Deliberate Errors......

Error 1

E-L equation for fields:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$$

Error 2

$$\hat{C}\,|\nu_L + \nu_L^c\rangle = |\nu_L^c + \nu_L\rangle$$

or

$$\hat{C}\,|\nu_L\rangle = |\nu_L^c\rangle$$

obviously rubbish. Rather

$$(\nu_L^c)^c = \nu_L$$

ie charge conjugate of charge conjugate is 1

so $\nu = \nu_L + \nu_L^c \Rightarrow \nu^c = \nu_L^c + \nu_L = \nu$

ie. $\hat{C}\,\overline{\nu_L^c}^T = \hat{C}\left(\overline{\hat{C}\,\overline{\nu_L}^T}\right)^T = \nu_L$

$\hat{C}$ would turn ν_L into $\bar{\nu}_L$ (L particle into L antiparticle)

Last Lecture

* Dirac Mass term: $m_D \overline{\Psi_R} \Psi_L$

  $\Rightarrow$ Mass couples chiralities

* ν_R does not exist so $m_\nu = 0$!

* but $m_\nu > 0$ (neutrino oscillations!)

** - ν_R exists but doesn't interact
** - A R-H field can be made from ν_L

* Majorana Neutrino:

  $\nu_L^c \equiv \hat{C} \overline{\nu_L}^T$ is right handed

* $\nu = \nu_L + \nu_L^c$ is "the" neutrino

* Neutrino = Antineutrino

* Can now form the mass term

$$\mathcal{L}_{maj} = m_L \overline{\nu_L^c} \nu_L$$

:) (?)

So: $L_{maj} = m_L \overline{\nu_L^c} \nu_L$ is a good mass term.

Unhappily not. In fact this term ~~do~~ can't exist in the SM. The reason is that this term violates gauge invariance.

Meaning? Essentially it doesn't conserve charge.

Aside: Look at Dirac mass term for electrons.

$m_e \bar{e}_R e_L$: Feynman Diagram: $\overset{e_L}{\text{———}}\times\overset{e_R}{\text{———}}$

Now: e_L is part of a weak isospin doublet with weak isospin = $1/2$

$$I_W = \frac{1}{2} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{array}{l} \rightarrow I_{W,3} = 1/2 \\ \rightarrow I_{W,3} = -1/2 \end{array}$$

e_R is a weak isospin singlet. $I_{W,3} = 0$ since it doesn't interact with the weak current.

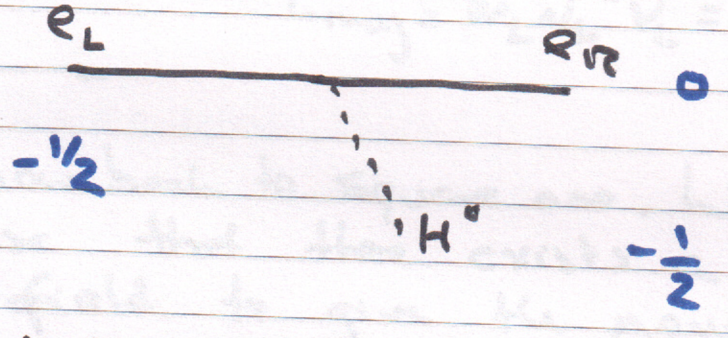
	e_L	×	e_R
$I_{W,3}$	$-\frac{1}{2}$		0

$\Rightarrow$ weak isospin not conserved in the Dirac mass term.

The Higgs theory introduces another doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \begin{array}{l} \rightarrow I_{w,3} = \tfrac{1}{2} \\ \rightarrow I_{w,3} = -\tfrac{1}{2} \end{array} \qquad I_w = \tfrac{1}{2}$$

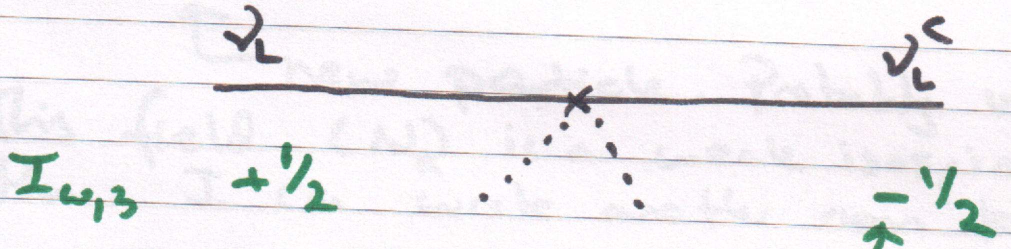
One obtains the mass term when you "break symmetry". ~~then~~ The underlying interaction is now



$\Rightarrow$ weak isospin is conserved with the addition of the Higgs.

Now consider the left-handed Majorana term

$$m_L \overline{\nu_L^c} \nu_L$$



since I've taken the charge conjugate

To get the isospins to be conserved I need a field with $I_{w,3} = +1$ on the RHS.

The SM needs extension, usually to a Higgs triplet

$$\begin{pmatrix} H^{++} \\ H^{+} \\ H^{0} \end{pmatrix} \quad \text{with } I_W = 1$$

This doesn't seem to exist. So we can set $M_L = 0$ so

$$L_{maj} = M_L \overline{\nu_L^c} \nu_L = 0 \text{ is SM}$$

We are now back to square one. We are forced to suppose that there exists a right-handed neutrino field to give the neutrino mass.

$$\nu = \underset{LH}{\nu_L} + \underset{RH}{\nu_L^c}$$

$$N = N_R^c + N_R$$

← new particle. Probably very heavy.

This field (N_R) is a weak isospin singlet then I can write another mass term

$$L_{maj,R} = M_R \overline{N_R^c} N_R$$

which doesn't break gauge invariance.

What is the most general mass term we can write down?

$$\nu \;:\; \nu_L \quad + \quad \nu_L^c$$

$$N \;:\; N_R^c \quad + \quad N_R$$

$$\mathcal{L} = \underbrace{m_D \bar{\nu}_L N_R + m_D \overline{N_R^c}\, \nu_L^c}_{\substack{\text{Dirac mass terms} \\ m_D \simeq 1\,\text{MeV}}} + \underbrace{M_R \overline{N_R^c} N_R}_{\substack{\text{Majorana term} \\ M_R \simeq\, ?}}$$

$$= \underbrace{\begin{pmatrix} \bar{\nu}_L & \overline{N_R^c} \end{pmatrix}}_{\text{LH fields}} \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}}_{\text{mass matrix}} \underbrace{\begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}}_{\text{RH fields}}$$

The mass matrix is not diagonal. Hence the ν_L, N_R fields are <u>not</u> mass eigenstates.

We've written the theory in a chiral basis, not in terms of physical particles. In order to ~~\$~~ find the physical particles I have to diagonalise the mass matrix. ~~to get a dia~~ In so doing I will find that the physical states are ~~mixes~~ mixtures of the chiral states.

When I diagonalize I will find a Lagrangian which looks like

$$(\bar{\nu}_1 \ \bar{\nu}_2) \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

ν_1, ν_2 are mass states with masses m_1, m_2

The masses are

$$m_{1,2} = \frac{m_R}{2} \pm \frac{m_R}{2}\sqrt{1 + \left(\frac{4m_D^2}{m_R^2}\right)}$$

Supposition: Suppose m_R is really heavy

$m_D \simeq 1$ MeV like other Dirac masses.

Then: $m_1 \simeq \dfrac{m_D^2}{m_R}$ $\quad m_2 \simeq m_R$

with physical states: $\nu_1 = \nu + \left(\frac{m_D}{m_R}\right) N \simeq \nu$

$\nu_2 = N - \left(\frac{m_D}{m_R}\right) \nu \simeq N$

$m_R \gg m_D$: m_1 is very small and m_2 is very large. We can explain the lightness of the physical neutrino, ν, "naturally"

"See-saw Mechanism"