Neutrino Physics - Exam

1. (a) Describe the Water Cerenkov detection method that Super-Kamiokande uses to measure the products of neutrino interactions. What information can be obtained from such an experiment that a radiochemical experiment cannot.

   (b) A 10.98 GeV neutrino interacts with a proton at rest in Super-Kamiokande. The interaction products are a charged muon with energy of 9.3 GeV, a proton with energy 0.99 GeV and a charged pion with energy 1.57 GeV. The the mass of a muon is 0.105 GeV, the mass of a proton is 0.938 GeV and the mass of a charged pion is 0.139 GeV. Given that the refractive index of water is 1.33, state which of these particles can be detected by the Super-Kamiokande detector.

   (c) Elastic scattering of a neutrino from an electron is the main detection technique used by Super-Kamiokande to detect solar neutrinos. Suppose 22,000 events were observed by Super-Kamiokande in 1500 days (and nights) in 22 kton of water. Given an average neutrino cross section of $10^{-44} cm^{-2}$, what is the observed neutrino flux.

   (d) Describe the Solar Neutrino Problem, and discuss the role of the SNO experiment in its resolution.

2. (a) In the two neutrino flavour formalism, the neutrino mass eigenstates ν_1 and ν_2 are related to the the flavour states ν_α and ν_β by a 2×2 mixing matrix

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad (1)$$

   where θ is the mixing angle. Suppose a neutrino of flavour α is generated at time $t = 0$ and starts propagating along the x-axis. The mass states evolve during propagation in a vacuum according to

$$\nu_k(t, x) = \nu_k(0, 0)e^{i\phi_k(t,x)} \quad (2)$$

   with $k = 1, 2$ and where the phase $\phi_k(t, x)$ is defined to be $\phi_k(t, x) = E_k t - p_k x$.

   - Using this information, show that the survival probability

$$P(\nu_\alpha \to \nu_\alpha) = 1 - sin^2(2\theta)sin^2(\frac{\phi_1 - \phi_2}{2}) \quad (3)$$

   - Hence show that, if the mass of the neutrino states are much less than the neutrino energy ($m_k << E_k$), that the oscillation probability can be expressed by

$$P(\nu_\alpha \to \nu_\beta) = sin^2(2\theta)sin^2(\frac{\Delta m^2 L}{4E_\nu}) \quad (4)$$

     where L is the distance the neutrino has travelled, and $\Delta m^2 = m_1^2 - m_2^2$.

   - The oscillation probability is usually written as

$$P(\nu_\alpha \to \nu_\beta) = sin^2(2\theta)sin^2(\frac{1.27\Delta m^2 L}{E_\nu}) \quad (5)$$

where Δm^2 is measured in units of eV, L is measured in units of km, and E_ν is measured in units of GeV. Suppose that the squared mass splitted were known to $\Delta m^2 = 0.35eV^2$. If a detector is placed 120 km away from the source, what energy of neutrinos would built the accelerator to produce to be maximally sensitive to neutrino oscillations.

3. (a) The Homestake experiment pioneered the field of solar neutrino detection and measured the solar neutrino flux for about 30 years. Describe briefly the experiments and discuss the merit of radiohemical solar neutrino detection.

   (b) The observed rate of interactions was 2.56 ± 0.23 SNU (1 SNU = 10^{-36} interactions per target atom per second). Given an absorption cross section on ^{37}Cl of $\sigma = 10^{-42}\text{cm}^2$ compute the measured flux.

   (c) Given a predicted rate of 8.5 ± 1.8 SNU, compute the oscillation probability of an electron neutrino to a neutrino of a different flavour. Assuming oscillation in vacuum between two neutrino species and a mixing angle of $sin^2(2\theta) = 0.86$, what value of Δm^2 does Homestake suggest? The current best fit to solar neutrino data is $\Delta m^2_{12} = 7 \times 10^{-5}\text{eV}^2$. Compare this value to the one you just computed, and suggest reasons for any discrepancy.

4. (a) Define the terms *helicity* and *chirality*. Under what conditions are these two operators identical?

   (b) The operators $P_L = \frac{1}{2}(1-\gamma_5)$ and $P_R = \frac{1}{2}(1+\gamma_5)$ project the left- and right-handed chiral states out of a Dirac spinor.

   - Show that these two operators have the right properties to be projection operators. Specifically, show that $P^2_{L,R} = P_{L,R}$ and $P_L P_R = P_R P_L = 0$.
   - Show that the adjoint spinors $\overline{\psi_L}$ and $\overline{\psi_R}$ corresponding to the left- and right-handed chiral components $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$ are

$$\overline{\psi_L} = \overline{\psi}\frac{1}{2}(1+\gamma_5) \tag{6}$$

     and

$$\overline{\psi_R} = \overline{\psi}\frac{1}{2}(1-\gamma_5) \tag{7}$$

   - Hence, or otherwise, show that the Dirac current $\overline{\psi}\gamma^\mu\psi$ can be written as $\overline{\psi}\gamma^\mu\psi = \overline{\psi_L}\gamma^\mu\psi_L + \overline{\psi_R}\gamma^\mu\psi_R$.

   (c) Explain what is meant by the statement that charged current weak interactions have a V-A structure. What constraints does this impose on the helicity and chirality of neutrinos, which can only be produced in charged current weak interactions?

   (d) The pion is a spin-0 particle. Consider the decays

$$\pi^+ \rightarrow \mu^+\nu_\mu \tag{8}$$

   and

$$\pi^+ \rightarrow e^+\nu_e \tag{9}$$

. Experimentally it is found that whilst the first decay happens 99.99% of the time, the second occurs only 0.01% of the time. This is very strange as, up to mass thresholds, it is thought that muons and electrons behave the same way (this is called lepton universality), so each decay should happen closer to 50% of the time. Explain this apparent discrepancy, with reference to the helicity of the final state particles and without detailed mathematics.

5. (a)
   - What was the Atmospheric neutrino problem? How was it resolved.
   - Describe briefly the main components of an accelerator based neutrino beam.
   - Describe the difference between a wide-band and off-axis neutrino beam.

   (b) The neutrino burst detected from supernova SN1987A provided a lot of information about neutrinos.

   - Assuming that one of the SN1987A neutrinos was emitted at time t_0, show that the flight time of this neutrino to earth is given by

$$t_F \sim L\left(1 + \frac{m_\nu^2}{2E^2}\right) \tag{10}$$

     where m_ν is the mass of the neutrino, E is the neutrino energy
   - The time difference between two neutrinos arriving at earth from SN1987A can be expressed by

$$\Delta t = t_2 - t_1 = \Delta t_0 + \frac{Lm_\nu^2}{2}\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right) \tag{11}$$

     If the neutrino pulse was 10seconds long, obtain an estimate for the neutrino mass assuming that the supernova was 163,400 light years from Earth. Assume $\Delta t_0 = 0$.

   (c) In a conventional neutrino beam, neutrinos are generated from the decay (mostly) of high energy charged pions. Suppose that, in the laboratory frame of reference, a meson has energy E_M and is travelling parallel to the x-axis. Show that the energy E_ν and angle $cos(\theta_\nu)$ of the neutrino produced in the decay of this meson is related to the energy $E_\nu^{"*"}$ and angle, $cos(\theta_\nu^{"*"})$ of the neutrino in the meson rest frame by

$$E_\nu = \gamma E_\nu^*(1 + \beta cos\theta^*) \tag{12}$$

   and

$$cos(\theta_\nu) = \frac{cos\theta^* + \beta}{1 + \beta cos\theta^*} \tag{13}$$

   where $\gamma = \frac{E_M}{m_M}$ and $\beta = \frac{p_M}{E_M}$ where m_M is the mass of the meson. You will need

   to use the Lorentz transformations. The energy and momentum of a particle in an inertial frame S' which is moving with velocity $\mathbf{v}$ with respect to an inertial frame S are related to the energy and momenta of that particle in S by the

Lorentz transformation. If we assume that the S' is moving with respect to S along a common x/x' axis then the transformation is given by

$$\begin{pmatrix} E' \\ cp'_x \\ cp'_y \\ cp'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ cp_x \\ cp_y \\ cp_z \end{pmatrix} \tag{14}$$

where $\beta = \frac{v}{c} = \frac{p}{E}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

(d) The decay pipe in a particular experiment is 53 metres long. If the pions entering the decay pipe have an energy of 25 GeV, calculate the fraction of mesons which would *not* have decayed by the time they reached the end of the decay pipe. Note that the pion decay time *in the rest frame of the pion* is 2.6×10^{-8} second. You will need to transform this to the laboratory frame.