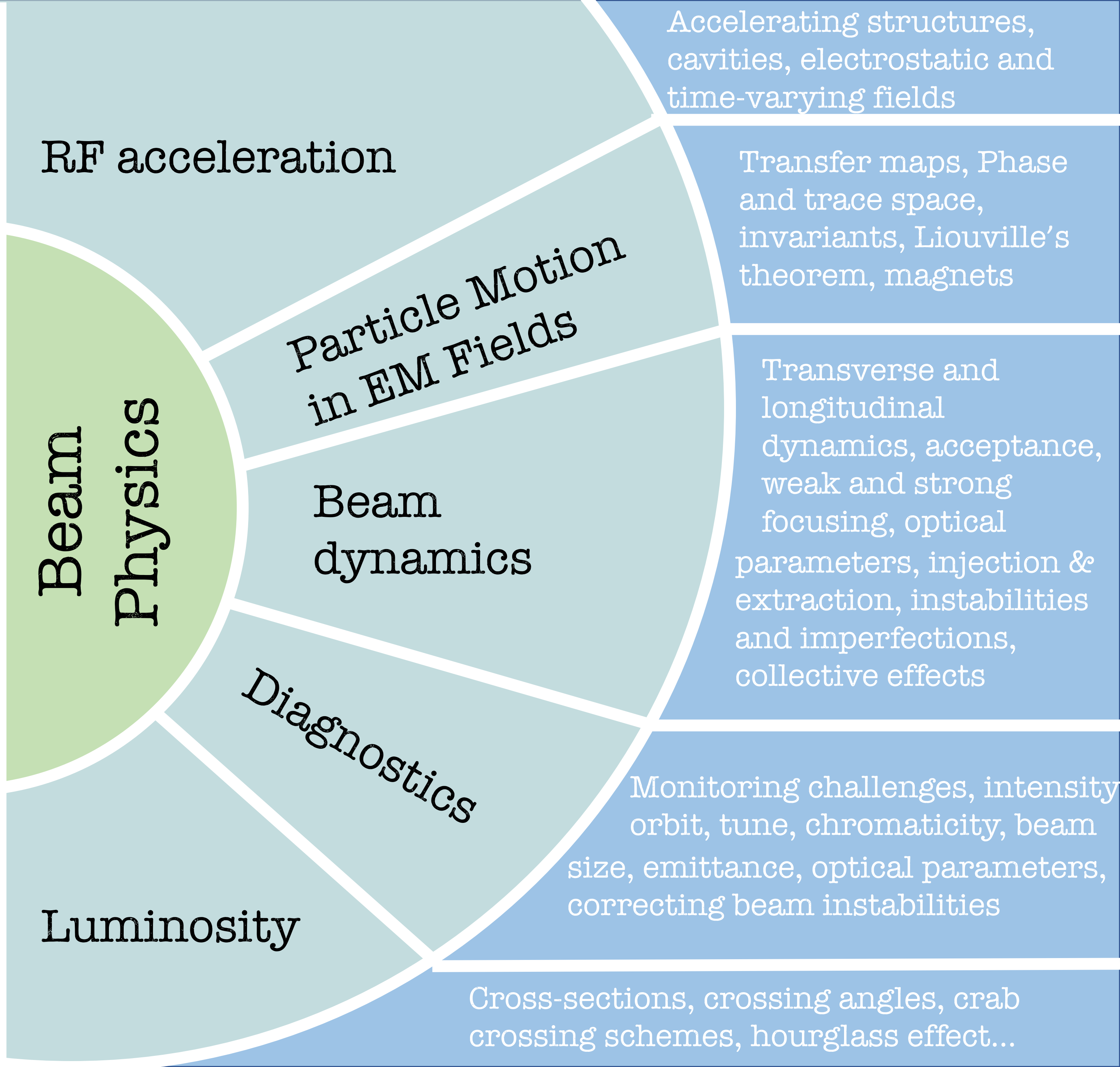
A Beam Physics Primer

V. Blackmore
Imperial College London
v.blackmore@imperial.ac.uk

Contents

- "Beam physics" is a large topic
  - Broad overview of the topic
  - See references at end for more detail
  - CERN Accelerator School (CAS) is an excellent starting point for those who are interested



Contents

- “Beam physics” is a large topic
  - Broad overview of the topic
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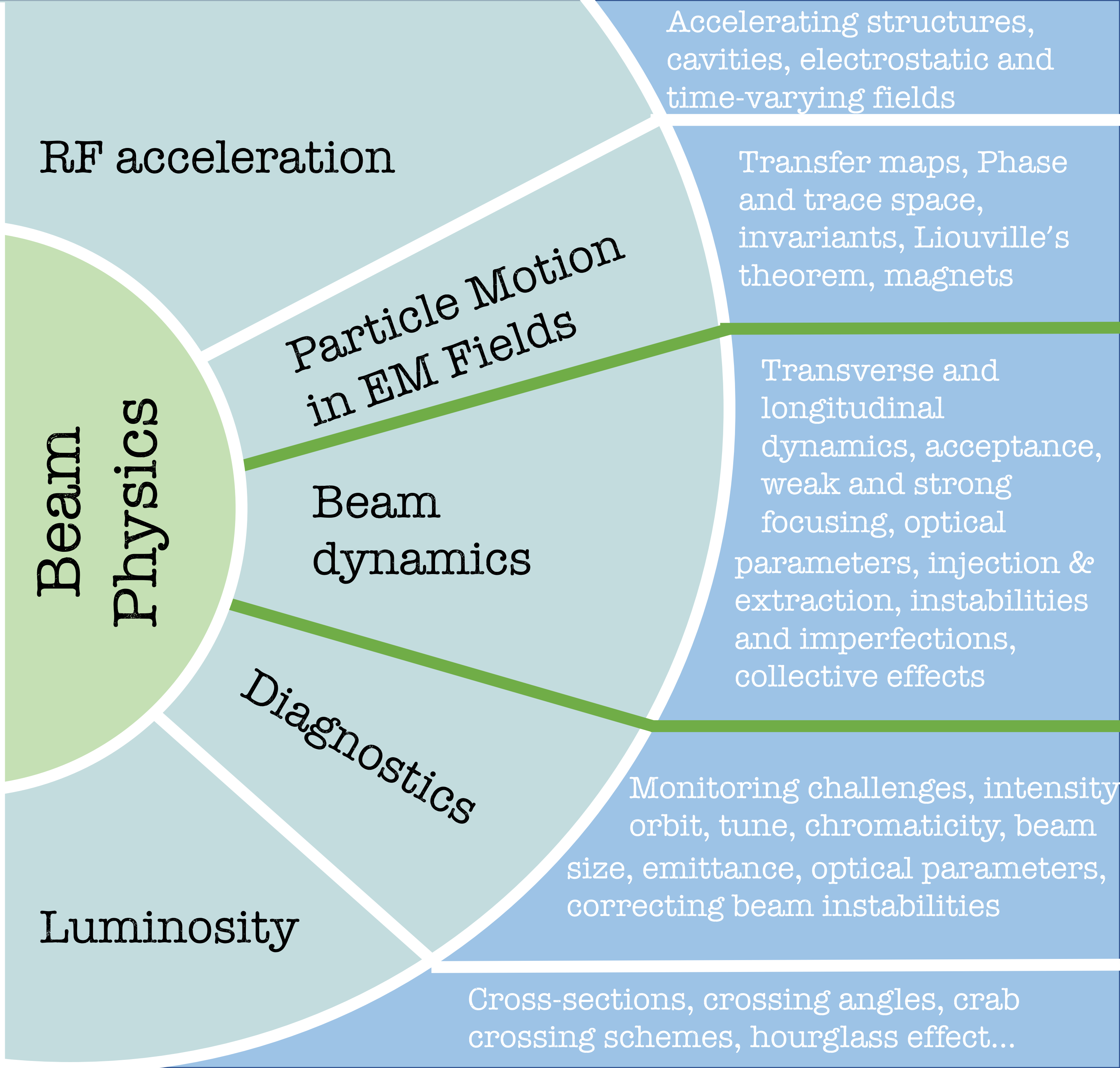
  - CERN Accelerator School (CAS) is an excellent starting point for those who are interested
- Will spend most of our time discussing the *dynamics* of beams

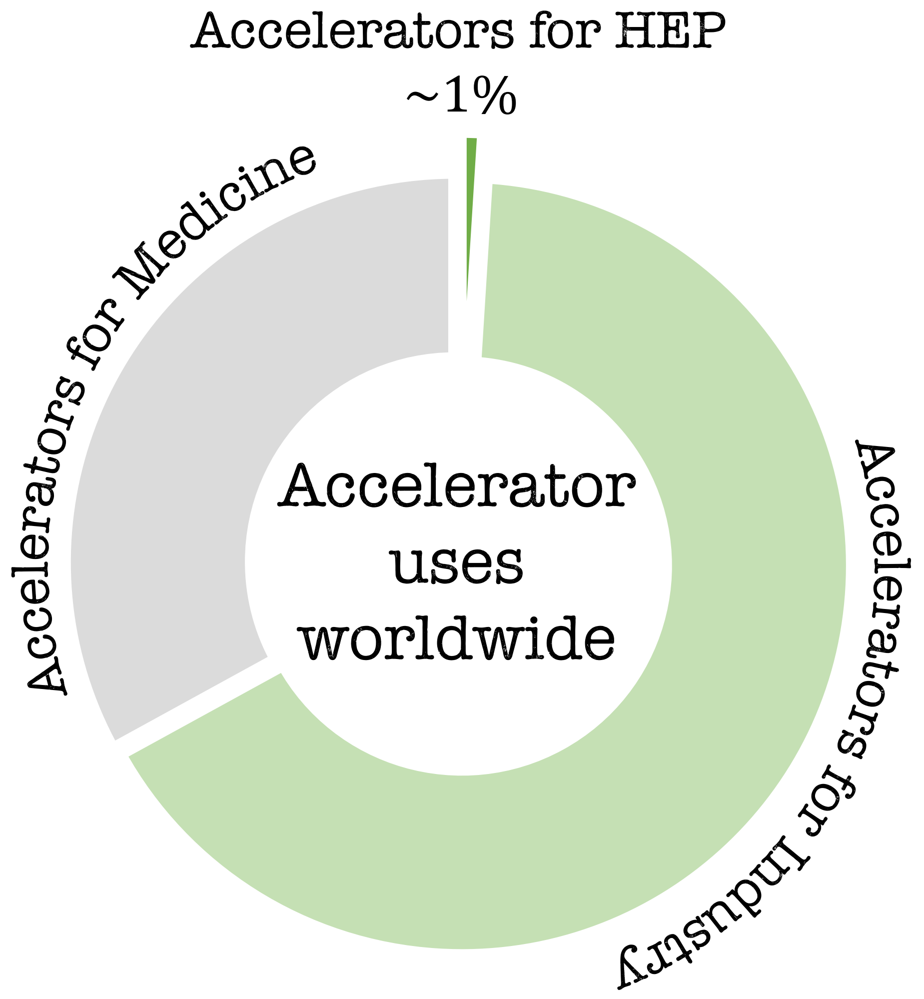


The Basics

A few definitions to help us not lose our way

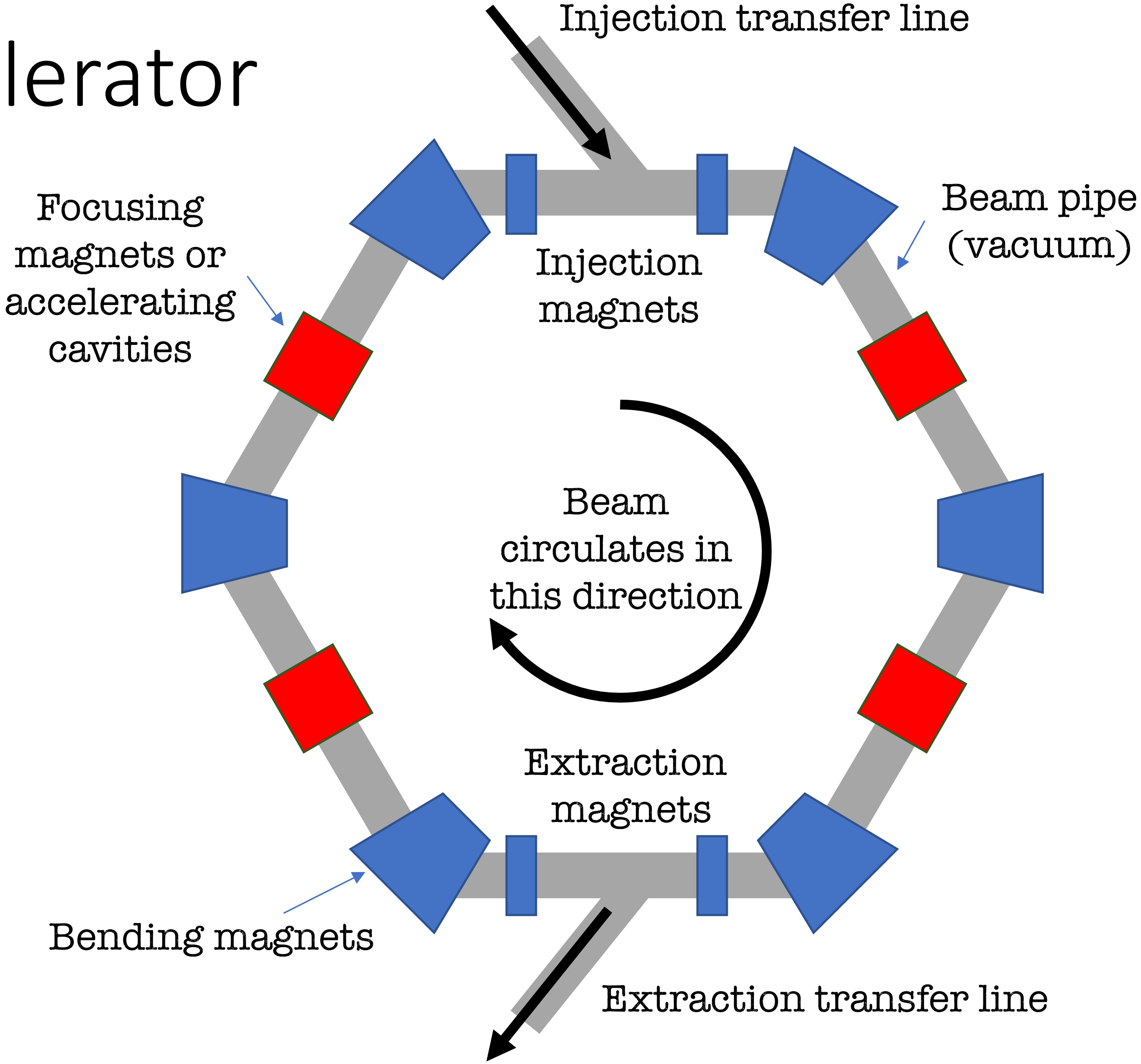
Why do we need accelerators?

- Accelerators provide energy to the particles we study
- HEP isn't the only user of accelerators
  - Industry and medicine are the majority user
- Increasing the particle energy, reduces the wavelength ($E = {}^{hc}/_{\lambda}$)
- Fixed target vs Collider?
  - Fixed target: easier to do, but less energy available for collision processes
  - Collider: more difficult, more available energy
- Beam physics depends on the type of accelerator: linear or circular



Anatomy of an accelerator

- Beam arrives through a transfer line (from another accelerator or source)
  - Injection
  - Magnetic field "kicks" incoming beam into circulating beam
- Beam is accelerated over may turns (in a 'circular' machine)
  - Dipoles!
- Is extracted (to be used elsewhere or for collisions)
  - Magnetic field "kicks" beam outgoing beam into extracting transfer line
  - Leaves through a transfer line

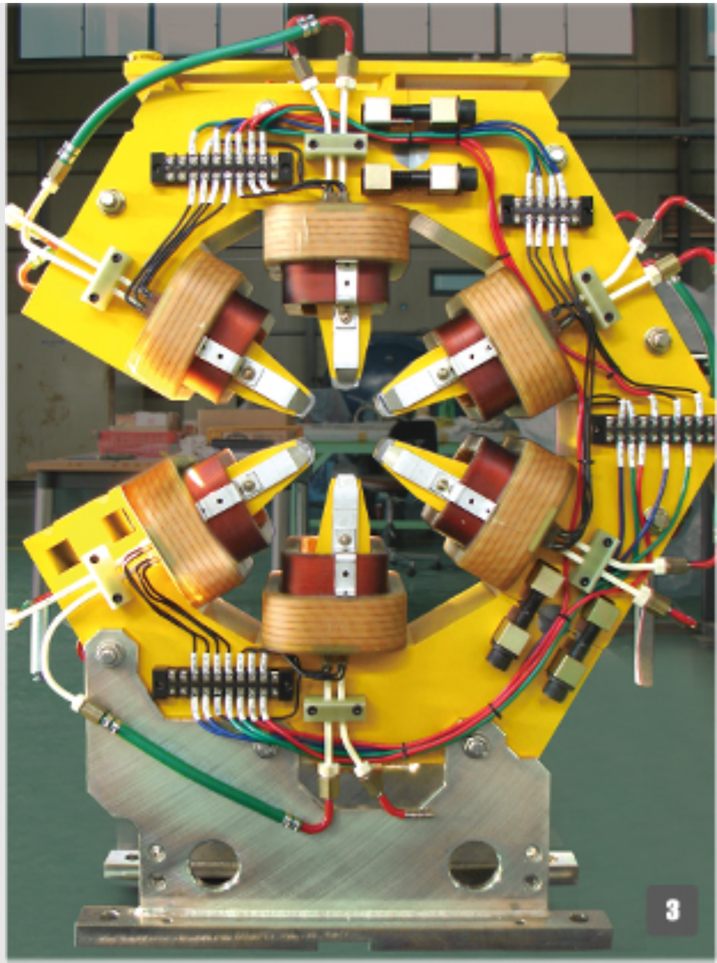


Types of Magnets

Dipole:
Bend beam

Quadrupole:
Focus beam

Sextupole:
Correct beam



Particle Motion

- Particles controlled by magnetic fields
  - Lorentz force: $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$
  - When not accelerating, $\boldsymbol{E} = 0$
  - Equilibrium between centrifugal and Lorentz force → particle path
  - $m\gamma v^2\boldsymbol{\kappa} + e(\boldsymbol{v}\times\boldsymbol{B}) = 0$
    - where $\boldsymbol{\kappa} = (\kappa_x, \kappa_y, 0)$
    - $\kappa_{x,y} = {}^{1}/_{\rho_{x,y}}$
    - $\rho_{x,y}$ is the bending radius of the trajectory
  - Then solve equations of motion for particle in some field

- ***Harmonic oscillator:***
  - $x'' + Kx = 0$
  - $K = k_0 + \kappa_{0x}^2 = \text{ constant}$
- Solution usually expressed in matrix formulation:
  - $\begin{bmatrix} x_2(z) \\ x_2'(z) \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x_1(z) \\ x_1'(z) \end{bmatrix}$
  - $\boldsymbol{M}$ depends on magnetic fields propagated through
- Transfer map → propagates particles through fields

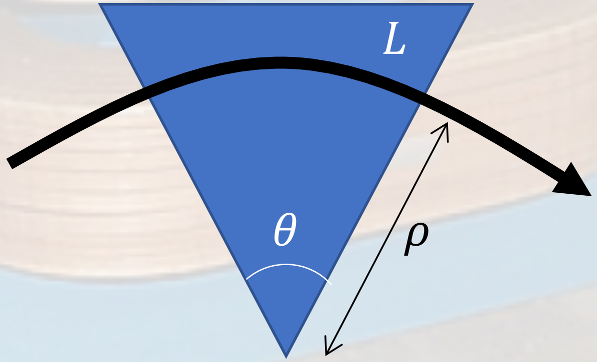
Drift (no magnetic field): $\boldsymbol{M_O} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$

Types of Magnets

Dipole:
Bend beam

$$\boldsymbol{M_B} = \begin{bmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta = \frac{L}{\rho}$$



Quadrupole:
Focus beam

$$\boldsymbol{M_F} = \begin{bmatrix} \cos\psi & \frac{1}{\sqrt{k}}\sin\psi \\ -\sqrt{k}\sin\psi & \cos\psi \end{bmatrix}$$

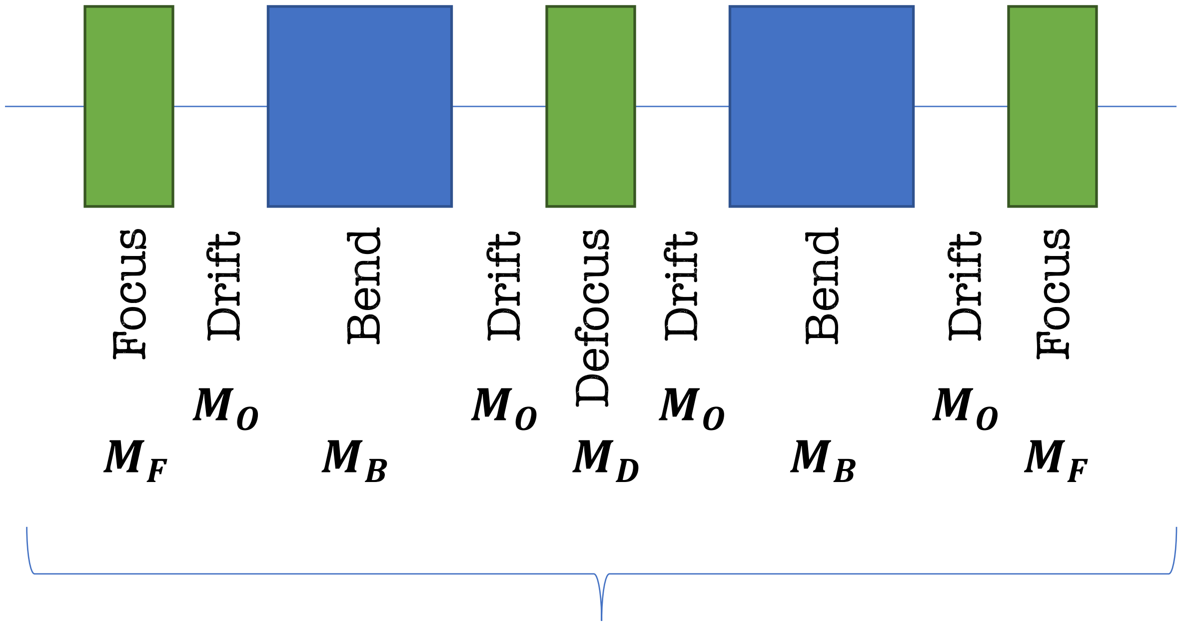
$$\boldsymbol{M_D} = \begin{bmatrix} \cosh\psi & \frac{1}{\sqrt{|k|}}\sinh\psi \\ \sqrt{|k|}\sinh\psi & \mathrm{cohs}\,\psi \end{bmatrix}$$

$$\psi = \sqrt{k}l$$
$$k = \text{quadrupole strength}$$
$$l = \text{quadrupole length}$$

Sextupole:
Correct beam

<u>Even more complicated...</u>

Transfer Maps



$$M_{Total} = M_F M_O M_B M_O M_D M_O M_B M_O M_F$$

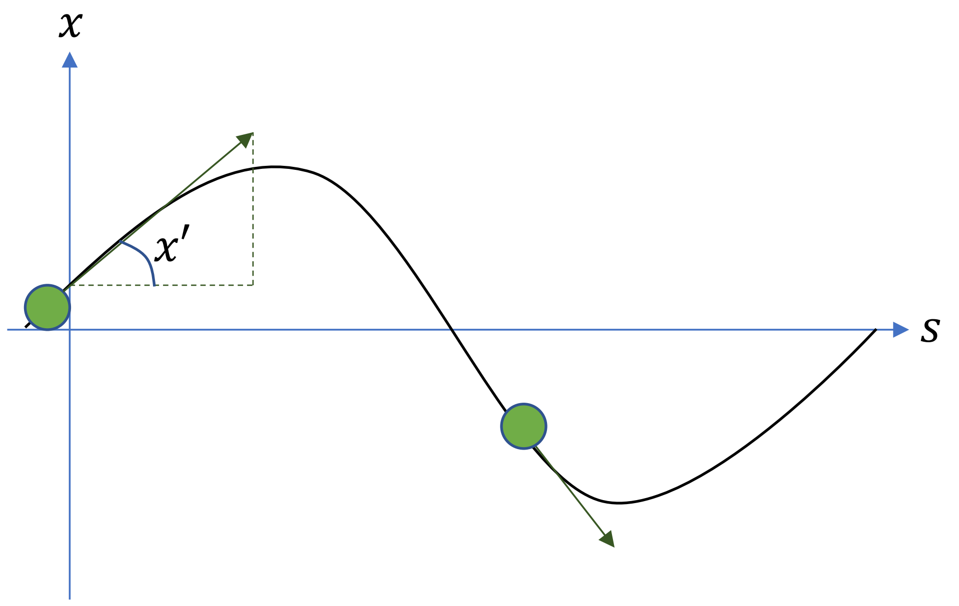
$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = M_{Total} \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

- In each accelerator element, the particle trajectory corresponds to movement like a harmonic oscillator
- Quadrupole magnets focus in **one plane only**, defocus in the other
  - Consider optics: focus-defocus-focus = overall focusing
  - Call this a ***FODO cell***
    - O = drift space

- Weak focusing vs. Strong focusing
- $x'' + (k + \frac{1}{\rho^2})x = 0$
- If $k = 0$ → **weak** focusing
  - ”Less complicated” than using all the cosh’s and sinh’s, but...
  - High energy → large machine
  - Eventually becomes unwieldy
- If $k \neq 0$ → **strong** focusing
  - All machines today use strong focusing

Particle Motion II

- Particles controlled by magnetic fields
  - Design about an idealised orbit, the *reference particle or reference orbit*
  - Particles with different initial conditions, in a homogenous magnetic field, will experience oscillations about this orbit, *betatron oscillations*
- Oscillation of a particle about the reference orbit describes an ellipse in trace space
  - Trace space: $(x, x'), (y, y')$ etc.
  - Phase space: $(x, p_x), (y, p_y)$ etc.



$s =$ trajectory of reference particle

$x =$ displacement of particle from reference trajectory

$$x' = \frac{dx}{ds}$$

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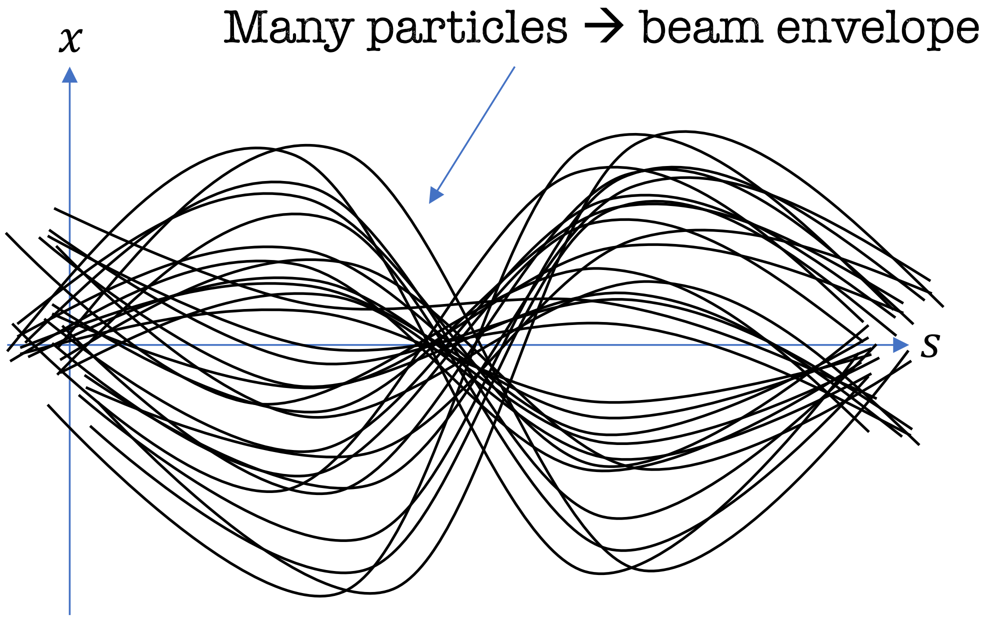

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Particle Motion II

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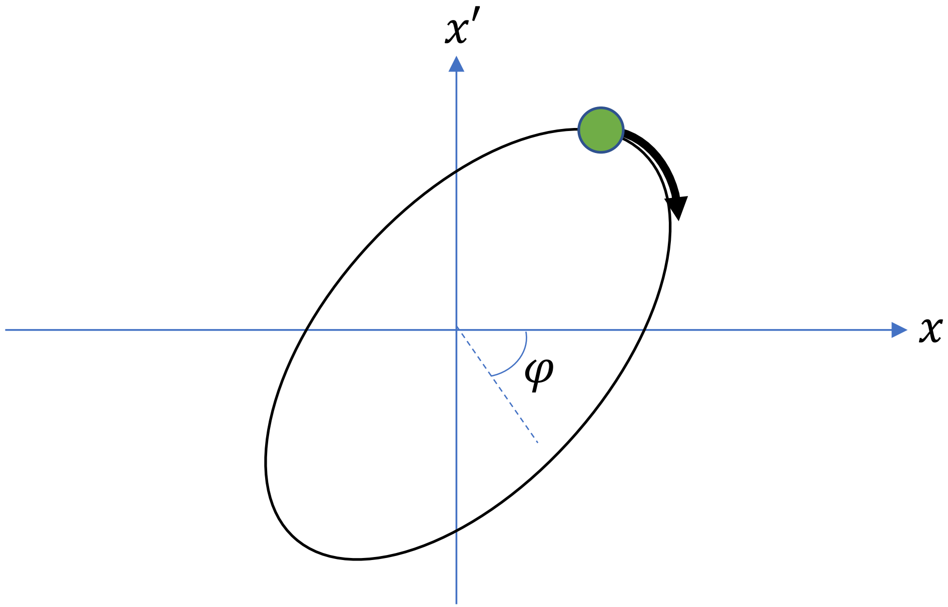


$x =$ displacement of particle from reference trajectory

$$x' = \frac{dx}{ds}$$

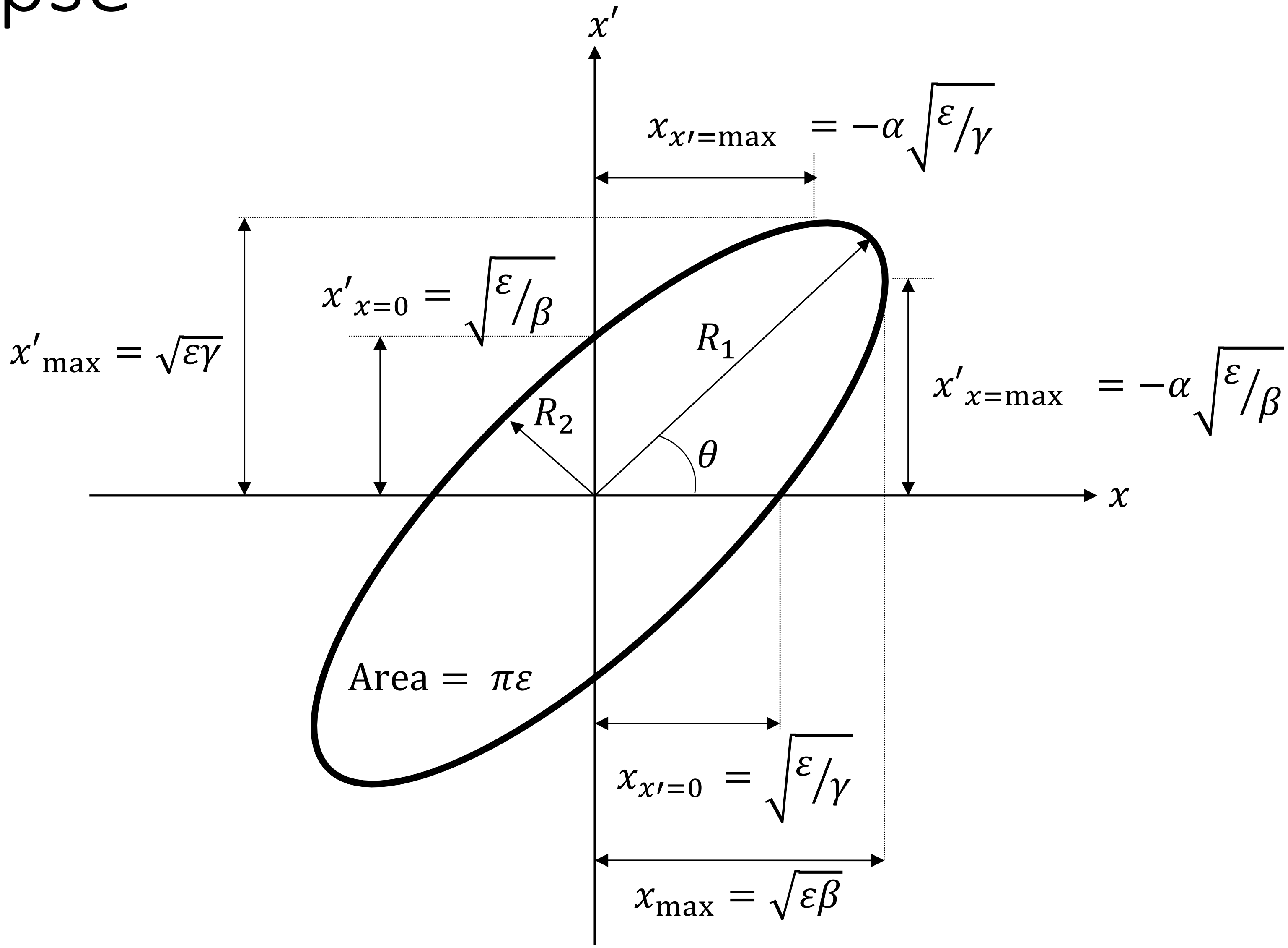
$\varphi = \omega t$, phase angle

Ellipse describes motion of particle in (x, x') space as it travels around the ring.

$$\varepsilon = \gamma\langle x^2\rangle + 2\alpha\langle xx'\rangle + \beta\langle x'^2\rangle$$

The machine ellipse

- ***Machine ellipse***
  - RMS ellipse around all particles in the beam
- ***Twiss parameters***
  - Defined by betatron motion of individual particles
  - $\alpha, \beta, \gamma, \varepsilon$
  - $\varepsilon =$ emittance, a conserved quantity
  - α, β, γ derive from the covariances of the particles
  - $x_{\max} = \sqrt{\varepsilon\beta} \equiv$ beam envelope



$$\varepsilon = \gamma\langle x^2\rangle + 2\alpha\langle xx'\rangle + \beta\langle x'^2\rangle$$

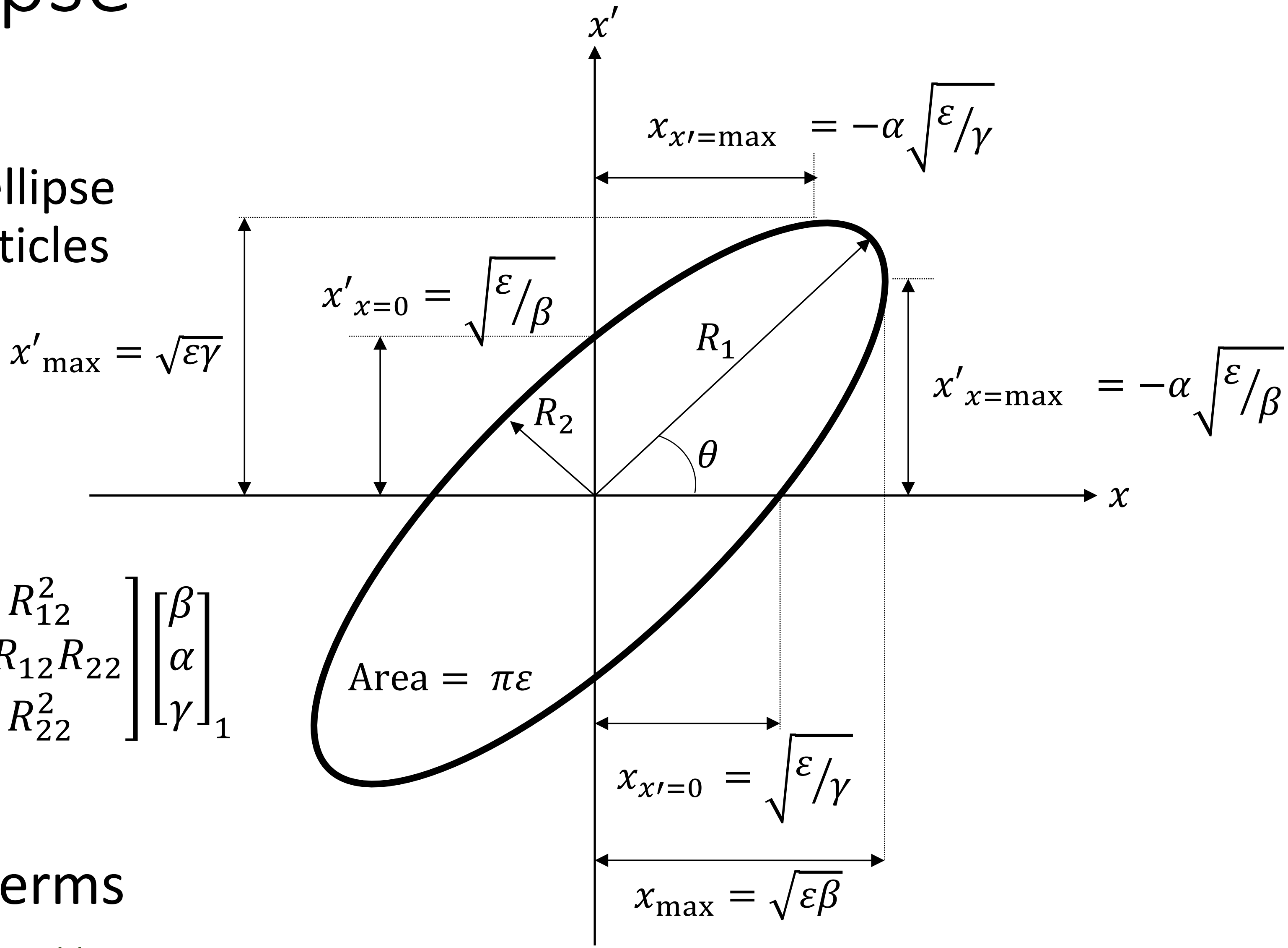
The machine ellipse

- Benefit:
  - Transport the machine ellipse instead of individual particles

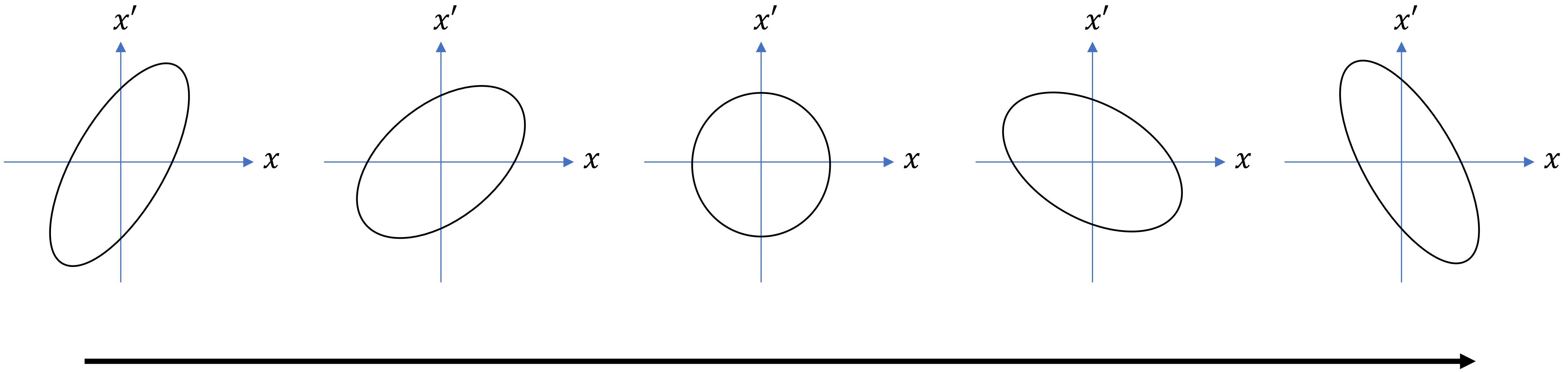
$$M_{1\to 2} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

$$\begin{bmatrix} \beta \\ \alpha \\ \gamma \end{bmatrix}_2 = \begin{bmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -2R_{11}R_{21} & 1 + 2R_{12}R_{22} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \\ \gamma \end{bmatrix}_1$$

- Can even express $\boldsymbol{M}$ in terms of Twiss parameters *(but won't)*



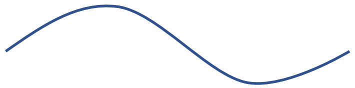
The machine ellipse

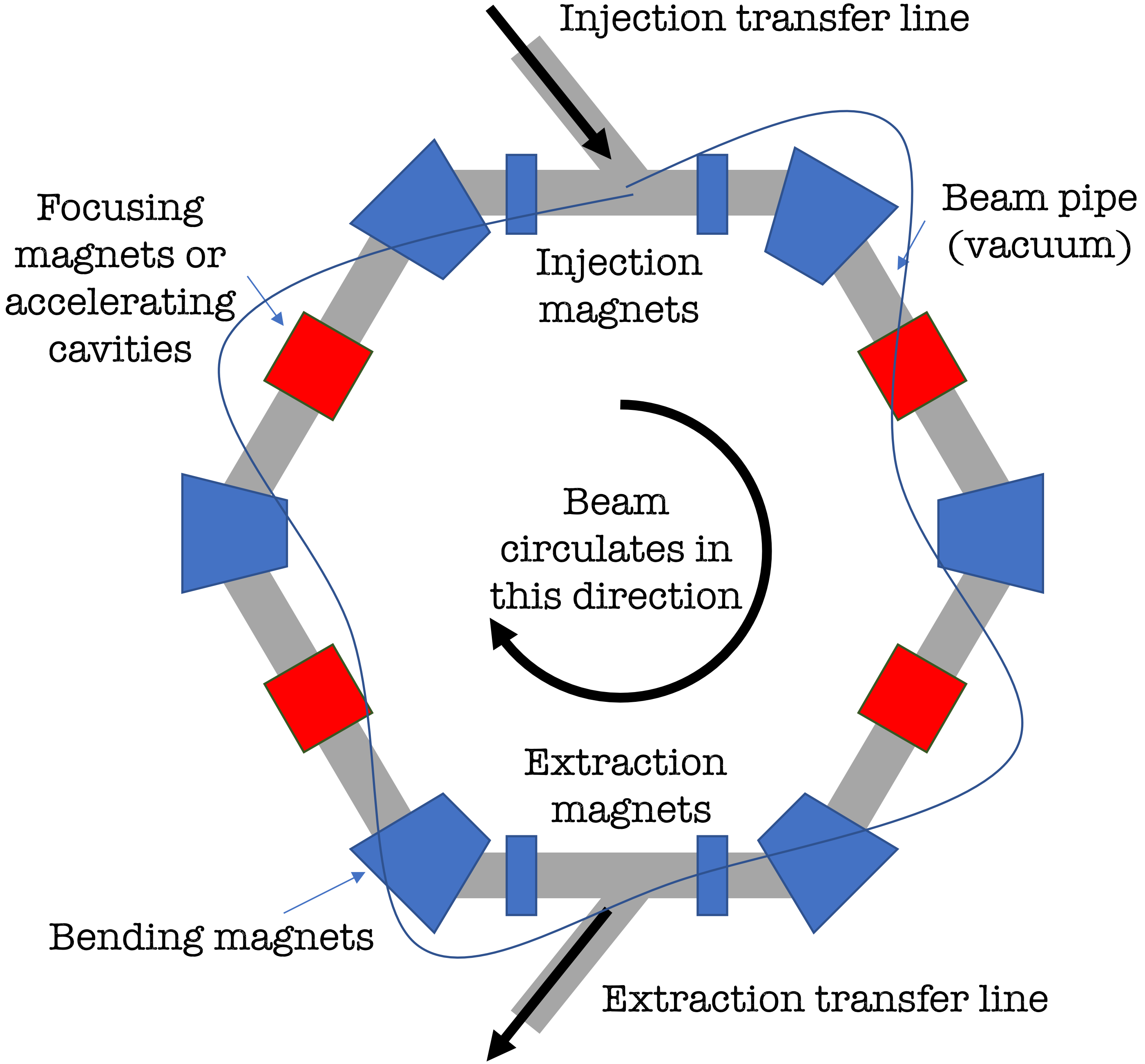


- Machine ellipse rotates as the beam crosses different accelerating structures
- Area of ellipse is conserved →Liouville's theorem
  - Valid as long as the particles are acted on by *conservative* forces

- Transfer maps used to go from *a* → *b*, or to go from *a* → *a* ("one-turn" map)

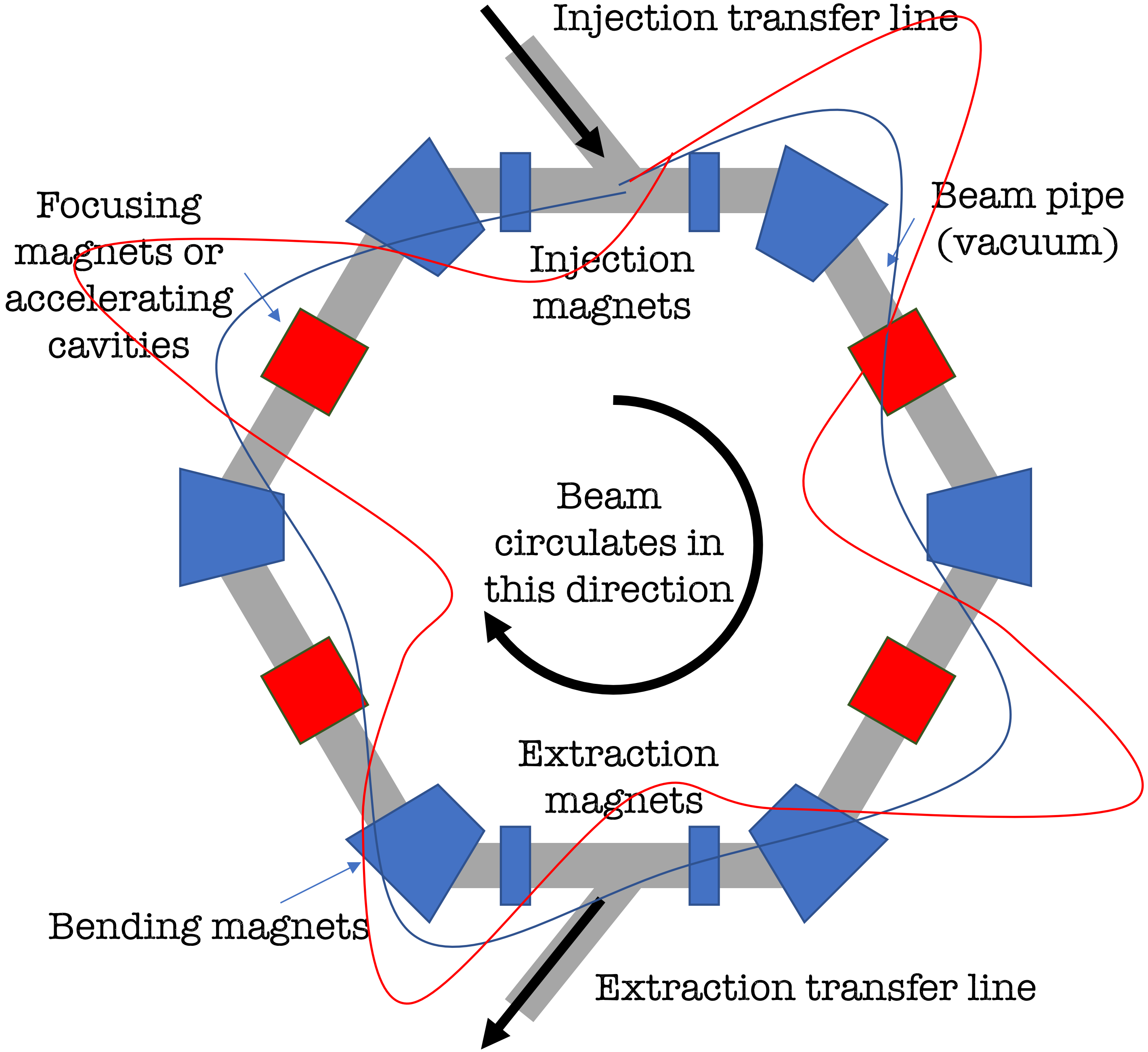
The first instability

- Accelerator structures aren't perfect
  - **Turn one:** 
  - Orbit start matches orbit end



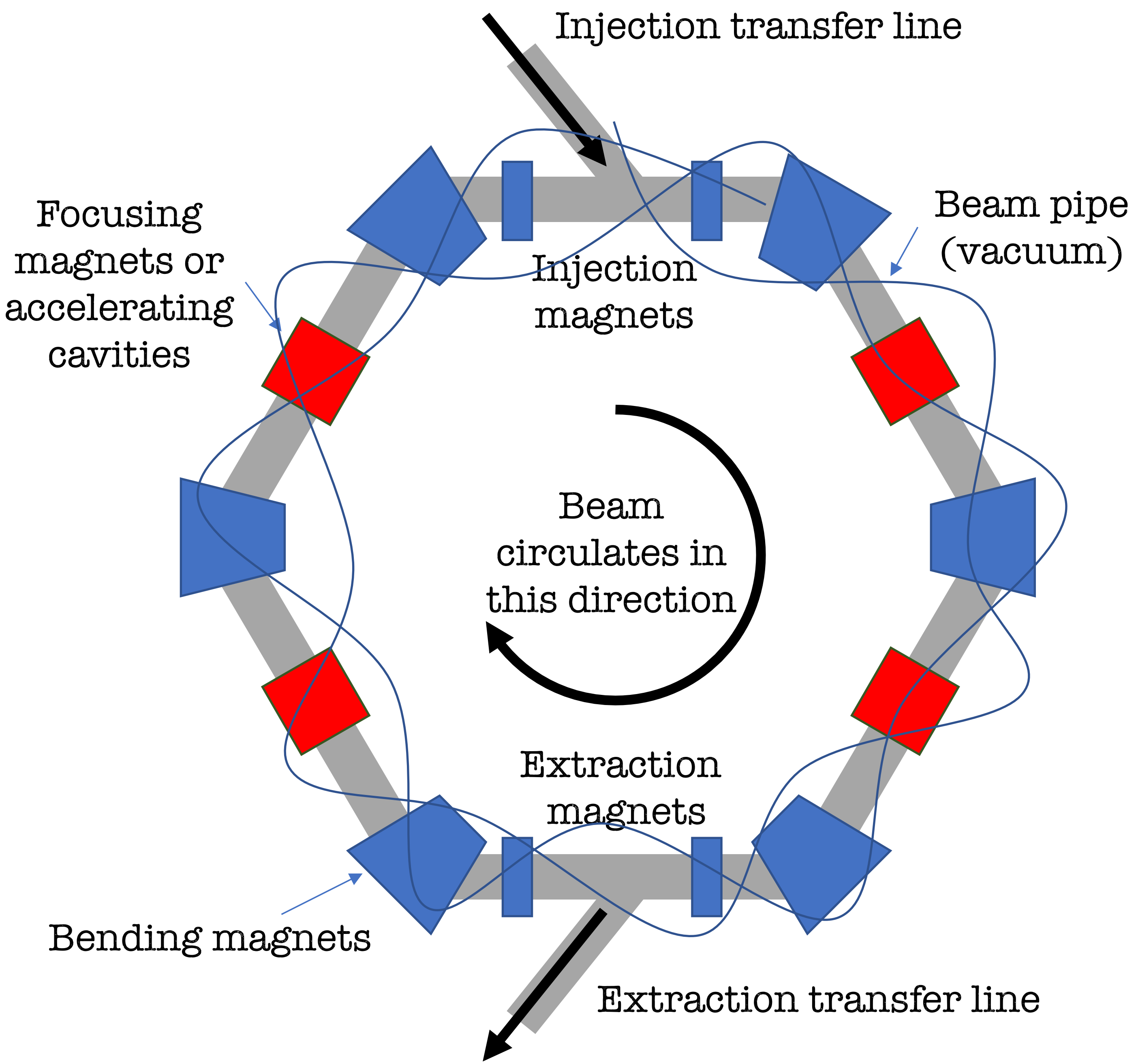
The first instability

- Accelerator structures aren’t perfect
  - **Turn one:**
  - Orbit start matches orbit end
  - **Turn two:**
  - Amplification of errors → Resonance
  - Probably going to lose this beam
    - *Unhappy physicists ☹*



The first instability

- Accelerator structures aren't perfect
    - **Don't** want beam to make an **integer** number of oscillations per turn
- Tune: Oscillations/turn
    - Horizontal, vertical, and longitudinal tunes to worry about!
- Want to avoid resonances
    - Frequency of transverse (betatron) oscillations **must not** be equal to (or an integer multiple of) the revolution frequency



Design your own 'accelerator'

- Know the basic (linear) transfer map through different fields
  - Horizontal and vertical particle motion independent
  - Can track particles from $a \rightarrow b$
  - Can study the machine ellipse
- Avoid resonances
- Emittance, $\varepsilon_{x,y}$, of beam is a constant of motion
  - Smaller emittance → better beam quality
- $\beta_{x,y}$ defines beam envelope or physical size
  - Can use this to check your beam doesn't clip anywhere

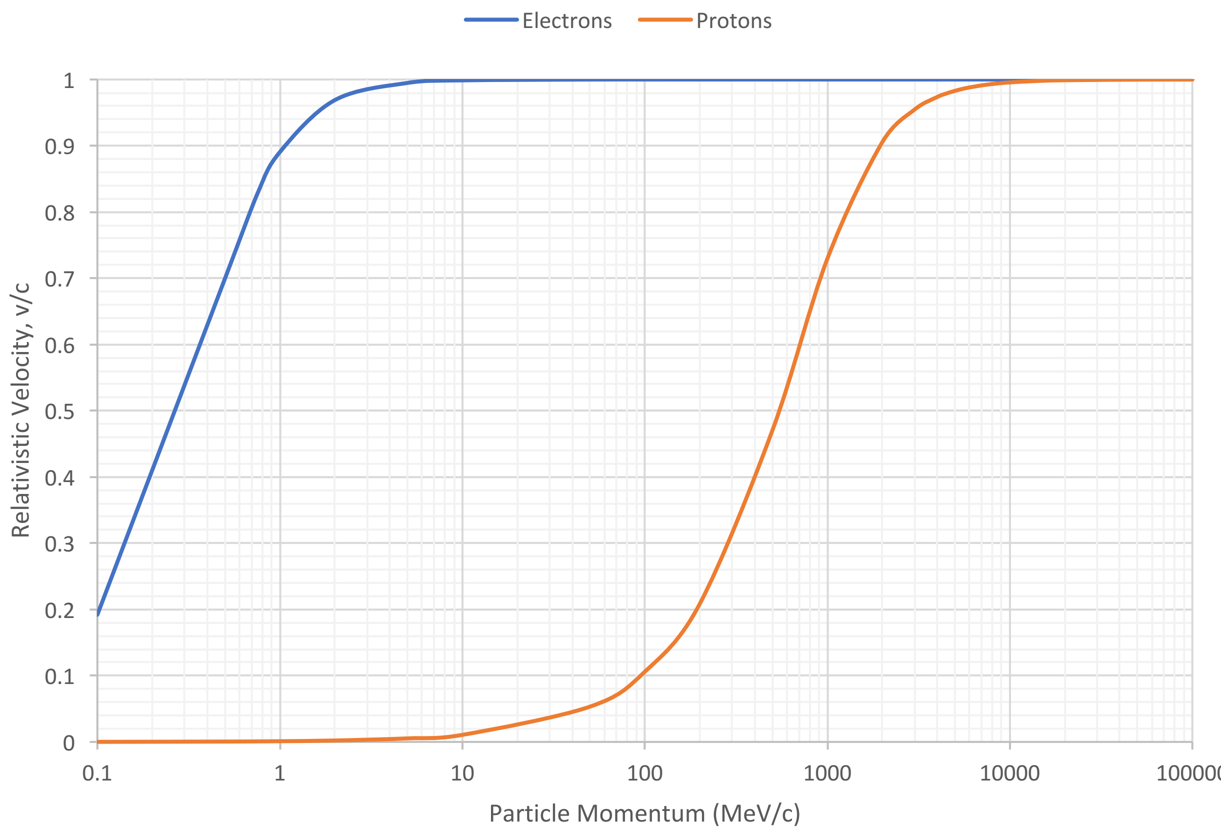
- What magnetic field strengths might you need?
  - Magnetic rigidity, $B\rho = \frac{p}{q}$
  - $p =$ momentum
  - $\rho =$ radius of curvature
  - High momentum → large $B\rho$
  - High momentum → large machine radius
- Machine size is dictated by desired beam energy and magnet capability

Accelerating particles

Have the basics of beam control, but can we also accelerate?

Particle differences

- Electrons and protons require slightly different approaches
  - Depends on the evolution of the particle velocity along the system
  - Electron rest mass: 0.511MeV
  - Proton rest mass: 938 MeV
- Magnetic fields need to scale with increased momentum



Reminder: 1eV = energy gained by 1q of charge when accelerated across a potential difference of 1V

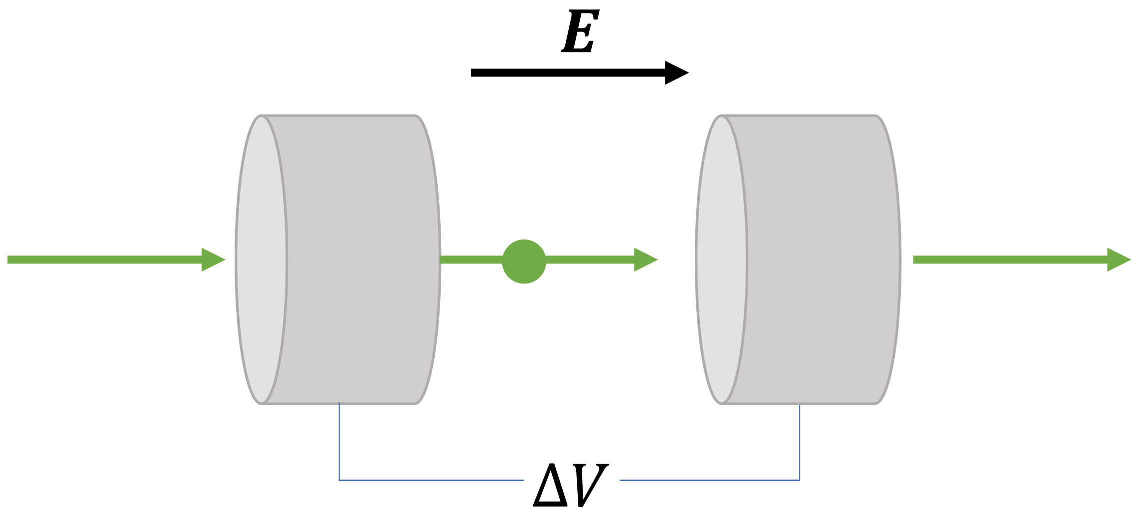
Using the Force

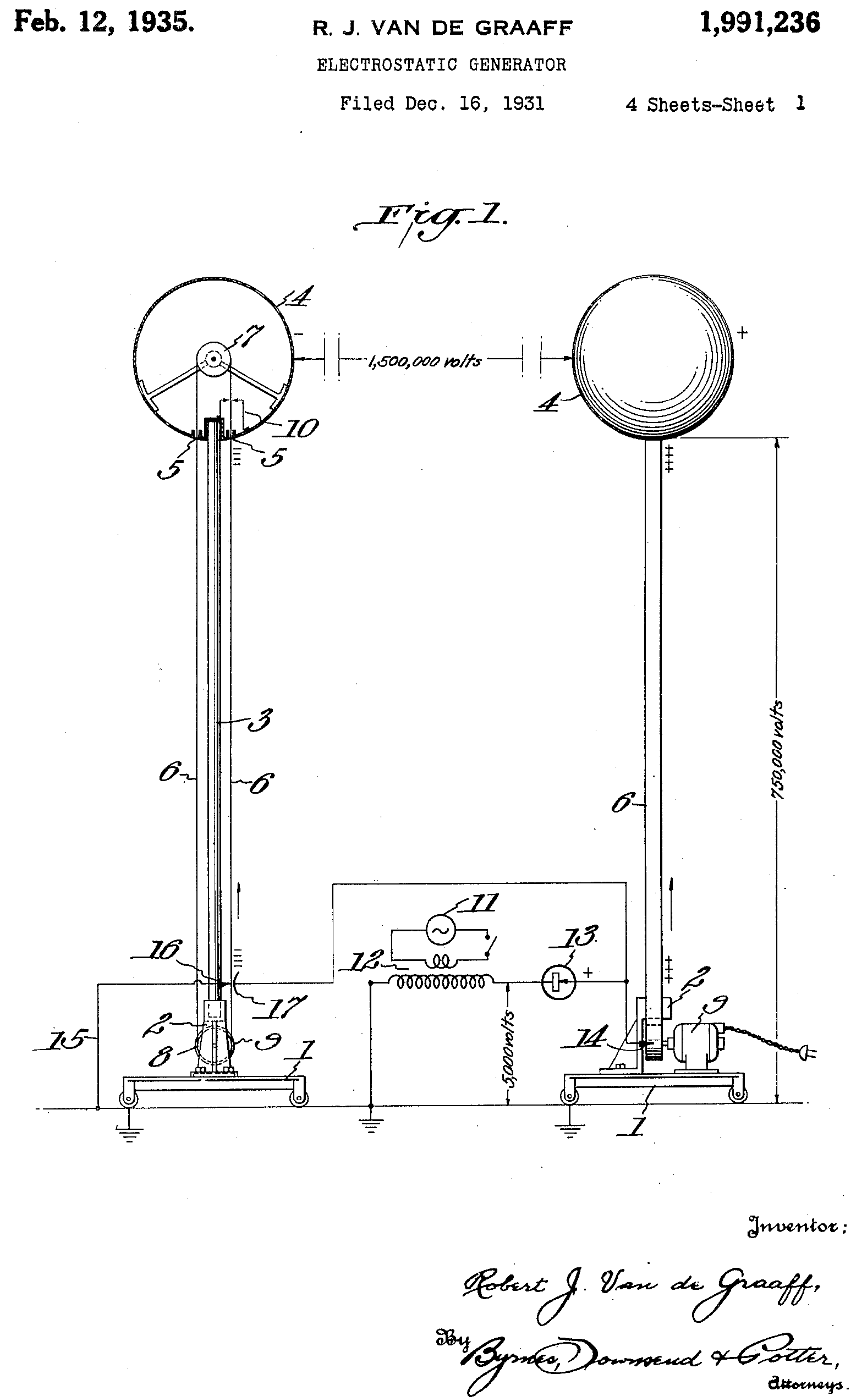
- Back to the Lorentz force: $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$
  - Want acceleration in the direction of motion:
  - $\boldsymbol{E} \neq 0$, ideally $\boldsymbol{E} = (0,0,E_z)$ → changes particle momentum
  - $\boldsymbol{v}\times\boldsymbol{B} = 0$ in the *direction of motion*
- Acceleration increases momentum ($F = {}^{dp}/_{dt}$)
  - Increases kinetic energy of charged particles
  - $E^2 = m^2 + p^2$ *(in natural units)*
  - Rate of energy gained per unit acceleration length: ${}^{dE}/_{dz} = qE_z$
- We always want ***more energy***, so how do we get it?
  - LHC: 7→14 TeV (protons)
  - CLIC: 3 TeV (electrons)
  - HE/VHE-LHC: 33/100 TeV

Why is this so different? *Synchrotron radiation!*

Acceleration methods

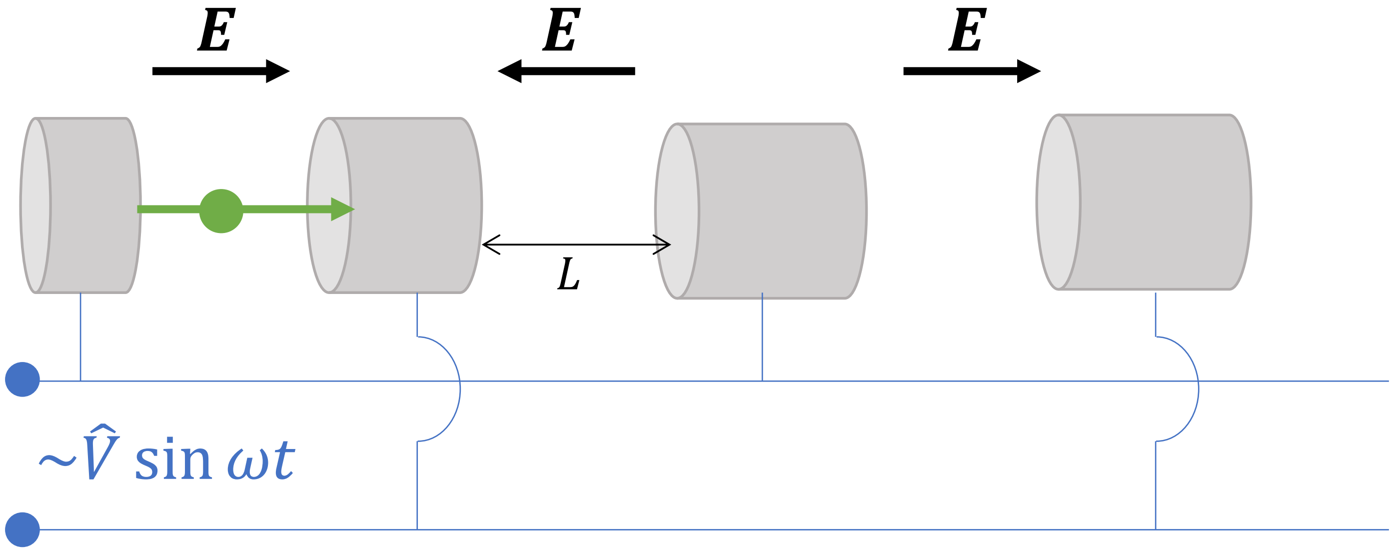
- Simplest: Electrostatic acceleration
  - e.g. Van-de-Graaf generators
  - Used for initial acceleration
  - Limited by insulation problems
  - Max voltage ~10 MV



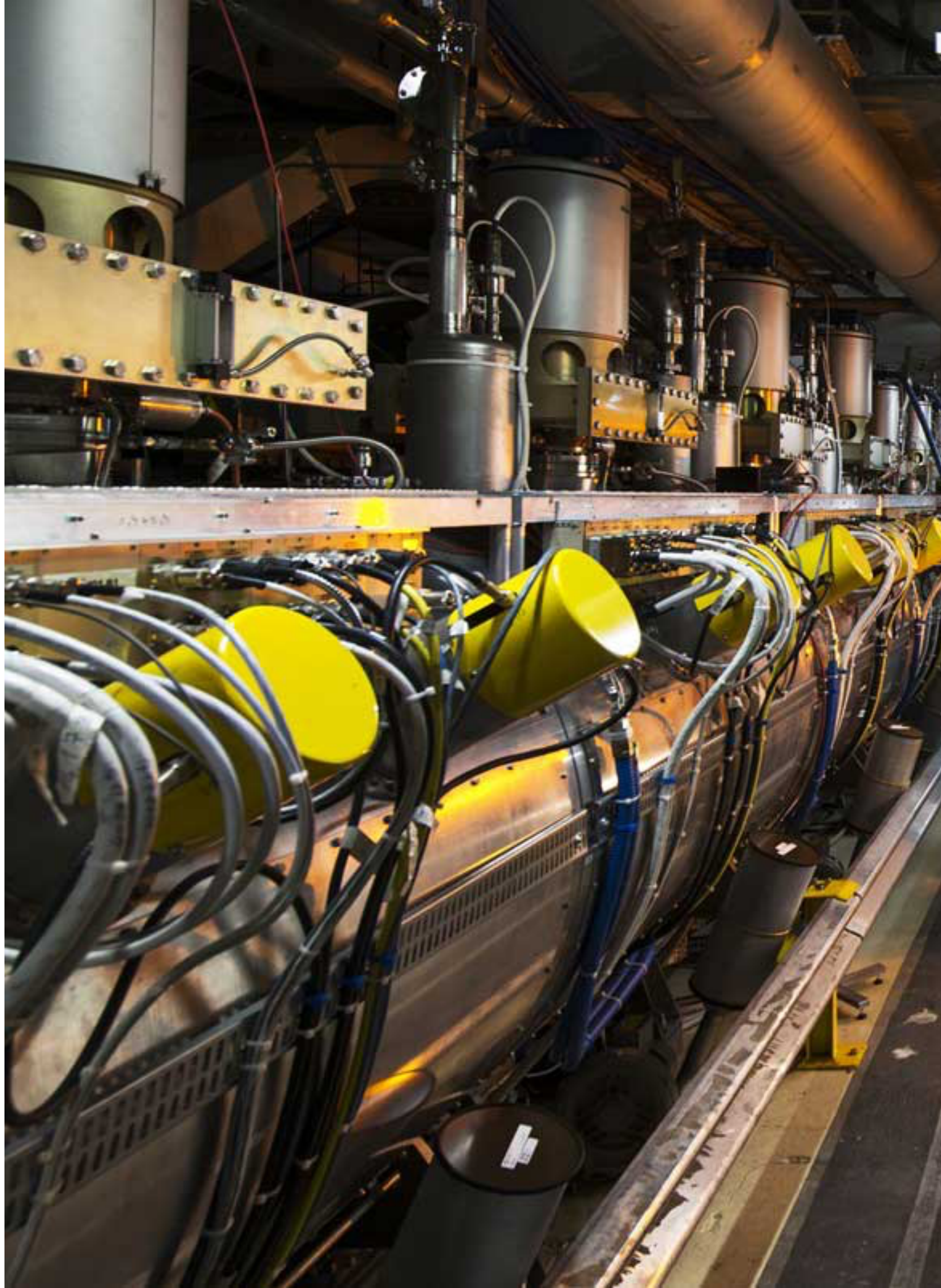


Acceleration methods

- Time-varying fields
    - Radio-Frequency (RF) Acceleration
    - Wideroe-type
    - Drift tubes and gaps, alternating E-field
    - $L = vT/2$, sychronism condition
        - T = RF period, v = particle velocity

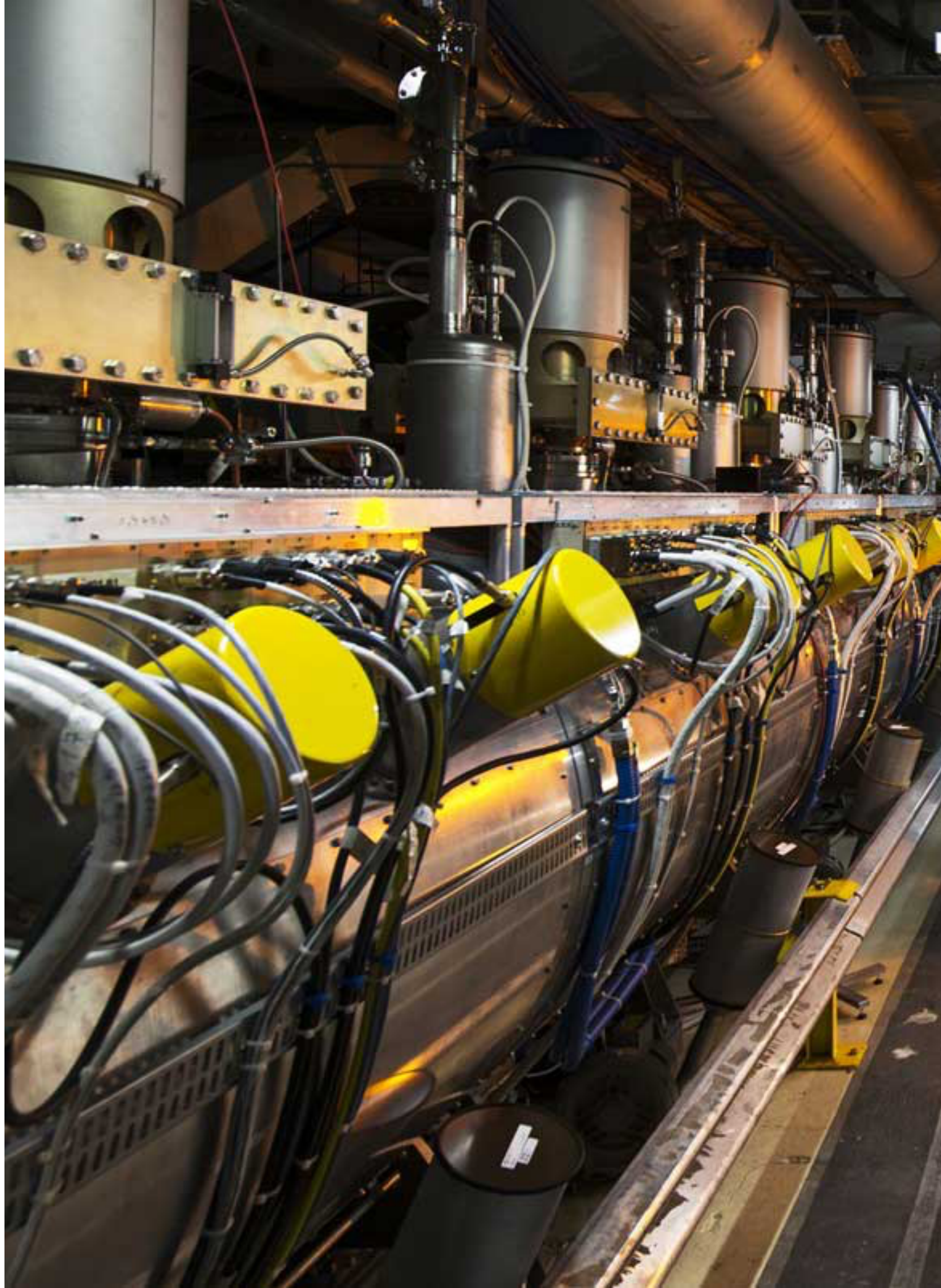


RF cavities in the LHC



Acceleration methods

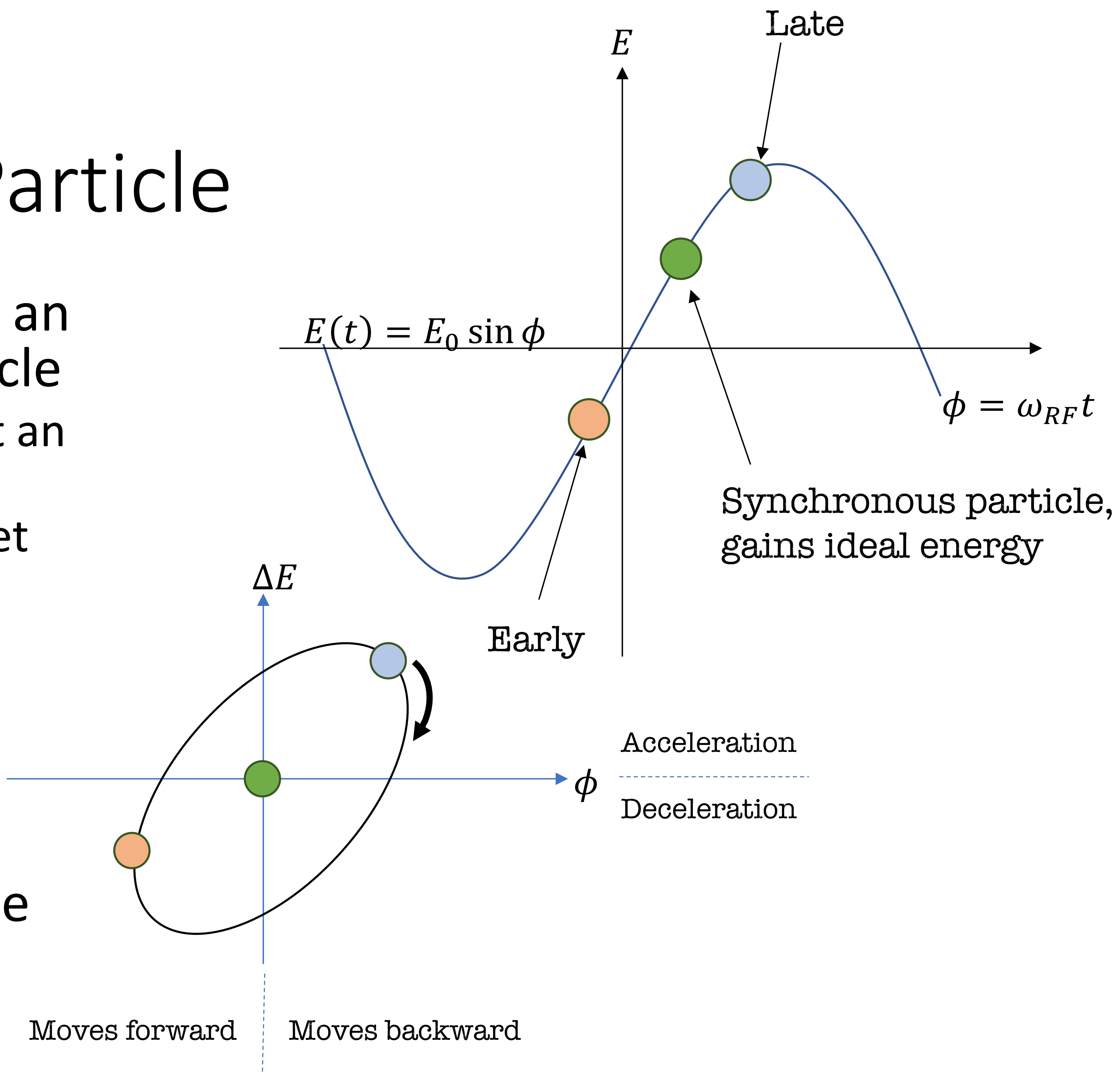
- Higher velocity → longer drift space
  - Inefficient
  - Circulating current on electrodes loses energy proportional to RF frequency
- Solution: ***RF cavity*** with resonant frequency matched to RF generator frequency
  - Constrains electromagnetic power in a resonant volume
  - Can be independently powered
  - π-mode: Field direction alternates from one cavity to the next
  - 2π-mode: Field does not alternate



RF cavities in the LHC

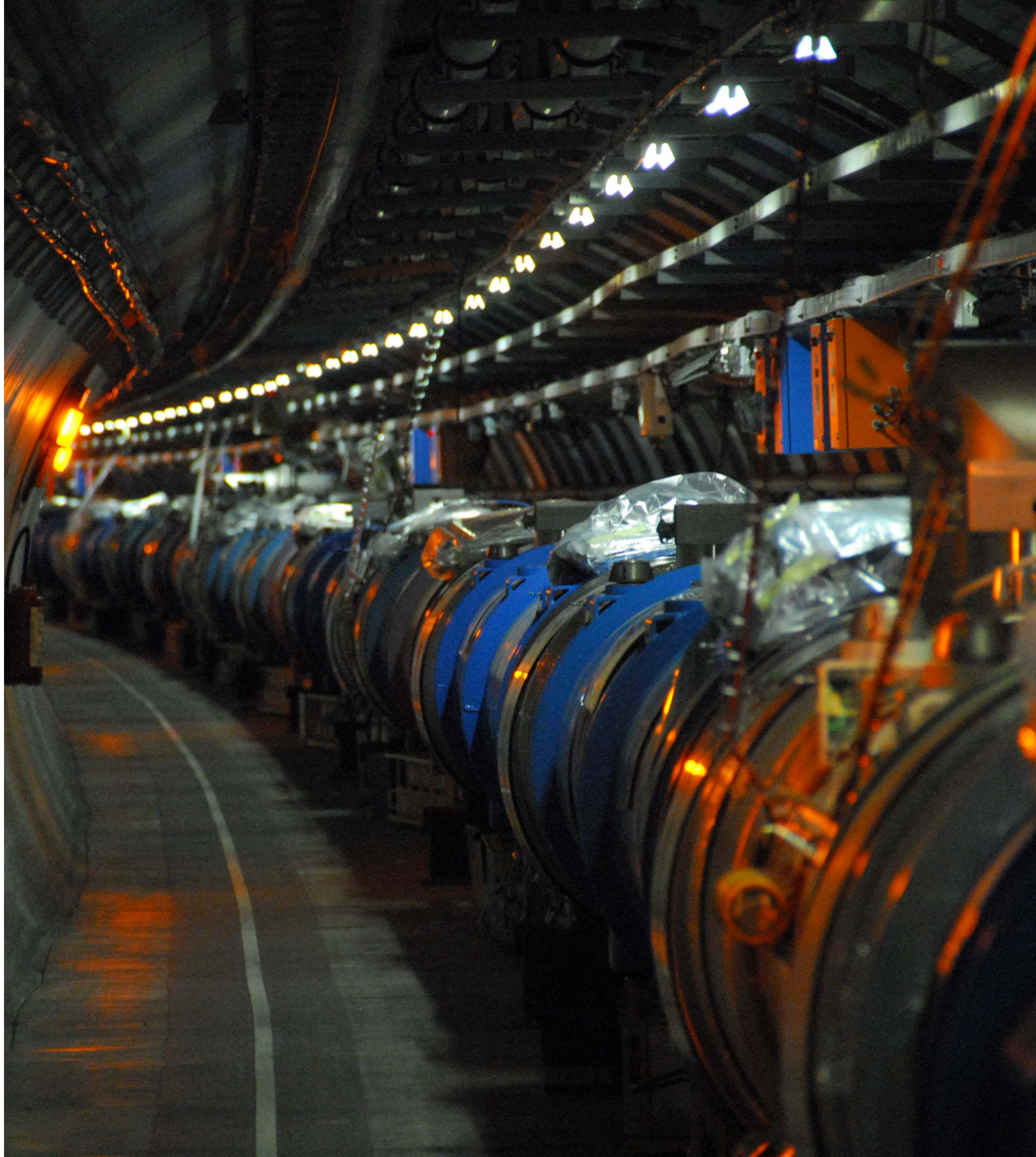
The Synchronous Particle

- Beams are bunched around an idealised synchronous particle
  - Particles that arrive ‘late’ get an energy boost
  - Particles that arrive ‘early’ get less energy
- Longitudinal phase space describes this motion
  - Before had (x, x')
  - Now have $(\phi, \Delta E)$
- Particles oscillate around the synchronous phase



The Synchrotron

- Increase particle energy → increase magnetic fields
- ***Synchrotron***: synchronise RF field with increase in magnetic field
  - Variable RF frequency
  - Variable magnetic field
  - Keeps particles within the same ‘ring’
- Can juggle (x, x'), (y, y'), $(\phi, \Delta E)$
  - Complicated, but there are even bigger challenges ahead...



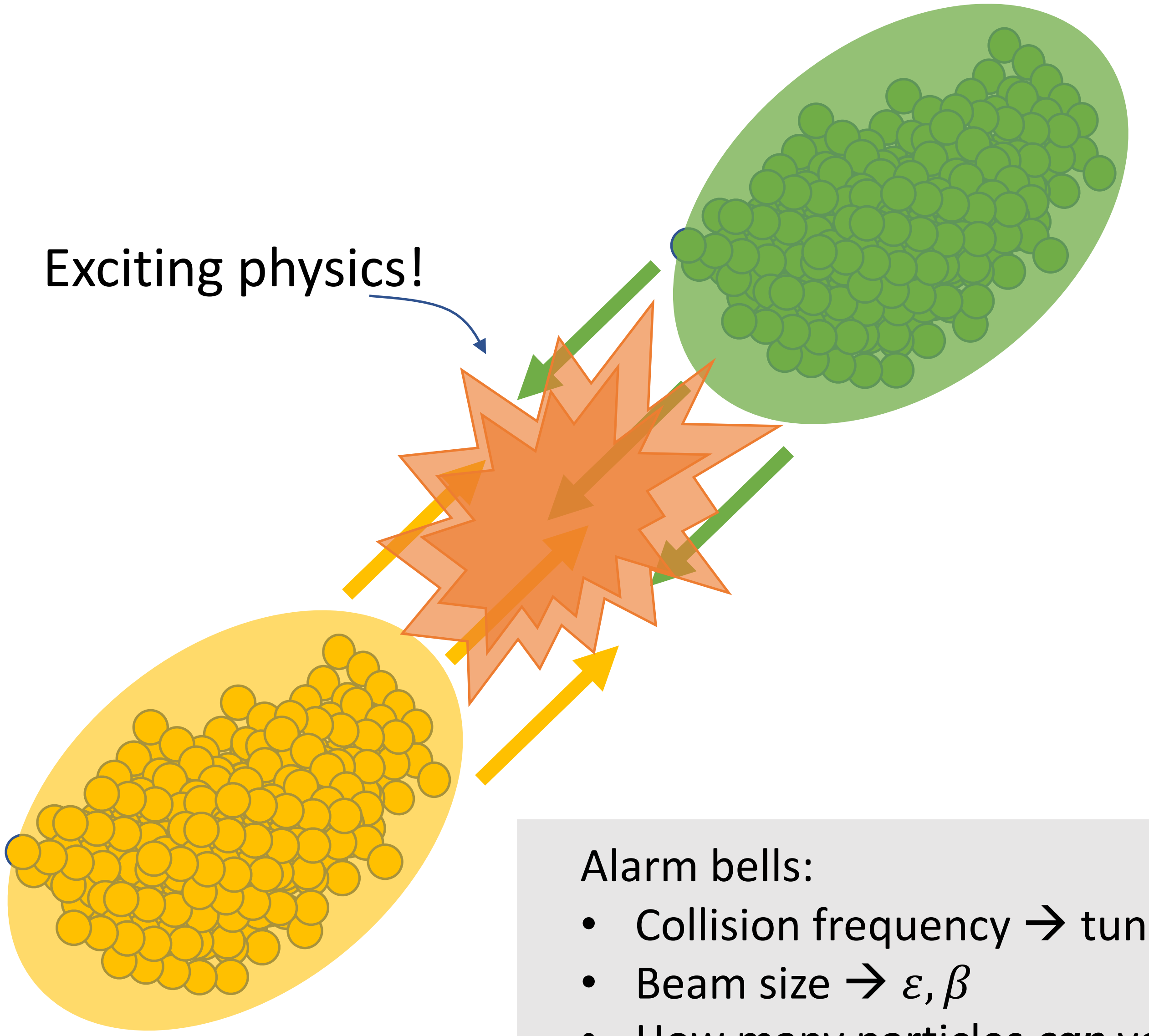
The LHC Synchrotron

What does a HEP physicist care about?

Data, data, data!

Luminosity

- Every accelerator is concerned with its ***luminosity***
  - Number of expected events, $N_e = \sigma_e \int \mathcal{L}(t)dt$
  - σ_e = some rare cross-section
  - Luminosity, $\mathcal{L} \cong f_c \frac{n_1 n_2}{4\pi\sigma_x\sigma_y}$
  - f_c, collision frequency
  - $n_{1,2}$, particles per bunch
  - $\sigma_{x,y}$, rms transverse beam size
    - Assumes identical colliding bunches

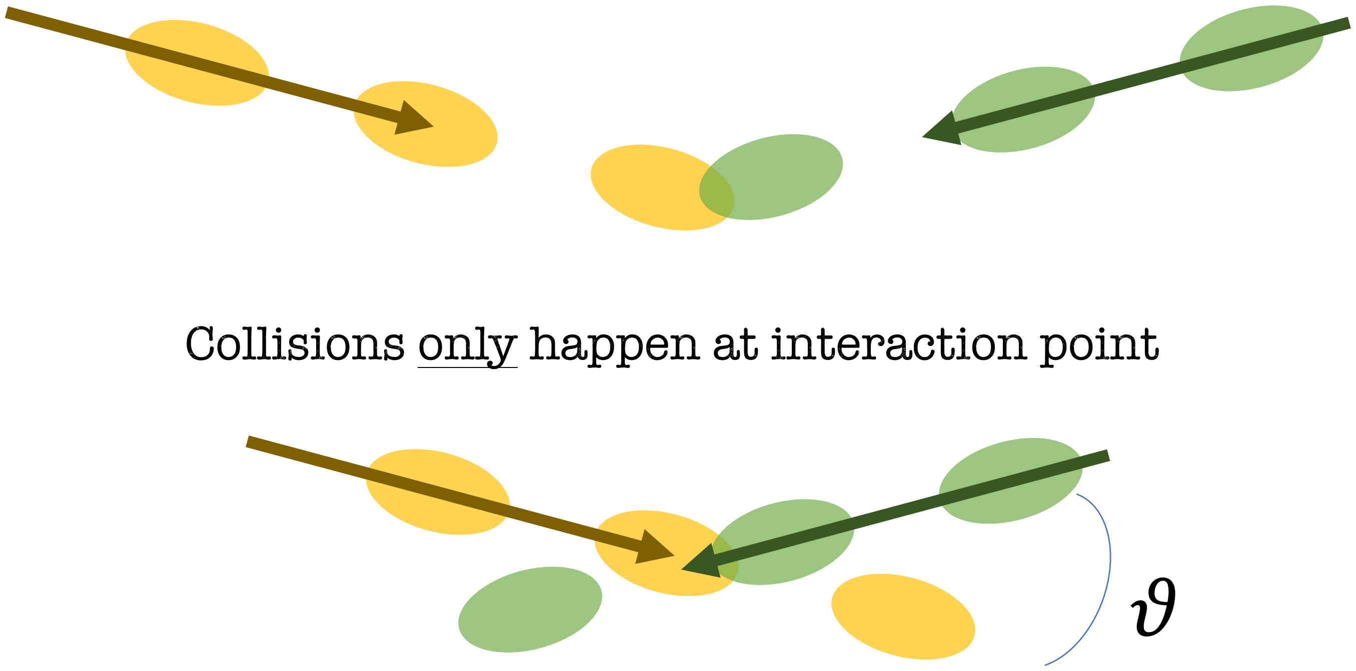


Alarm bells:

- Collision frequency → tune
- Beam size → ε, β
- How many particles ***can*** you get in a bunch...?

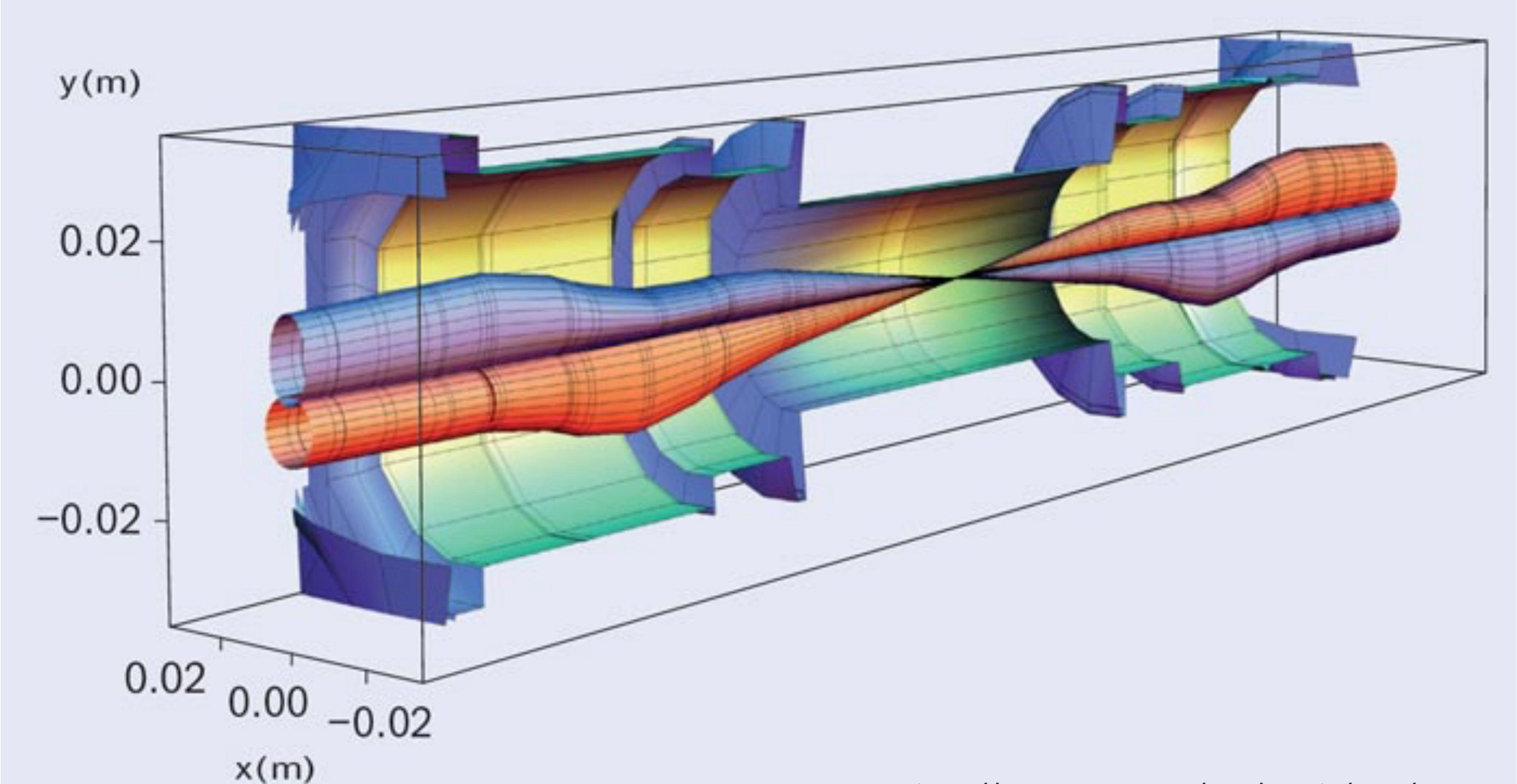
Luminosity II

- The *truth* is that detectors are *large*
  - Only want **one** interaction point in the detector
  - Need to introduce a crossing angle, ϑ
  - Reduces luminosity, as particles no longer cross the entire length of the 'opposite' bunch
  - $\mathcal{L} \cong f_c \frac{n_1 n_2}{4\pi\sigma_x\sigma_y} \frac{1}{\sqrt{1+\left(\frac{\sigma_z}{\sigma_x}\tan\frac{\vartheta}{2}\right)^2}}$
  - σ_z, rms beam length
    - Valid if ϑ is small and $\sigma_z \gg \sigma_{x,y}$



Beam envelope at interaction point 1 (ATLAS) of the LHC

This requires some **serious** beam gymnastics!

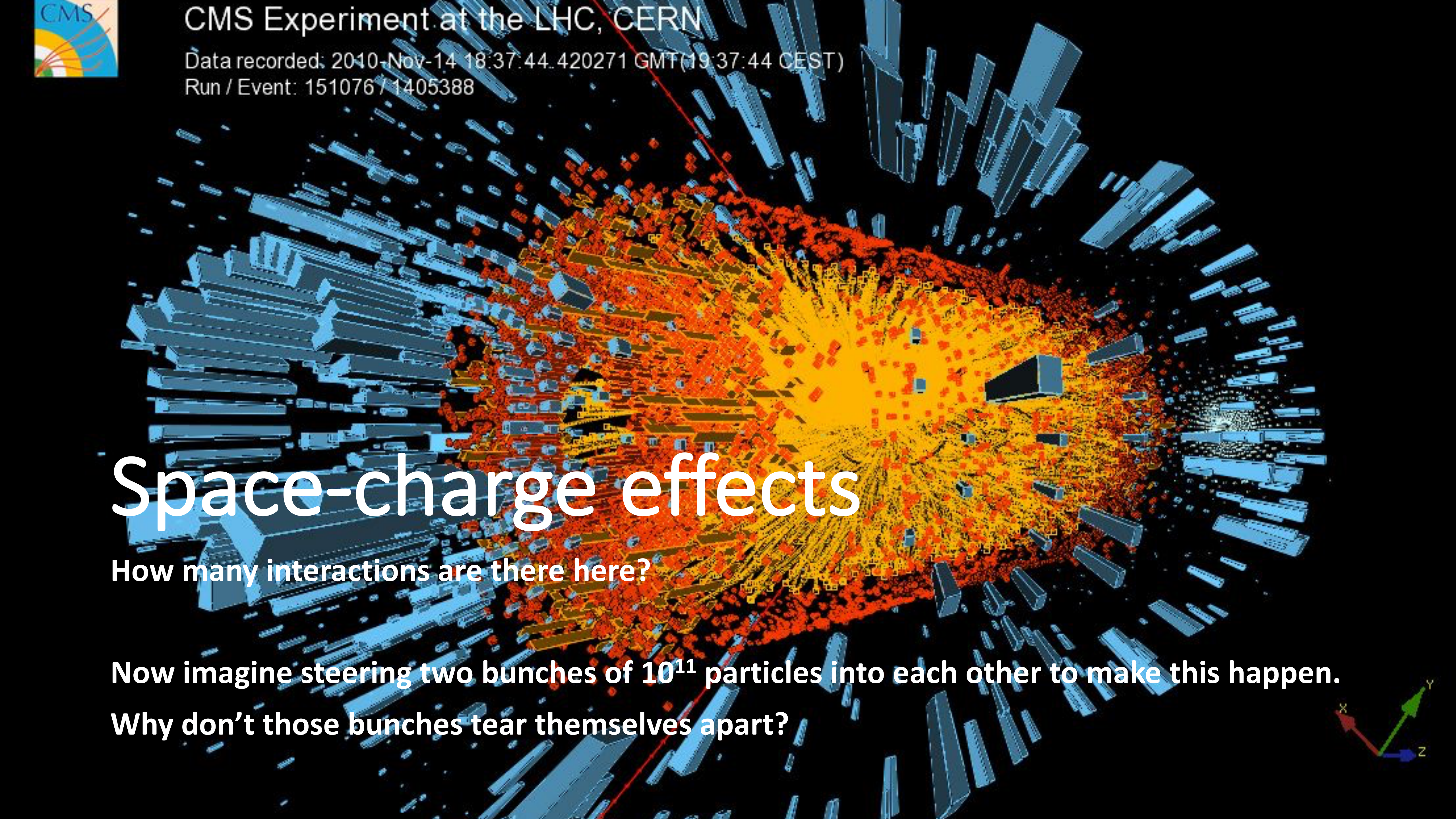


http://cerncourier.com/cws/article/cern/54381

The balancing act

- Every dimension of the beam is critical
  - Small is best!
- Nothing is *really* independent
  - Altering particle energy alters path length and revolution frequency
  - Must keep an eye on the tune as energy increases
- Detectors are large
  - Can’t really focus the beam *inside* the detector
  - Lots of fancy work must be done to get those beams colliding

- But if that was all, life would be “easy”...



Space-charge effects

How many interactions are there here?

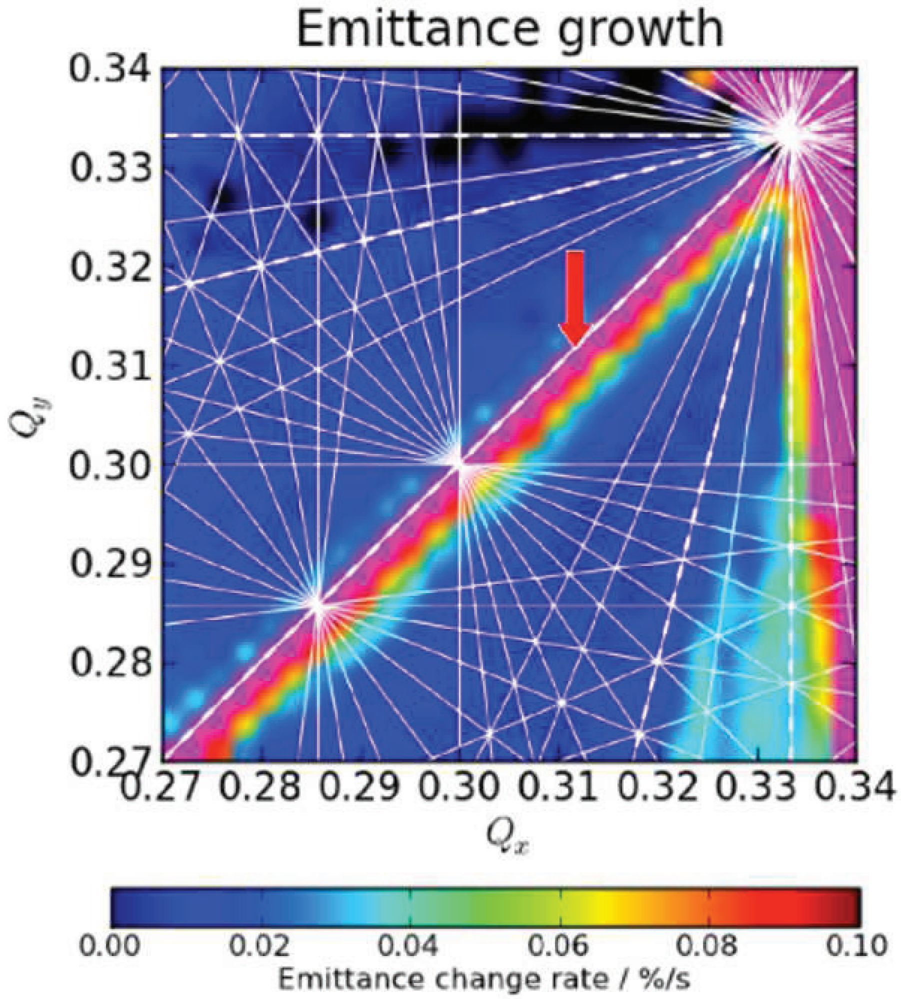
Now imagine steering two bunches of 10^{11} particles into each other to make this happen.

Why don't those bunches tear themselves apart?

Tune studies with beam-beam effects in LHC, S. Paret et al, IPAC 2013

Space-charge effects

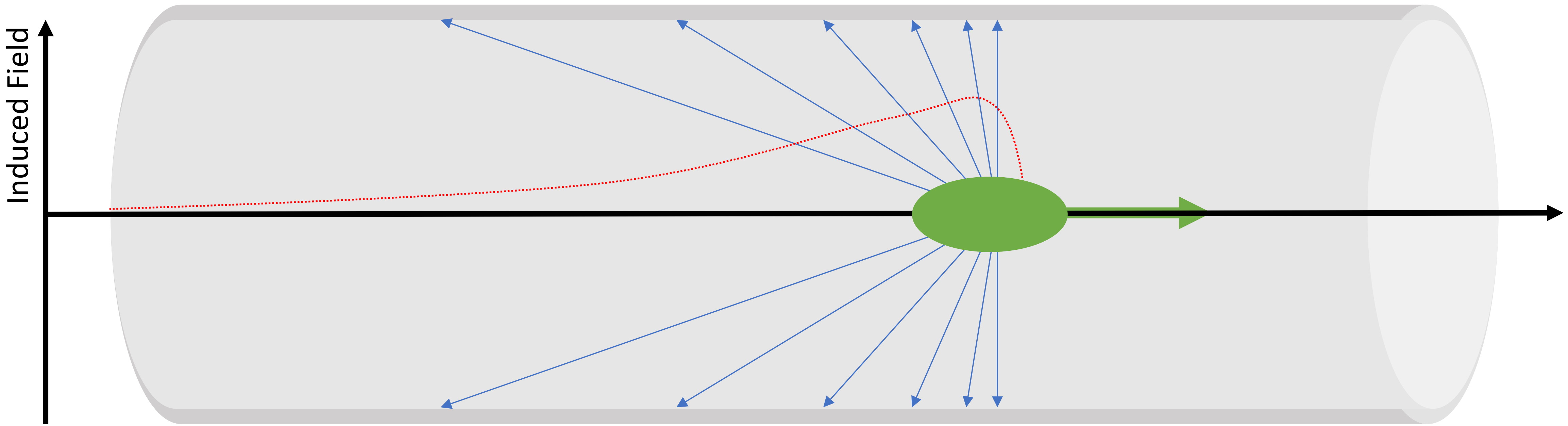
- Particles in a bunch repel each other
  - Defocuses beam
  - Force felt by a particle depends on where it is in the bunch
  - Introduces a position-dependent tune shift
  - $dQ_{x,y} \sim \frac{n}{\sigma_z \varepsilon_{x,y}} \left(\frac{E_0}{E}\right)^3$
  - Difficult to avoid resonances
    - Energy dependent
    - Long bunches are easier to handle
    - But have to balance with luminosity goals



Emittance change near resonant tune lines due to space-charge induced tune-shift

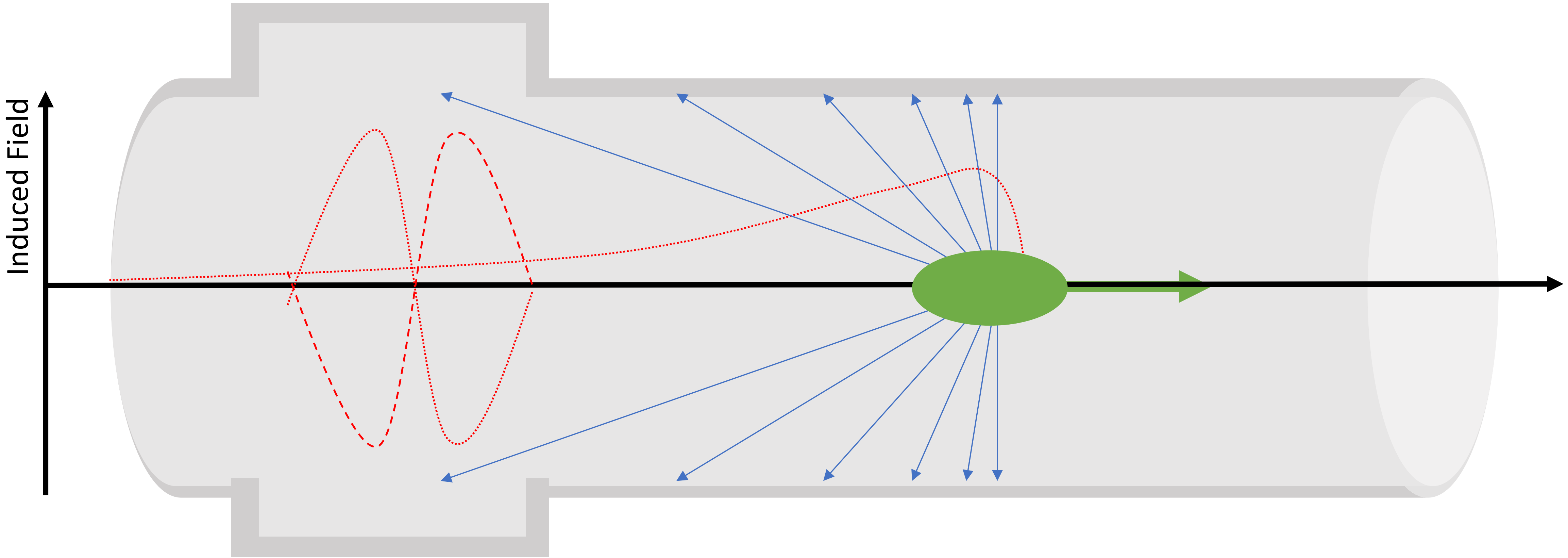
Induced wakefields

- Have ~10^{11} charged particles hurtling down a metal beam pipe
- Surface charge induced on beam pipe
- Produces a wakefield



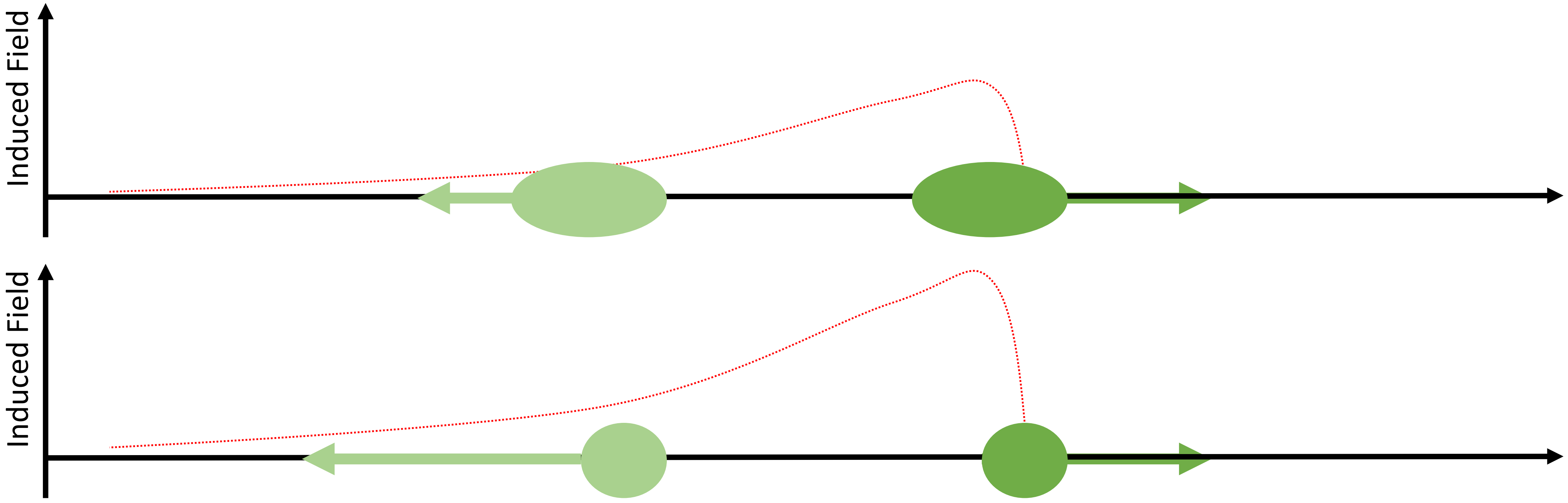
Induced wakefields

- Cavity-like structures create standing waves
- Wakefield is intensified



Induced wakefields

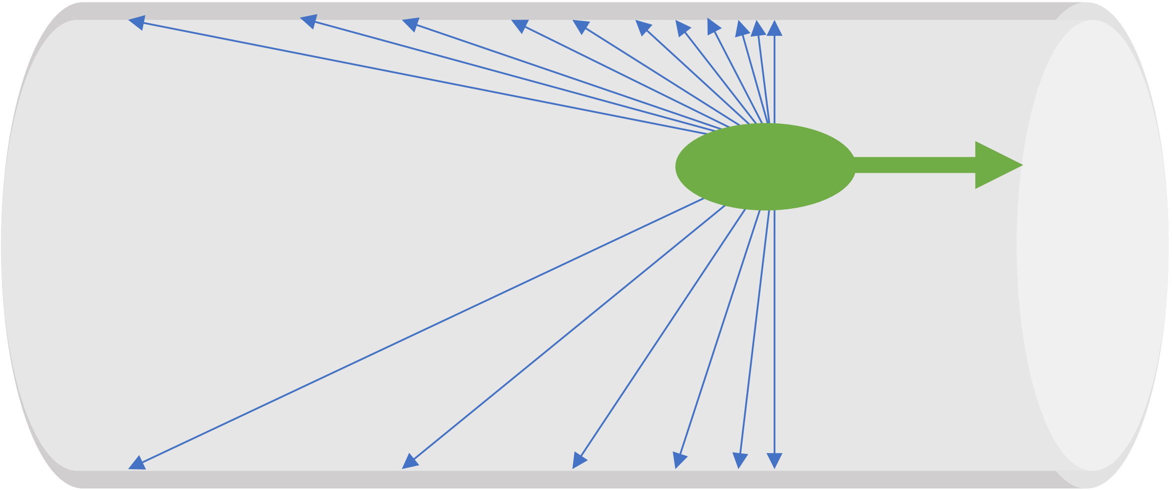
- Longer bunches produce less-intense wakefields
- Bunches that follow are decelerated in the wakefield

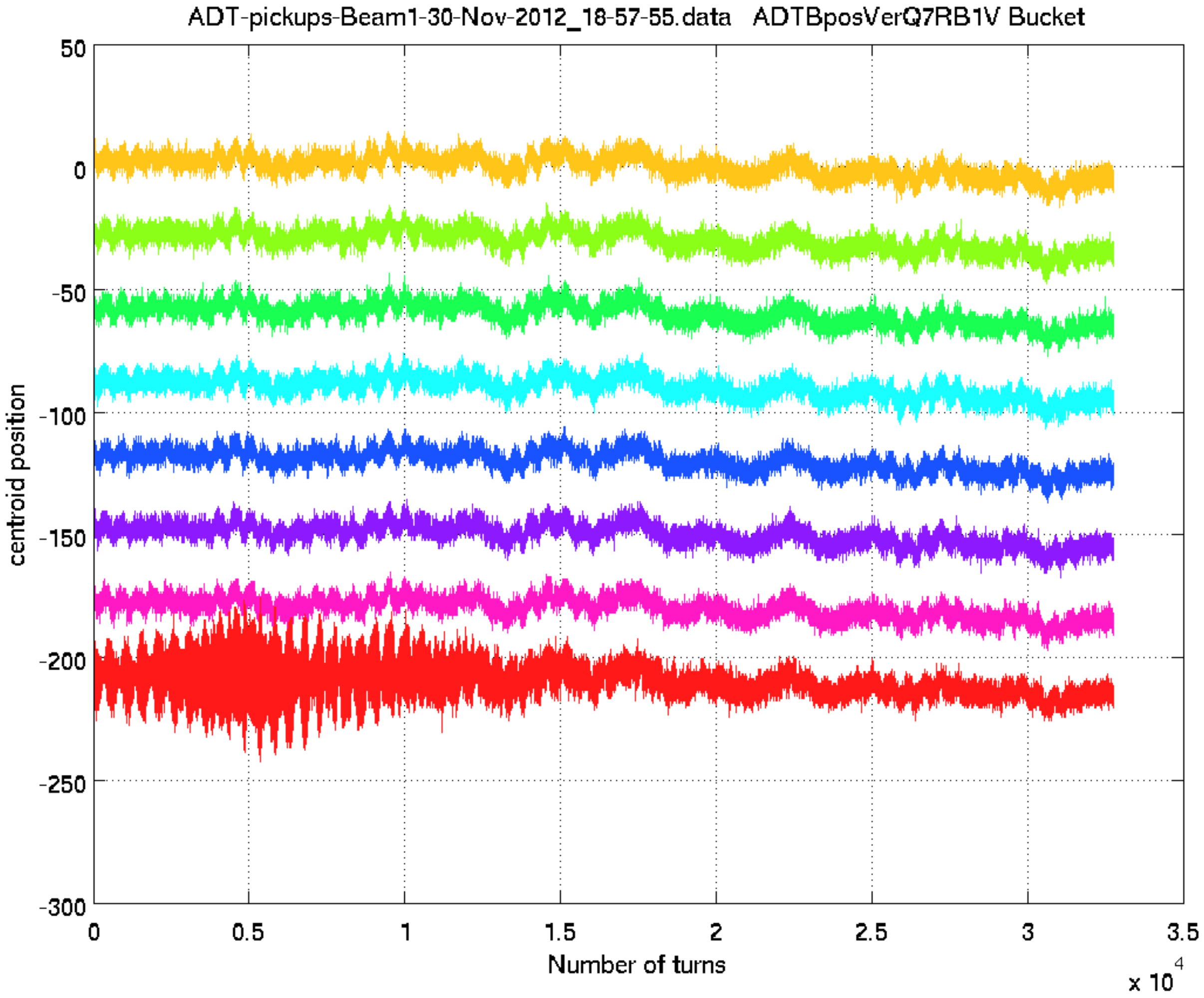


Induced wakefields

Source

- What happens if the beam is not centered?
- Transverse wakefield, kicks following bunches





Fighting instabilities

From 'Physics of Collective Beam Instabilities in High Energy Accelerators', A. Chao

- Many instabilities to discover
  - Finding instabilities and correcting for them is usually what drives an improvement in machine performance
  - Cycle of (a) find instability, (b) correct instability, (c) find new instability...
- Smooth beam pipes minimise wakefields
- Maximise bunch length
  - Keep luminosity in mind
- Create a frequency spread
  - Make bunches be out-of-step with each other
  - Exploit momentum differences, non-linear magnets
- Actively damp beams with feedback
  - Measure and "kick" bunches back into place

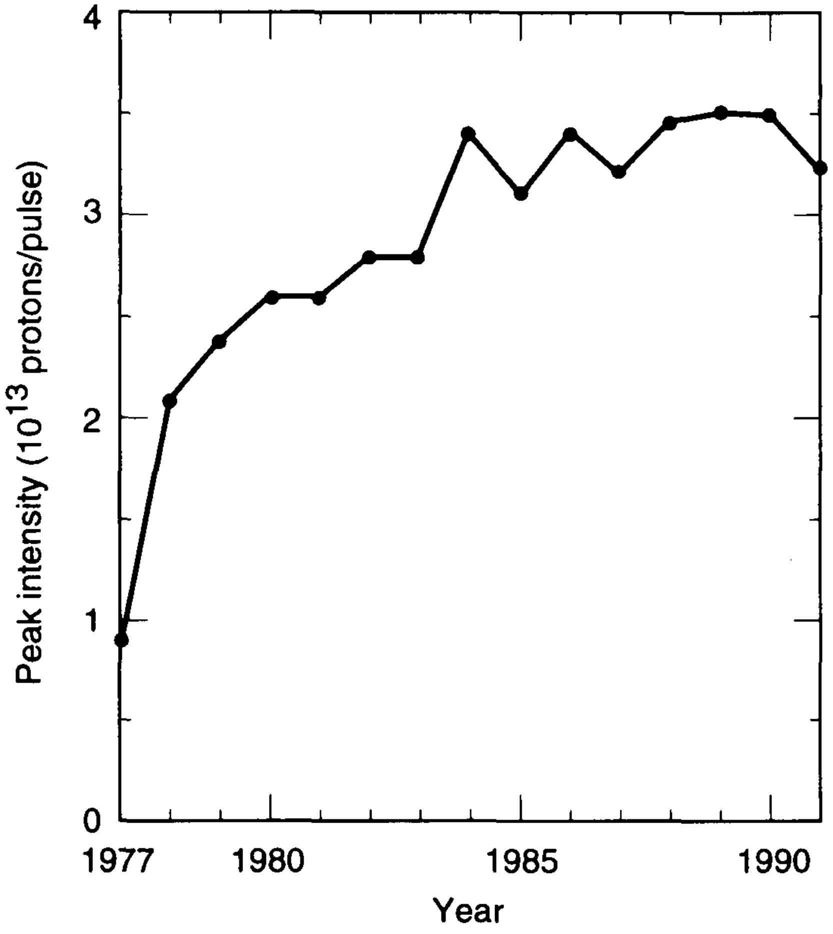


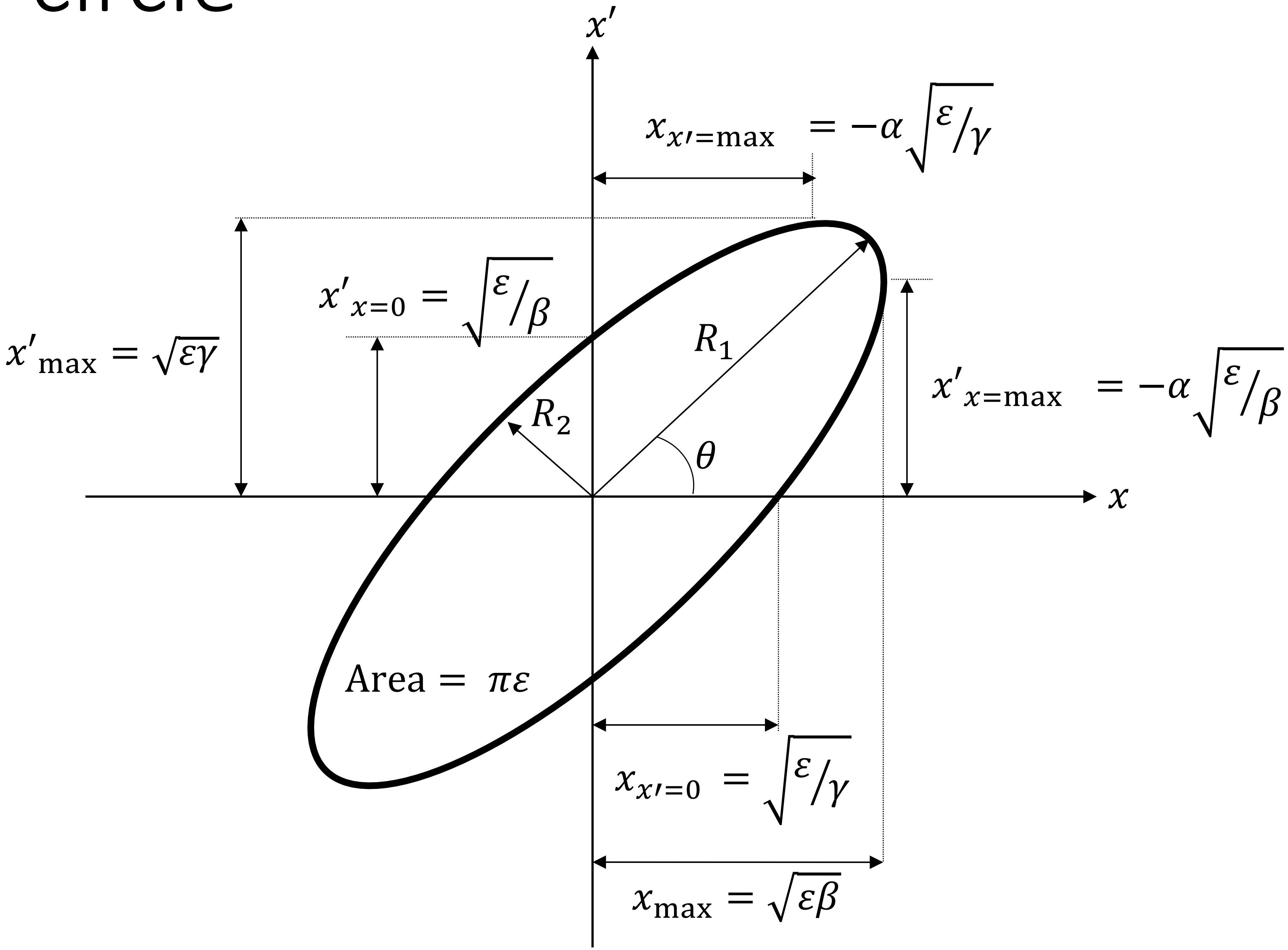
Figure 1.1. Peak beam intensity of the CERN Super Proton Synchrotron from 1977 to 1991. (Courtesy Jacques Gareyte, 1991.)

Back to emittance

$$\varepsilon = \gamma\langle x^2\rangle + 2\alpha\langle xx'\rangle + \beta\langle x'^2\rangle$$

Coming back full-circle

- Emittance is a conserved quantity
  - Under conservative forces
- Beam size at IP depends on emittance
- With all these instabilities and non-linear effects, is emittance something we can't effect?
  - Have seen space-charge induced tune shift changes emittance... how?



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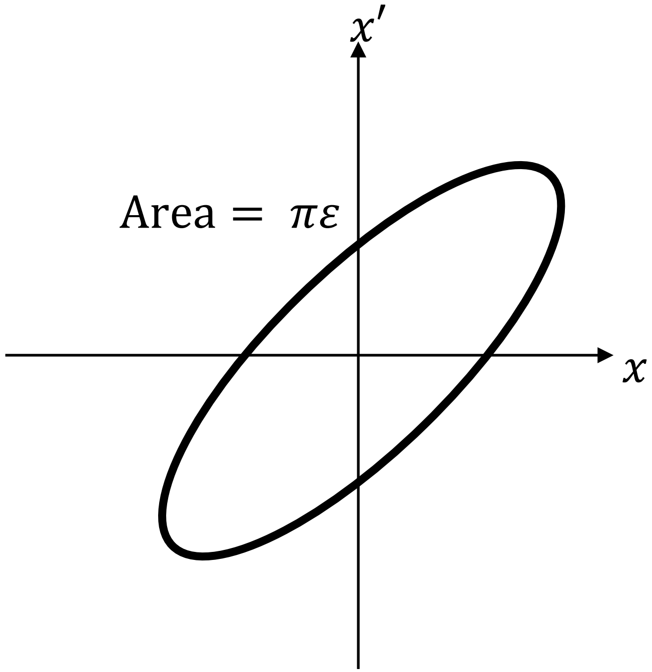
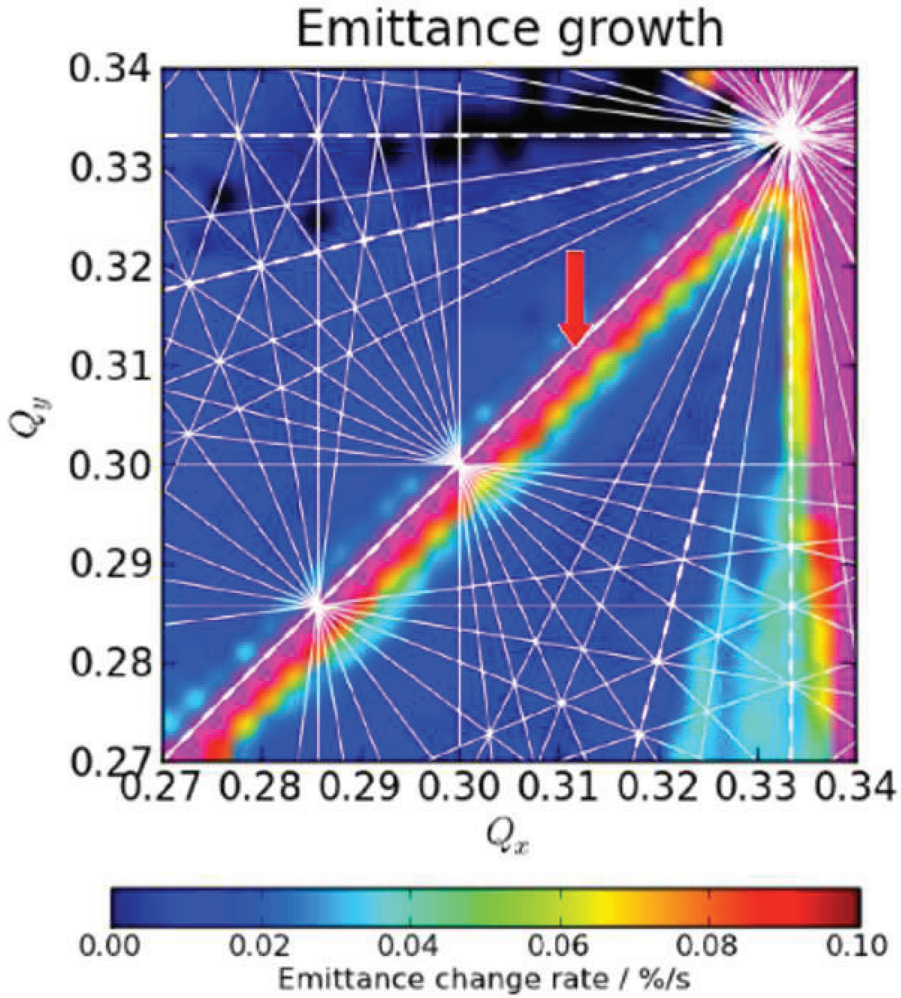


Tune studies with beam-beam effects in LHC, S. Paret et al, IPAC 2013



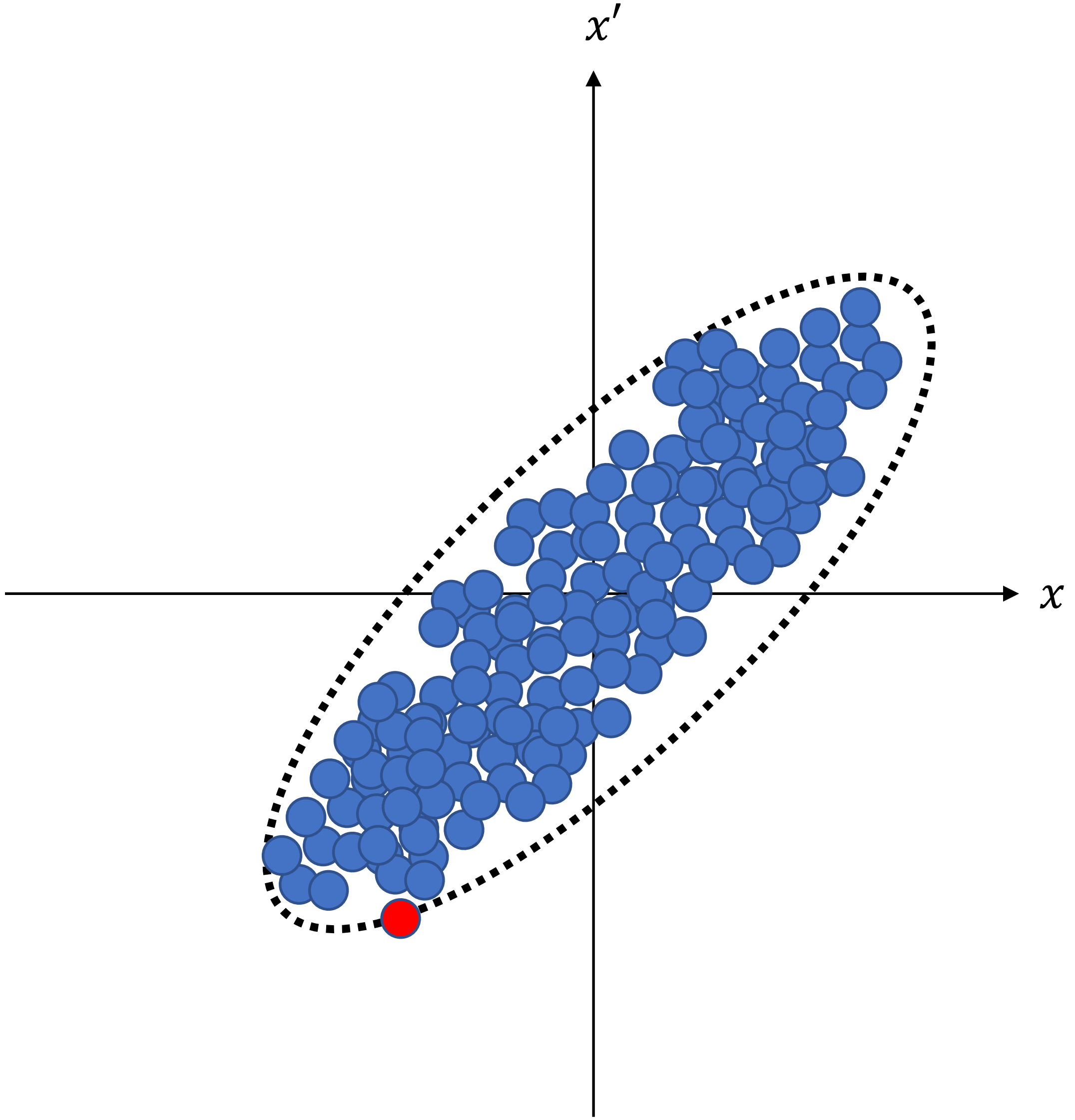
Emittance change near resonant tune lines due to space-charge induced tune-shift

Area $= \pi\varepsilon$

Emittance growth

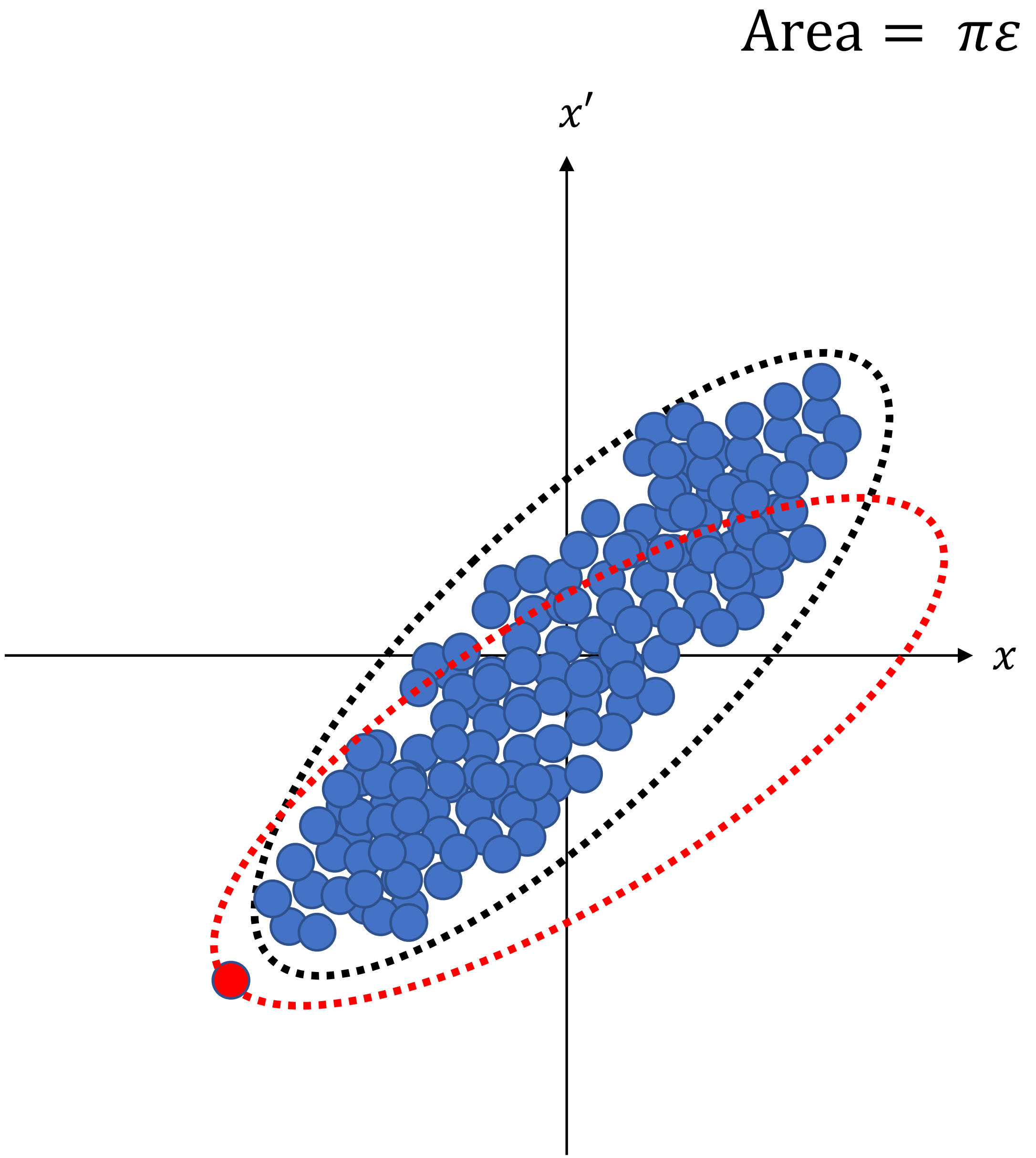
- Multiple ways:
  - Tune shift due to space-charge effects
  - Path length changes
  - Chromaticity
  - Wakefield effects
  - Optical mismatch

- Start with a beam contained within the nominal machine ellipse
- The particle undergoes transverse oscillations about the ideal orbit
  - Turn after turn it will follow the black dotted line (in an ideal universe)
  - So far, so expected



Emittance growth

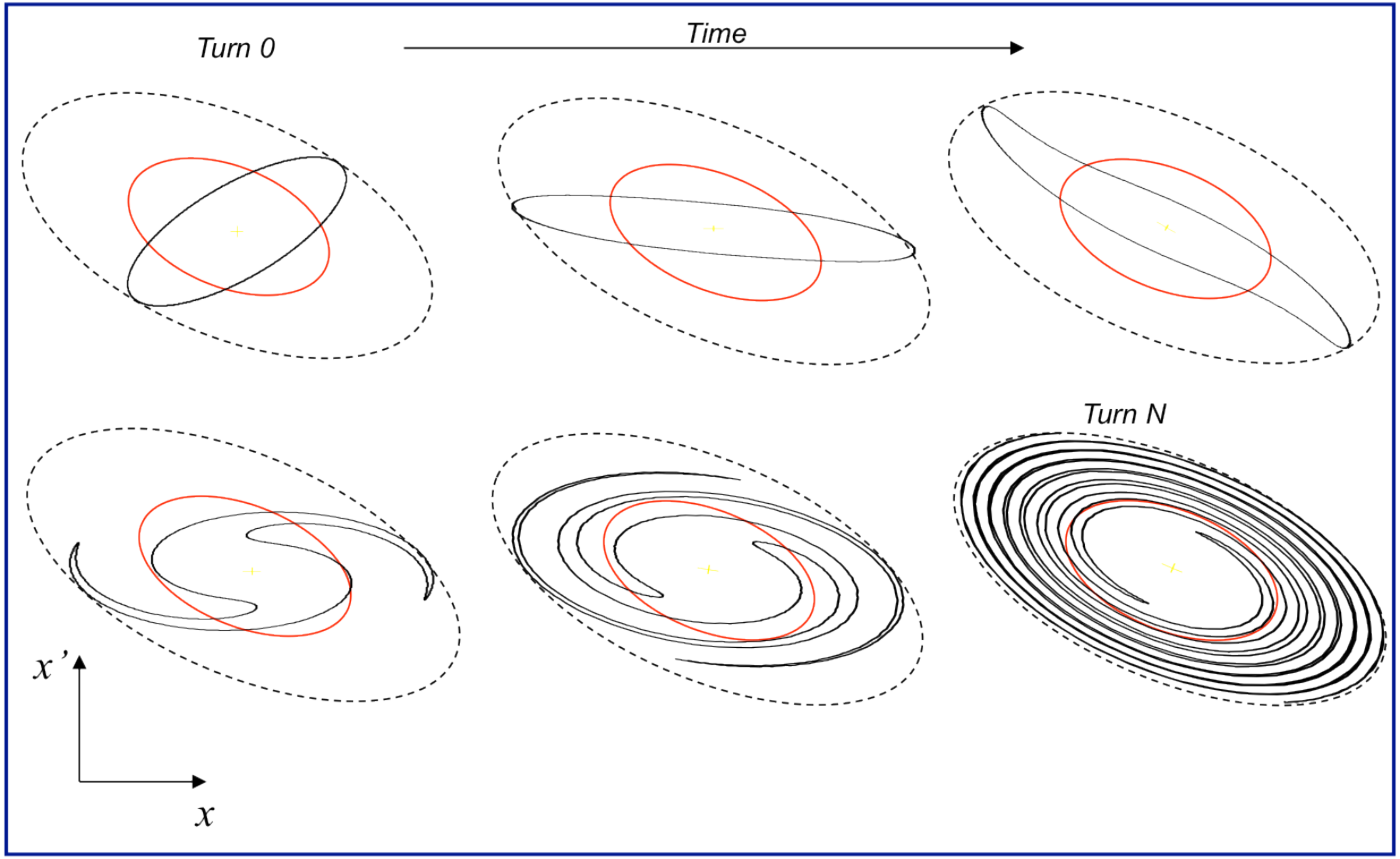
- In the ideal case, the ● particle will remain on its trace-space ellipse.
- Assume the particle is ‘pushed out’ by space-charge.
    - It has the same momentum as before, but new initial conditions
    - It will follow a different ellipse around the reference particle
- But this is happening to every particle.
- The particle distribution ’filaments’ across phase space
- Eventually reaches a new, larger, equilibrium



Emittance growth

Source

- Optical mismatch
  - Ideal machine ellipse = red
  - Incoming beam ellipse (e.g. on injection...)
  - After multiple turns, beam occupies a much larger area of trace space



Emittance reduction

- A lot of effort is spent *reducing* the emittance of beams before they're accelerated to high energy
  - Smaller emittance = better beam quality and cheaper acceleration
- Reduction = reduction in the area (volume) occupied by particles in trace-space
  - Also called "cooling"
- Requires non-conservative forces
  - e.g. synchrotron radiation for electrons (damping rings), stochastic cooling for protons

- Emittance growth due to instabilities is not a lost cause
- Assumed acceleration of electrons/protons
  - BUT what if you want to accelerate muons?
  - Produced occupying large phase space volume
  - Need to reduce emittance from the start, and ***fast*** (muons decay)
  - Muon Ionisation Cooling Experiment → a talk for another time, but ask S. Boyd for more

In Summary…

... we’ve barely touched the surface, but...

Summary

- HEP drives accelerator development
  - Every machine strives for higher energy, and encounters its own unique set of instabilities
- Particle motion can be described using transfer maps
  - Machine ellipse crucial for understanding emittance and beam size
- RF cavities provide energy, synchronised with an increase in magnetic field strength
  - Synchrotron!
- Careful balancing act between maximising luminosity, and minimising instabilities
  - Tune
  - Space-charge effects
  - Wakefields
- Lots of instabilities cause emittance growth
  - Creates problems keeping beam size small
  - Need to “cool” beams
- Different emittance reduction techniques for different particles
  - Some are ‘easier’ than others!

References

- The most recent CERN Accelerator School talks: https://indico.cern.ch/event/575505/timetable/
  - Always a good place to start
- *An introduction of Particle Accelerators*, E. Wilson
  - Approachable, a good starting book
- *[Your favourite book on electromagnetism]* – I have a preference for Griffith's, but everyone has their own
- *The physics of Particle Accelerators,* K. Wille
  - Next step up from Wilson
- *Fundamentals of Beam Physics,* F. Rosenzweig
  - Next step up from Wilson
- *The physics of Collective Beam Instabilities in High Energy Accelerators*, A. Chao
- *Particle Accelerator Physics*, H. Wiedemann
  - All the maths you ever dreamed of
- *Classical electrodynamics,* Jackson
  - The electromagnetism bible