Mass in the Standard Model

(very abridged)

In Classical Mechanics, the equations of motion derive from the Principle of Least Action

Equations of motion come when you extremise

$$S = \int L(x, t, \dot{x})\, dt$$

where $L(x, t, \dot{x})$ is the Lagrangian: $L = KE - PE$

Extremising the action $\equiv$ Solving the Euler-Lagrange equation

$$\text{E-L:} \quad \frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

In QFT we do the same thing to get the equations of motion. Here, the Lagrangian is a function of fields

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

and must obey the E-L equation

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$$

For a free fermion, the Lagrangian is

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$$

Free Dirac ~~Ferio~~ Fermion Lagrangian

Consider $\bar{\Psi}$:
$$\frac{\partial \mathcal{L}}{\partial \bar{\Psi}} = (i\gamma^\mu \partial_\mu - m)\Psi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\Psi})} = 0$$

so E-L: $(i\gamma^\mu \partial_\mu - m)\Psi = 0$

Dirac equation is equation of motion for free Dirac fermion described by the Lagrangian

$$L = \bar{\Psi}(\underbrace{i\gamma^\mu \partial_\mu}_{KE} - m)\Psi$$

Mass term: $m\bar{\Psi}\Psi$

Decomposing into chiral fields:

$$m\bar{\Psi}\Psi = m(\bar{\Psi}_L + \bar{\Psi}_R)(\Psi_L + \Psi_R)$$
$$= m\bar{\Psi}_L\Psi_L + m\bar{\Psi}_L\Psi_R + m\bar{\Psi}_R\Psi_L + m\bar{\Psi}_R\Psi_R$$

Claim: $\bar{\Psi}_L\Psi_L = \bar{\Psi}_R\Psi_R = 0$

Mass term : $m\overline{\Psi}_L \Psi_R + m\overline{\Psi}_R \Psi_L$

Mass couples the chiral sectors of the Standard Model.

1) $m\overline{\Psi}_L \Psi_R$ isn't allowed in the SM as it violates gauge invariance.

2) The Higgs theory says I can. Higgs theory also gives a mass term that looks like

$$y \langle H \rangle \overline{\Psi}_L \Psi_R$$

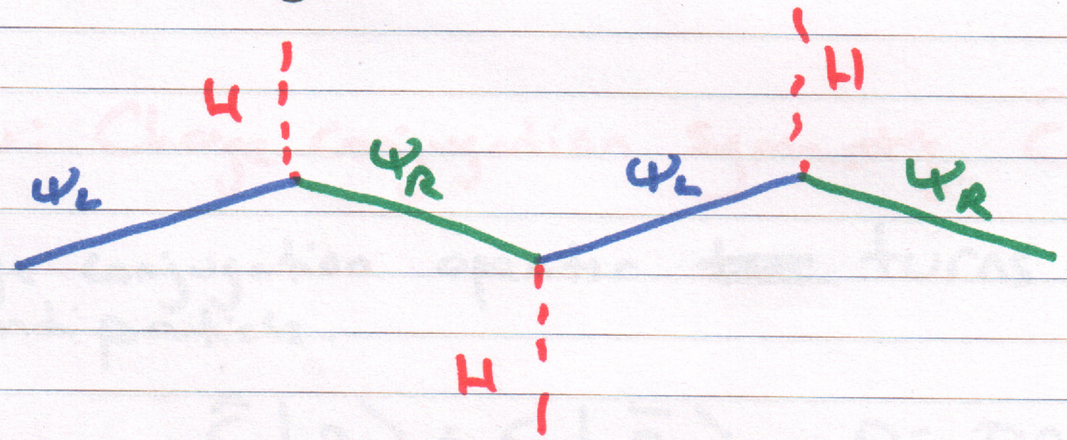
Arbitrary constant

Higgs Vacuum Expectation Value

$$\text{mass } m = y \langle H \rangle$$

The penalty for Higgs theory is an extra term

$$y \overline{\Psi}_L H \Psi_R$$



Higgs field flips chiralities and gives mass to fermions.

The term: $m\bar{\Psi}\Psi$ is a "Dirac mass term"

m is a Dirac mass.

~~Ro~~ So what? For most particles this is fine
as they have L- and R- chiral states.

Except for the neutrino which doesn't have a
R-chiral state. So $m_\nu = 0$.

But $m_\nu \neq 0$?!

Two loopholes:

1. ν_R does exist. It gives the neutrino mass but doesn't interact with any of the forces. "Sterile neutrino". In principle undetectable directly.

2. Maybe you can form a mass term just from ν_L

Reminder: Charge Conjugation symmetry, $\hat{C}$.

Charge conjugation operator ~~turns~~ turns particle to antiparticle.

$$\hat{C}|p\rangle = c|\bar{p}\rangle \qquad p = \text{particle}$$

Only the neutrino, which is neutral, can be an eigenstate of $\hat{C}$.

Eigenstates of $\hat{C}$ are called Majorana particles. A majorana particle is it's own antiparticle.

The Majorana Neutrino

Proposed in the 1930's by Ettore Majorana to see if he could make a mass ~~te~~ term just out of ν_L.

He noted that

$$\hat{C}\,\overline{\nu_L}^{T} \equiv \nu_R \Rightarrow \text{Right-chiral field}$$

Then he defined the "Neutrino field" as

$$\nu = \nu_L + \hat{C}\overline{\nu_L}^{T} = \nu_L + \nu_L^{c}$$

charge Conjugate field for ν_L.

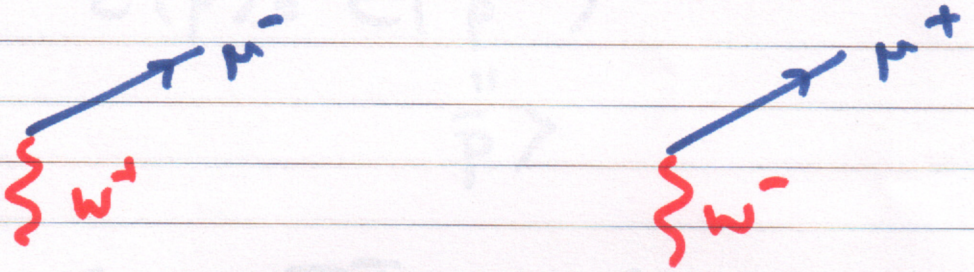
Now: $\hat{C}|\nu\rangle = \hat{C}|\nu_L + \nu_L^{c}\rangle = c|\nu_L^{c} + \nu_L\rangle$

$= c|\nu\rangle$

Show that $\nu = \nu_L + \nu_L^c$ is a Majorana fermion.

Neutrino $\equiv$ Anti-neutrino

Up to now, $\nu_\mu \neq \bar{\nu}_\mu$. We define $\nu_\mu, \bar{\nu}_\mu$ by the charge of the W boson and charged lepton that they interact with.



ν_L couples to W^+ $\qquad$ $\nu_R = \bar{\nu}$ couples to W^-

But only L-chiral fields couple to W^+ and only R-chiral fields couple to W^-. So one can view this as: the L-~~ca~~ component of ν will couple to μ^- and the R- " of ν " " " μ^+. You don't need "distinct" particles - just the neutrino field will do.

$$\nu = \nu_L + \nu_L^c$$

So what mass term can I form now?

$$L_{maj.} = m_L \overline{\nu_L^c} \nu_L + h.c.$$

is a good Majorana mass term Yay!

$$\hat{C} | \nu_L \rangle = C | \nu_L^c \rangle$$

$$\hat{C} | p \rangle = C | p^c \rangle$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad \| $$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad \bar{p} \rangle$$

$$\hat{C} \nu_L^c = \underline{\hat{C} \overline{\nu_L}^T} = \nu_L^c \equiv \nu_R$$