricity
- Although individual causes of nonlinearity are difficult to isolate, it is closely related to stiffness properties
- Experimental data consistent with mathematical theory
- Investigate alternative FUTs in future, such as electromagnetic-driven devices

<table>
<thead>
<tr>
<th>Transducer and Associated Operating Mode</th>
<th>Frequency Reduction (Hz) 20 $V_{p.p}$ to 40 $V_{p.p}$</th>
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</thead>
<tbody>
<tr>
<td>Small Aluminium FUT: (1,0) Mode</td>
<td>200</td>
</tr>
<tr>
<td>Large Aluminium FUT: (1,0) Mode</td>
<td>200</td>
</tr>
<tr>
<td>Brass FUT: (0,0) Mode</td>
<td>300</td>
</tr>
<tr>
<td>Titanium FUT: (0,0) Mode</td>
<td>400</td>
</tr>
<tr>
<td>Titanium FUT: (1,0) Mode</td>
<td>1400</td>
</tr>
</tbody>
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I would like to thank Jonathan Harrington of the University of Warwick for valuable technical contributions to this investigation, and acknowledge the Engineering and Physical Sciences Research Council (EPSRC) Grant Number EP/N025393/1 for funding this research.