Averaging Results

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WHEPPXI
Workshop on High Energy Physics Phenomenology

January 2010
Current chairs:
- Alan Schwartz (Belle), Gianluca Cavoto (BaBar)

HFAG operates as a set of quasi-autonomous subgroups:
- Oscillations & lifetimes (Olivier Schneider)
- Semileptonic (Christoph Schwanda)
- Rare decays (Paoti Chang)
- Unitarity Triangle (Tim Gershon)
- B to Charm (Simon Blyth)
- Charm (Alan Schwartz & Jon Coleman)
- Tau (Swagato Banerjee)
• History:
  • evolved from LEP B physics averaging group
  • founded at first CKM workshop (CERN 2002)
  • initial chairs:
    – David Kirkby (BaBar) & Yoshi Sakai (Belle) (2002-2005?)
    – succeeded by Soeren Prell (BaBar) & Simon Eidelman (Belle) (2006-2007?)
  • initially four subgroups; additional groups added according to demand
  • manpower within subgroups evolves at a rate that differs significantly between subgroups
Heavy Flavor Averaging Group

- Webpage:

- Documentation:
  - preprints updated irregularly
    - end of 2005 update hep-ex/0603003
    - winter 2005 update hep-ex/0505100
    - summer 2004 update hep-ex/0412073
  - end of 2009 update in preparation
Relations with other groups

PDG
- Some HFAG members are also PDG members/contributors
- HFAG provides some averages for PDG (at their request)

Experiments (BaBar, Belle, CDF, D0, LEP, CLEO, BES …)
- Subgroups contain representatives of relevant experiments
- Close relations (heritage of LEP B physics WG)

CKMfitter & UTfit
- Some HFAG members are CKMfitter/UTfit members
- Aim for strict independence (but friendly relations)

Theorists
- Discussions warmly encouraged
- Care to avoid bias possible due to preferences for particular theoretical models
HFAQ: Unitarity Triangle Parameters

- **Mission**

  - Provide world averages for measurements related to the angles of the Unitarity Triangle. In particular, we provide world averages of measurements of time-dependent CP violation, and related parameters. We also prepare averages of time-independent measurements that have a clean relation to the UT angles (especially in the B→DK system).
  - Identify common experimental and theoretical uncertainties and treat them coherently in the averages. Rescale the measurements if updated input parameters are available.
  - In cases where effective UT angles are measured no attempts are made to derive the fundamental quantities, if this requires input from QCD calculations.
  - If straightforward, an interpretation of the results is given.

- **Members**

<table>
<thead>
<tr>
<th>Contact Persons</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tim Gershon (*)</td>
<td>BABAR and LHCb</td>
</tr>
<tr>
<td>Gianluca Cavoto</td>
<td>BABAR</td>
</tr>
<tr>
<td>Owen Long</td>
<td>BABAR</td>
</tr>
<tr>
<td>Achille Stocchi</td>
<td>BABAR</td>
</tr>
<tr>
<td>Yoshi Sakai</td>
<td>Belle</td>
</tr>
<tr>
<td>Karim Trabelsi</td>
<td>Belle</td>
</tr>
<tr>
<td>Diego Tonelli</td>
<td>CDF</td>
</tr>
<tr>
<td>Hal Evans</td>
<td>DO</td>
</tr>
<tr>
<td>David Asner</td>
<td>CLEOc</td>
</tr>
</tbody>
</table>
• Mission

Provide world averages for measurements related to the angles of the Unitarity Triangle. In particular, we provide world averages of measurements of time-dependent CP violation, and related parameters. We also prepare averages of time-independent measurements that have a clean relation to the UT angles (especially in the B→DK system).

Identify common experimental and theoretical uncertainties and treat them coherently in the averages. Rescale the measurements if updated input parameters are available.

In cases where effective UT angles are measured no attempts are made to derive the fundamental quantities, if this requires input from QCD calculations.

If straightforward, an interpretation of the results is given.

• Members

Φs (Bs → J/ψφ) handled by HFAG lifetimes & oscillations (actually handled directly by CDF & D0 at present)
- Averaging procedure
  - **For measurements with Gaussian uncertainties**
    - perform simultaneous average of all physics parameters that are determined in the fits taking (linear) correlations into account (NB. *different to PDG*)
    - encourage experiments to use parameters that have Gaussian uncertainties, and to report all physics parameters and correlations
  - **For measurements with non-Gaussian uncertainties (ie. asymmetric errors)**
    - perform uncorrelated averages using the PDG prescription
  - **Correlations between measurements from different experiments are handled**
    - can arise due to dependence on external nuisance parameters
    - rescaling of external parameters can be handled
  - **If measurements do not agree, we discuss with the experiments and try to find the cause**
    - we do NOT inflate the uncertainties (NB. the PDG does)
• Averaging procedure
  • **Standard minimum $\chi^2$ procedure**
    - $i$ independent measurements of parameter $x$
    - Values $x_i$ and uncertainties $\sigma_i$
      \[
      \sum_i \left( x_i - \bar{x} \right)^2 / \sigma_i^2
      \]
    - Generalisation to $i$ sets of measurements of correlated parameters
      - $x_i$ is now a vector, $C_i$ the covariance matrix ($\sigma_{ia} = \sqrt{(C_{ia})}$)
        \[
        \sum_i \left( x_i - \bar{x} \right)_a (C_i^{-1})_{ab} \left( x_i - \bar{x} \right)_b
        \]
  • Solved analytically
• “The PDG prescription:”
  • See section 5.2 of the PDG RPP

When experimenters quote asymmetric errors $(\delta x)^+$ and $(\delta x)^-$ for a measurement $x$, the error that we use for that measurement in making an average or a fit with other measurements is a continuous function of these three quantities. When the resultant average or fit $\bar{x}$ is less than $x - (\delta x)^-$, we use $(\delta x)^-$; when it is greater than $x + (\delta x)^+$, we use $(\delta x)^+$. In between, the error we use is a linear function of $x$. Since the errors we use are functions of the result, we iterate to get the final result. Asymmetric output errors are determined from the input errors assuming a linear relation between the input and output quantities.
• Inclusion of results
  - We include all published and many preliminary results (NB. different to PDG)
  
  - We strongly encourage written documentation to accompany preliminary results
    - preferably collaboration authored (ie. not proceedings)
    - preferably available on arXiv (not hidden on web-pages)
  
  - We exclude preliminary results which remain unpublished for a long time (> 2 years) and/or for which no publication is planned
Example: $B^0 \rightarrow \pi^+\pi^-$

- Statistically dominated
- No need to worry about correlations of systematic uncertainties

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$S_{CP}$ ($\pi^+\pi^-$)</th>
<th>$C_{CP}$ ($\pi^+\pi^-$)</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>$-0.68 \pm 0.10 \pm 0.03$</td>
<td>$-0.25 \pm 0.08 \pm 0.02$</td>
<td>$-0.06$ [stat]</td>
<td>arXiv:0807.4226</td>
</tr>
<tr>
<td>Belle</td>
<td>$-0.61 \pm 0.10 \pm 0.04$</td>
<td>$-0.55 \pm 0.08 \pm 0.05$</td>
<td>$-0.15$ [stat]</td>
<td>PRL 98 (2007) 211801</td>
</tr>
<tr>
<td>Average</td>
<td>$-0.65 \pm 0.07$</td>
<td>$-0.38 \pm 0.06$</td>
<td>$-0.08$</td>
<td>HFAG correlated average $\chi^2 = 5.8/2$ dof (CL=0.055 $\Rightarrow$ 1.9$\sigma$)</td>
</tr>
</tbody>
</table>

Figures:

- $\pi^+\pi^- S_{CP}$
- $\pi^+\pi^- C_{CP}$
- HFAG

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Averaging Results
Example: $B^0 \rightarrow \pi^+ \pi^-$

- Recall: we do NOT inflate uncertainties

\[ \pi^+ \pi^- S_{CP} \text{ vs } C_{CP} \]

Contours give $2\Delta \ln L = \Delta \chi^2 = 1$, corresponding to 68.7% CL for 2 dof
Example: $b \rightarrow qqs$ penguins

$$\sin(2\beta^\text{eff}) = \sin(2\phi_1^\text{eff}) \hspace{1cm} \text{vs} \hspace{1cm} C_{CP} = -A_{CP}$$

Contours give $-2\Delta_n L = \Delta \chi^2 = 1$, corresponding to 60.7% CL for 2 dof.
Example: $B^0 \rightarrow D^{*+}D^{*-}$

- Correlations with additional non-CP parameters

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$S_{CP} (D^{<em>+}D^{</em>-})$</th>
<th>$C_{CP} (D^{<em>+}D^{</em>-})$</th>
<th>$R_{\perp} (D^{<em>+}D^{</em>-})$</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>$-0.71 \pm 0.16 \pm 0.03$</td>
<td>$0.05 \pm 0.09 \pm 0.02$</td>
<td>$0.17 \pm 0.03$</td>
<td>(stat)</td>
<td>PRD 79, 032002 (2009)</td>
</tr>
<tr>
<td>Belle</td>
<td>$-0.96 \pm 0.25 \pm 0.12$</td>
<td>$-0.15 \pm 0.13 \pm 0.04$</td>
<td>$0.12 \pm 0.04 \pm 0.02$</td>
<td>(stat)</td>
<td>arXiv:0901.4057</td>
</tr>
</tbody>
</table>

Average: $-0.77 \pm 0.14$ $-0.02 \pm 0.08$ $0.16 \pm 0.02$ (stat)

HFAG correlated average $\chi^2 = 2.9/3$ dof (CL=0.41 $\Rightarrow$ 0.8σ)
Example: $B^0 \rightarrow J/\psi K_S$

- Systematic correlations (external nuisance parameters) taken into account

| Parameter: $\sin(2\beta) \equiv \sin(2\phi_1)$ |
|----------------|----------------|----------------|----------------|----------------|
| **Mode**       | **BaBar**      | **Belle**      | **Average**    | **Reference**  |
| Charmonium: $J/\psi K_S$ ($\eta_{CP}=1$) | $0.657 \pm 0.036 \pm 0.012$ | $0.643 \pm 0.038_{\text{stat}}$ | - | BaBar (PRD 73 (2006) 072009) Belle (PRL 98 (2007) 031802) |
| Charmonium: $J/\psi K_L$ ($\eta_{CP}=+1$) | $0.694 \pm 0.061 \pm 0.031$ | $0.641 \pm 0.057_{\text{stat}}$ | - | |
| $J/\psi K^0$ | $0.666 \pm 0.031 \pm 0.013$ | $0.642 \pm 0.031 \pm 0.017$ | $0.655 \pm 0.0244_{(0.022_{\text{statonly}})}$ | CL = 0.62 |
| $\psi(2S) K_S$ ($\eta_{CP}=1$) | $0.897 \pm 0.100 \pm 0.036$ | $0.718 \pm 0.090 \pm 0.031_{N(BB)=657M}$ | $0.798 \pm 0.071_{(0.067_{\text{statonly}})}$ | BaBar (PRD 73 (2006) 072009) Belle (PRD 77 (2008) 051103R) |
| $\chi_c K_S$ ($\eta_{CP}=1$) | $0.614 \pm 0.180 \pm 0.040$ | - | - | |
| $\eta_c K_S$ ($\eta_{CP}=1$) | $0.925 \pm 0.160 \pm 0.057$ | - | - | BaBar (PRD 73 (2006) 072009) |
| $J/\psi K^{*0} | K^{*0} \rightarrow K_S T^0$ ($\eta_{CP}=1$-2$|A_L|^2$) | $0.601 \pm 0.239 \pm 0.087$ | - | - | |
| All charmonium | $0.687 \pm 0.028 \pm 0.012$ | $0.650 \pm 0.029 \pm 0.018$ | $0.670 \pm 0.023_{(0.020_{\text{statonly}})}$ | CL = 0.52 |
| $\chi_c K_S$ ($\eta_{CP}=+1$) | $0.69 \pm 0.02 \pm 0.07^{(*)}$ | - | - | BaBar (PRD 80 (2009) 112001) |
| $J/\psi K_S$, $J/\psi \rightarrow$ hadrons ($\eta_{CP}=+1$) | $1.56 \pm 0.04 \pm 0.02^{(**)}$ | - | - | BaBar (PRD 69 (2004) 052001) |
| All charmonium (incl. $\chi_c K_S$ etc.) | $0.691 \pm 0.031_{(0.028_{\text{statonly}})}$ | $0.650 \pm 0.029 \pm 0.018$ | $0.672 \pm 0.023_{(0.020_{\text{statonly}})}$ | CL = 0.30 |
Example: $B^0 \rightarrow J/\psi K_S$

- Systematic correlations (external nuisance parameters) taken into account

\[ \sin(2\beta) \equiv \sin(2\phi_1) \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value (HFAG FCP 2009 Preliminary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar $\chi_{e0}$ $K_S$</td>
<td>$0.690 \pm 0.520 \pm 0.040 \pm 0.070$</td>
</tr>
<tr>
<td>BaBar $J/\psi$ (hadronic) $K_S$</td>
<td>$1.560 \pm 0.420 \pm 0.210$</td>
</tr>
<tr>
<td>Belle $J/\psi K^0$</td>
<td>$0.642 \pm 0.031 \pm 0.017$</td>
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<tr>
<td>Belle $\psi(2S) K_S$</td>
<td>$0.718 \pm 0.090 \pm 0.031$</td>
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<td>Average HFAG</td>
<td>$0.672 \pm 0.023$</td>
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Averaging Results
When Averages Get Difficult

- Experiments measure different parameters
- Badly behaved parameters, eg. due to
  - low statistics
  - choice of parametrisation
    - NB. “good” choice of parameters from a physics perspective may be a statistically “bad” choice
  - “badly behaved” could mean
    - Non-Gaussian (including non-linear correlations)
    - Dependence of uncertainty of one parameter on central value of another
- Complicated dependence on external nuisance parameters
Example: $A_{FB} (B \rightarrow K^{*}ll)$ distributions

Experiments are using different binnings – cannot be combined
Example: $B_s \rightarrow J/ψφ$
Example: $B_s \rightarrow J/\psi \phi$

- Very complicated expression
- Assumes no direct CP violation (full expression even more complicated!)
- Dependence of physical observables on $\phi_s$ goes as
  - $\cos(\phi_s) \sinh(\Delta \Gamma_s t/2)$
  - $\sin(\phi_s) \sin(\Delta m_s t)$
  - and similar expressions in interference terms with further dependence on strong phase differences
- Fit performed with free (physics) parameters $\phi_s, \Delta \Gamma_s, R_\perp, R_\parallel, \delta_\perp, \delta_\parallel (\tau_s, \Delta m_s)$
- Non-Gaussian effects not surprising (unavoidable?)
\[ \Phi_s (B_s \rightarrow J/\psi \phi) \]

Tevatron measurements using tagged $B_s \rightarrow J/\psi \phi$

Angular analyses of vector-vector final state

Results depend on $\Delta \Gamma$

CDF note 9787

D0 5928-CONF

3166\(\pm\)56 $B_s \rightarrow J/\psi \phi$ events

1967\(\pm\)65 $B_s \rightarrow J/\psi \phi$ events

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Averaging Results
Tevatron measurements using tagged $B_s \rightarrow J/\psi \phi$

Angular analyses of vector-vector final state

Results depend on $\Delta \Gamma$

Combined CDF + D0 result

$2.1\sigma$ from SM

68% CL interval: $\Phi_s$ in $[0.27, 0.59] \cup [0.97, 1.30]$ rad

95% CL interval: $\Phi_s$ in $[0.10, 1.42]$ rad

Allowed values in “reasonable” new physics models
$B_s \to J/\psi\phi$: Latest Combination of results

Latest combination huge improvement on previous efforts

The two experiments perform very similar analyses

Two dimensional ($\Delta \Gamma_s$ vs. $\phi_s$) log-likelihoods are added

But:

- Non-Gaussian regime
  - Uncertainty on $\phi_s$ strongly depends on value of $\Delta \Gamma_s$
- $B_s \to J/\psi\phi$ is not a two-dimensional problem
  - Consistency of results on other variables?
  - Higher dimensional combination would be better
  - Most practical way is simultaneous fit of both data sets

complicated reparametrisation could improve matters?

- Work ongoing at the Tevatron ...

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Averaging Results
Example: $B \rightarrow DK, \ D \rightarrow K_S \pi^+\pi^-$

- Three physics parameters $(r_B, \delta_B, \gamma)$
- Dependence of observables as
  \[ x_\pm = r_B \cos(\delta_B \pm \gamma), \ y_\pm = r_B \sin(\delta_B \pm \gamma) \]
- Using these parameters addresses the problem that $\sigma(\gamma) \sim (r_B)^{-1}$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experiment</th>
<th>$x_+$</th>
<th>$y_+$</th>
<th>$x_-$</th>
<th>$y_-$</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DK$^-$</td>
<td>BaBar N(BB)=383M</td>
<td>$-0.067 \pm 0.043 \pm 0.014 \pm 0.011$</td>
<td>$-0.015 \pm 0.055 \pm 0.006 \pm 0.008$</td>
<td>$0.090 \pm 0.043 \pm 0.015 \pm 0.011$</td>
<td>$0.053 \pm 0.056 \pm 0.007 \pm 0.015$</td>
<td>(stat) (syst) (model)</td>
<td>PRD 78 (2008) 034023</td>
</tr>
<tr>
<td>DK$^-$</td>
<td>Belle N(BB)=657M</td>
<td>$-0.107 \pm 0.043 \pm 0.011 \pm 0.055$</td>
<td>$-0.067 \pm 0.059 \pm 0.018 \pm 0.063$</td>
<td>$0.105 \pm 0.047 \pm 0.011 \pm 0.064$</td>
<td>$0.177 \pm 0.060 \pm 0.018 \pm 0.054$</td>
<td>(stat) (model)</td>
<td>arXiv 0803.3375</td>
</tr>
<tr>
<td>Average</td>
<td>No model error</td>
<td>$-0.087 \pm 0.032$</td>
<td>$-0.037 \pm 0.041$</td>
<td>$0.104 \pm 0.033$</td>
<td>$0.111 \pm 0.042$</td>
<td>(stat+syst)</td>
<td>HFA correlated average $\chi^2 = 3.1/4$ dof (CL=0.54 $\Rightarrow$ 0.60)</td>
</tr>
</tbody>
</table>

Figures:
NB. The contours in these plots do not include model errors.

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Averaging Results
Example: $B \rightarrow DK, \ D \rightarrow K_S \pi^+ \pi^-$

- Three physics parameters ($r_B$, $\delta_B$, $\gamma$)
- Dependence of observables as $x^\pm = r_B \cos(\delta_B \pm \gamma)$, $y^\pm = r_B \sin(\delta_B \pm \gamma)$

Using these parameters addresses the problem that $\sigma(\gamma) \sim (r_B)^{-1}$.

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Averaging Results
Example: $B \rightarrow DK, \ D \rightarrow K_S \pi^+\pi^-$

- Problem is the complicated dependence on the Dalitz plot model
  - Effectively, a 4-dimensional nuisance parameter ...
    
    (A typical nuisance parameter is 0-dimensional)

- ... that depends on position in the $(x_\perp, y_\perp)$ plane

- Experiments use different models & assign different uncertainties

- Ideally, HFAG should
  - Rescale results to a common model

  - Then perform the average

  - Assign a model uncertainty to the result of the average
Example: $B \rightarrow DK$, $D \rightarrow K_S \pi^+ \pi^-$

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  - Effectively, a 4-dimensional nuisance parameter ...
    (A typical nuisance parameter is 0-dimensional)

- ... that depends on position in the $(x_\perp, y_\perp)$ plane

- Experiments use different models & assign different uncertainties

- Ideally, HFAG should
  - Rescale results to a common model
    almost impossible – do nothing
  - Then perform the average
    OK
  - Assign a model uncertainty to the result of the average
    very difficult – do nothing
http://ckm2010.warwick.ac.uk/
University of Warwick, UK, September 6-10, 2010

Please come!