Extracting weak phases from Dalitz plots

Tim Gershon
University of Warwick

Workshop on Amplitude Analysis in Hadron Spectroscopy
ECT* Trento

26th January 2011
Current status of CP violation and the CKM matrix

\[ V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \]

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

- CKM mechanism confirmed
  - All measurements of quark mixing & CP violation consistent with CKM paradigm
  - Several possible hints for effects of physics beyond the SM (\(A_{SL}, \beta_s, K^*\pi^+\pi^-, B \to \tau\nu\))
  - Large contributions from new physics not excluded
Experimental situation

- B factories have completed data taking
  - publication rate still impressive
- Next generation experiments will provide much larger data samples of b and c hadrons
  - exciting new possibilities for analysis
- LHCb (CERN) – taking data since 2009
  - main focus of this talk
- Super B factories – data taking anticipated ~2015+
  - SuperKEKB/Belle2 (KEK, Japan)
  - SuperB (LNF, Frascati)
The LHCb detector

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Weak Phases from Dalitz Plots
LHCb first physics

“Prompt charm production in pp collisions at $\sqrt{s} = 7$ TeV”
LHCb-CONF-2010-013

“Measurement of the $J/\psi$ production cross-section at $\sqrt{s} = 7$ TeV in LHCb”
LHCb-CONF-2010-010

“Measurement of $\sigma(pp \to b\bar{b}X)$ at $\sqrt{s} = 7$ TeV in the forward region”
Copious yields at LHCb (e.g. in charm)

\[ D^{\pm} \rightarrow D\pi^{\pm}; \ D \rightarrow K\pi \]

\[ D^{\pm} \rightarrow D\pi^{\pm}; \ D \rightarrow KK \]

\[ D^{\pm} \rightarrow KK\pi^{\pm} \]

Copious samples of charm already available
- e.g. \(10^5\) \(D^{\pm} \rightarrow D\pi^{\pm};\ D \rightarrow KK\) events in 34/pb
- c.f. Belle: \(\sim 3 \times 10^5\) in 384/fb

Challenge is to control systematics to necessary level
- work in progress – expect world's best results in 2011
Introduction

• **Dalitz plot analyses have been around for a long time**
  - “On the analysis of tau-meson data and the nature of the tau-meson.”
    - R.H. Dalitz, Phil. Mag. 44 (1953) 1068

• **Only more recently have they been used to obtain information about weak phases (CP violation)**
  - No observation (5σ) of direct CPV in any Dalitz plot analysis yet

• **I will distinguish between methods that provide “qualitative” measures of CP violation and those that allow “quantitative” extraction of weak phases**
  - *i.e.* Unitarity Triangle angles $\alpha$, $\beta$, $\gamma$ as well as $\beta_s$ and $\varphi_D$
  - I will focus mainly on the “quantitative” approaches and on B physics
Qualitative measures of CP violation in Dalitz plots

An incomplete bibliography (more literature outside B physics)

- “B meson CP violation without flavor identification”

- “CP violation in B mesons using Dalitz plot asymmetries”

- “Direct CP violation in untagged B meson decays”
  - S.Gardner, PLB 553 (2003) 261

- “Observing direct CP violation in untagged B meson decays”

- “A New 'Miranda' Procedure for Dalitz CP Studies”
  - I.Bediaga et al., PRD 80 (2009) 096006
“Miranda” procedure a.k.a. Dalitz plot anisotropy

\[ D_P S_{CP} \equiv \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}} \]

Toy model (using \( B^+ \to K^+\pi^+\pi^- \))

- Without CP violation: Gaussian
- With CP violation: Not Gaussian

- Good model-independent way to identify CP violation
- Could be sufficient to identify non-SM physics in, e.g., charm decays
- Constant (DP independent) systematic asymmetries can be accounted for
- Can isolate region of the Dalitz plot where CP violation effects occur

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Weak Phases from Dalitz Plots

But does not provide quantitative measure of weak phase
Quantitative methods –

Time-dependent Dalitz plot analyses
Snyder-Quinn method for $\alpha$

Measuring $CP$ asymmetry in $B \rightarrow \rho \pi$ decays without ambiguities

Arthur E. Snyder and Helen R. Quinn
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 24 February 1993)

- Methods to measure $\alpha$ exploit time-dependent $CP$ violation in $B_d$ decays via $b \rightarrow u$ transitions (eg. $B_d \rightarrow \pi^+\pi^-$)

- Penguin “pollution” can be subtracted using Gronau-London isospin triangles built from $A(\pi^+\pi^-)$, $A(\pi^+\pi^0)$, $A(\pi^0\pi^0)$

- Expect discrete ambiguities in the solution for $\alpha$

- Ambiguities can be resolved if you measure both real and imaginary parts of $\lambda = (q/p)(\bar{A}/A)$
  - ie. measure $\cos(2\alpha)$ as well as $\sin(2\alpha)$
**B → π⁺π⁻π⁰ − B factory results**

- Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004

\[ B \rightarrow \pi^+\pi^-\pi^0 \]

**FIG. 10:** Proper time distributions of good tag \((r > 0.5)\) regions for \(f_{tag} = B^0\) (upper) and \(f_{tag} = \bar{B}^0\) (middle upper), in \(\rho^+\pi^-\) (left), \(\rho^-\pi^+\) (middle), \(\rho^0\pi^0\) (right) enhanced regions, where solid (red), dotted, and dashed curves correspond to signal, continuum, and BB PDFs. The middle lower and lower plots show the background-subtracted asymmetries in the good tag \((r > 0.5)\) and poor tag \((r < 0.5)\) regions, respectively. The significant asymmetry in the \(\rho^-\pi^+\) enhanced region (middle) corresponds to a non-zero value of \(U_{13}^\ast\).
B → π⁺π⁻π⁰ − B factory results

• Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004

\[ \rho^+\pi^+ A_{CP}, \rho^+\pi^+ C, \rho^0\pi^0 C \]

\[ \neq 0 \text{ at } 3\sigma \]
B → π⁺π⁻π⁰ – B factory results

• Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004

<table>
<thead>
<tr>
<th>Experiment</th>
<th>A⁻⁺ (ρ⁺⁻π⁻⁺)</th>
<th>A⁺⁻ (ρ⁻⁺π⁺⁻)</th>
<th>Correlation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar N(BB)=375M</td>
<td>−0.37 ±0.16 −0.10 ± 0.09</td>
<td>0.03 ± 0.07 ± 0.04</td>
<td>0.62</td>
<td>PRD 76 (2007) 012004</td>
</tr>
<tr>
<td>Belle N(BB)=449M</td>
<td>0.08 ± 0.16 ± 0.11</td>
<td>0.21 ± 0.08 ± 0.04</td>
<td>0.47</td>
<td>PRL 98 (2007) 221602</td>
</tr>
</tbody>
</table>
| Average    | −0.18 ± 0.12 | 0.11 ± 0.06 | 0.40 | HFAG correlated average
  \chi^2 = 4.0/2 dof (CL=0.14 ⇒ 1.5σ) |

ρ⁺⁻π⁻⁺ DCPV A⁻⁺ vs A⁺⁻

≠0 at 3σ
(same as before)
B → π⁺π⁻π⁰ – B factory results

• Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004

Contour from $B \rightarrow \pi^{+}\pi^{-}\pi^{0}$ only

Including also information on $B^{+} \rightarrow \rho^{+}\pi^{0}$ and $B^{+} \rightarrow \rho^{0}\pi^{+}$
$B \to \pi^+\pi^-\pi^0$ – B factory results

- Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004
B → π⁺π⁻π⁰ – model dependence

• Nominal model
  – includes (ρ(770), ρ(1450), ρ(1450)) x (+,−,0)
  – Gounaris-Sakurai lineshape
  – largest source of model dependence from varying parameters of ρ' & ρ''

• Possible contributions from π⁺π⁻ S-wave (σ or nonresonant)?

• Not apparent in the (π⁺π⁻π⁰) data so far
Cross-check model (and extract $\gamma$?) from $B^+ \rightarrow \pi^+\pi^+\pi^-$

- Exploit interference between $b \rightarrow c\bar{c}d$ and $b \rightarrow u\bar{u}d$
  
  $B^+ \rightarrow \chi_{c0}\pi^+$ and charmless $B^+ \rightarrow \pi^+\pi^+\pi^-$ (eg. $B^+ \rightarrow \rho^0\pi^+$)

- Most recent analysis – no signal for $\chi_{c0}\pi^+$ → no sensitivity

But significant S-wave contribution ($NR + f_0(1370)$) seen
Cross-check model (and extract γ?) from $B^+ \rightarrow \pi^+\pi^+\pi^-$

TABLE III: Summary of measurements of branching fractions (averaged over charge conjugate states) and CP asymmetries. The first error is statistical, the second is systematic and the third represents the model dependence. Also included are 90% CL upper limits of the branching fractions of the components that do not have statistically significant fit fractions.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fit Fraction (%)</th>
<th>$B(B^\pm \rightarrow \text{Mode})(10^{-6})$</th>
<th>$A_{CP} (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm\pi^\pm\pi^\mp$</td>
<td>53.2 ± 3.7 ± 2.5</td>
<td>8.1 ± 0.7 ± 1.2</td>
<td>+18 ± 7 ± 5</td>
</tr>
<tr>
<td>$\rho^0(770)\pi^\pm; \rho^0(770) \rightarrow \pi^+\pi^-$</td>
<td>9.1 ± 2.3 ± 2.4</td>
<td>1.4 ± 0.4 ± 0.4</td>
<td>-6 ± 28 ± 20</td>
</tr>
<tr>
<td>$f_2(1270)\pi^\pm; f_2(1270) \rightarrow \pi^+\pi^-$</td>
<td>5.9 ± 1.6 ± 0.4</td>
<td>0.9 ± 0.2 ± 0.1</td>
<td>+41 ± 25 ± 13</td>
</tr>
<tr>
<td>$f_0(1370)\pi^\pm; f_0(1370) \rightarrow \pi^+\pi^-$</td>
<td>18.9 ± 3.3 ± 2.6</td>
<td>2.9 ± 0.5 ± 0.5</td>
<td>+72 ± 15 ± 14</td>
</tr>
<tr>
<td>$\pi^\pm\pi^\mp\pi^\mp$ nonresonant</td>
<td>34.9 ± 4.2 ± 2.9</td>
<td>5.3 ± 0.7 ± 0.6</td>
<td>-14 ± 14 ± 7</td>
</tr>
<tr>
<td>$f_0(980)\pi^\pm; f_0(980) \rightarrow \pi^+\pi^-$</td>
<td>-</td>
<td>&lt; 1.5</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_{c0}\pi^\mp; \chi_{c0} \rightarrow \pi^+\pi^-$</td>
<td>-</td>
<td>&lt; 0.1</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_{c2}\pi^\mp; \chi_{c2} \rightarrow \pi^+\pi^-$</td>
<td>-</td>
<td>&lt; 0.1</td>
<td>-</td>
</tr>
</tbody>
</table>

But significant S-wave contribution ($\text{NR} + f_0(1370)$) seen

BaBar PRD 79 (2009) 072006
More time-dependent Dalitz plot analyses

\[ B_d^0(t) \rightarrow DPP \] time-dependent Dalitz plots,
\[ CP \]-violating angles 2\( \beta \), 2\( \beta + \gamma \), and discrete ambiguities

J. Charles 1, A. Le Yaouanc 2, L. Oliver 3, O. Pène, J.-C. Raynal

Laboratoire de Physique Théorique et Hautes Énergies 4, Université de Paris XI, Bâtiment 211, 91405 Orsay Cedex, France

- \( B_d \rightarrow D^+D^-\pi^0 \) (\( b \rightarrow ccd \) transition)
  - measures 2\( \beta \), never done yet

- \( B_d \rightarrow D^+D^-K_S \) (\( b \rightarrow ccs \) transition)
  - measures 2\( \beta \), never done yet (but note \( B_d \rightarrow D^{\ast+}D^{\ast-}K_S \) analyses by both B factories)

- \( B_d \rightarrow D_{CP}^+\pi^+\pi^- \) (\( b \rightarrow cud \) transition)
  - measures 2\( \beta \), never done yet

- \( B_d \rightarrow D^+\pi^-K_S \) (\( b \rightarrow cus \) transition)
  - measures 2\( \beta + \gamma \), done by BaBar
\[ \mathbf{B}_d \rightarrow \mathbf{D}\pi^+\pi^- \]

- Neutral D mesons conveniently reconstructed as either
  - \( \mathbf{D} \rightarrow \mathbf{K}\pi \) (quasi-flavour-specific); \( \mathbf{D} \rightarrow \mathbf{K}\mathbf{K}, \pi\pi \) (CP-eigenstate)

All data

Signal enhanced

Green dashed – background
Black dotted – signal

BABAR-CONF-10/004, arXiv:1007.4464
(see also Belle PRD 76 (2007) 012006 &
related analyses of \( \mathbf{B}^+ \rightarrow \mathbf{D}^-\pi^+\pi^+ \) )
Neutral $B$ mesons conveniently reconstructed as either $D \to K\pi$ (quasi-flavour-specific); $D \to KK, \pi\pi$ (CP-eigenstate).

Model agrees well with data but contains some interesting features.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Fit Fraction (%)</th>
<th>$B(B^0 \to \text{Mode}) \times B(R \to hh) \times 10^{-4}$</th>
<th>$B(B^0 \to \text{Mode}) \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive $B^0 \to D^0 \pi^+\pi^-$</td>
<td>...</td>
<td>...</td>
<td>8.81±0.18±0.76±0.78±0.11</td>
</tr>
<tr>
<td>$D_2^0$ (2460)$^-$ $\pi^+$</td>
<td>20.5±0.9±1.3±3.7</td>
<td>1.80±0.09±0.19±0.37±0.02</td>
<td>...</td>
</tr>
<tr>
<td>$D_0^+$ (2400)$^-\pi^+$</td>
<td>24.8±2.5±3.0±12.9</td>
<td>2.18±0.23±0.33±1.15±0.03</td>
<td>...</td>
</tr>
<tr>
<td>$\rho(770)^0 D^0$</td>
<td>33.4±2.0±5.2±10.0</td>
<td>2.94±0.19±0.53±0.92±0.04</td>
<td>2.98±0.19±0.53±0.93±0.04</td>
</tr>
<tr>
<td>$f_2(1270)D^0$</td>
<td>9.8±1.1±1.6±3.4</td>
<td>0.86±0.10±0.16±0.31±0.01</td>
<td>1.02±0.12±0.18±0.36±0.03</td>
</tr>
<tr>
<td>$D_s^0$ (2010)$^-\pi^+$</td>
<td>15.8±0.9±1.2±3.7</td>
<td>1.39±0.08±0.16±0.35±0.02</td>
<td>...</td>
</tr>
<tr>
<td>$D\pi$ nonresonant</td>
<td>18.4±2.3±4.3±13.6</td>
<td>1.62±0.21±0.41±1.21±0.02</td>
<td>...</td>
</tr>
<tr>
<td>K matrix total</td>
<td>25.6±2.5±3.2±6.1</td>
<td>2.26±0.22±0.34±0.58±0.03</td>
<td>...</td>
</tr>
</tbody>
</table>

First use of K matrix in B decays

BABAR-CONF-10/004, arXiv:1007.4464 (see also Belle PRD 76 (2007) 012006)
Time-dependent Dalitz plot analyses of charmless hadronic B decays

- **$B_d \to K_s K^+ K^-$ (mainly $b \to sss$)**
  - $\sin(2\beta^{\text{eff}})$ ambiguity for $\varphi K_s$ broken by interference with $K^+ K^-$ S-wave

- **$B_d \to K_s \pi^+ \pi^-$**
  - $b \to uus$ tree and $b \to sqq$ penguin
  - also useful for $\gamma$ measurement

\[ \sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}}) \]

Figure 6: The change in the value of $-2\log(L)$ as a function of $\beta_{\varphi}$, for (a) the whole DP, (b) the High-mass region, (c) $c(800)$, and (d) $c(2000)$.

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Weak Decays, CPV & CKM

Quantitative methods –
Weak phases from direct CP violation in Dalitz plot analyses
\[ \gamma \text{ from } B^+ \rightarrow K^+\pi^+\pi^- \]

- Can apply same argument as for \( B^+ \rightarrow \pi^+\pi^+\pi^- \), but now interference is between \( b \rightarrow ccs \) and \( b \rightarrow uus \)
  - \( B^+ \rightarrow \chi_{c0}K^+ \) and charmless \( B^+ \rightarrow K^+\pi^+\pi^- \) (eg. \( B^+ \rightarrow \rho^0K^+ \))


- Large penguin contribution with different weak phase
  - method is not theoretically clean
  - use flavour symmetries to reduce uncertainties
\[ \mathbf{B}^+ \rightarrow \mathbf{K}^+ \pi^+ \pi^- \]

Model includes:
- \( K^*(892)\pi^+ \), \( K_2^*(1430)\pi^+ \)
- \((K\pi)^*_0\pi^+ \) (LASS lineshape)
- \( \rho^0(770)K^+ \), \( \omega(782)K^+ \), \( f_0(980)K^+ \), \( f_2(1270)K^+ \), \( \chi_{c0}K^+ \)
- \( f_\chi(1300)K^+ \), phase-space nonresonant

BaBar PRD 78 (2008) 012004
See also Belle PRL 96 (2006) 251803
\[ B^+ \rightarrow K^+ \pi^+ \pi^- \]

**TABLE II: Summary of measurements of branching fractions (averaged over charge conjugate states) and CP asymmetries.** Note that these results are not corrected for secondary branching fractions. The first uncertainty is statistical, the second is systematic, and the third represents the model dependence. The final column is the statistical significance of direct CP violation determined as described in the text.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fit fraction (%)</th>
<th>( B(B^+ \rightarrow \text{Mode}) \times 10^{-6} )</th>
<th>( A_{CP} ) (%)</th>
<th>DCPV sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+ \pi^- \pi^+ ) total</td>
<td></td>
<td>54.4 ± 1.1 ± 4.5 ± 0.7</td>
<td>2.8 ± 2.0 ± 2.0 ± 1.2</td>
<td></td>
</tr>
<tr>
<td>( K^{*0}(892)\pi^+ ); ( K^{*0}(892) \rightarrow K^+ \pi^- )</td>
<td>13.3 ± 0.7 ± 0.7 ± 0.9</td>
<td>7.2 ± 0.4 ± 0.7 ± 0.5</td>
<td>+3.2 ± 5.2 ± 1.1 ± 0.7</td>
<td>0.9σ</td>
</tr>
<tr>
<td>( (K\pi)^{0}<em>{0} \pi^+ ); ( (K\pi)^{0}</em>{0} \rightarrow K^+ \pi^- )</td>
<td>45.0 ± 1.4 ± 1.2 ± 0.2</td>
<td>24.5 ± 0.9 ± 2.1 ± 0.11</td>
<td>+3.2 ± 3.5 ± 2.0 ± 1.9</td>
<td>1.2σ</td>
</tr>
<tr>
<td>( \rho^0(770)K^+ ); ( \rho^0(770) \rightarrow \pi^+ \pi^- )</td>
<td>6.54 ± 0.81 ± 0.58 ± 0.26</td>
<td>3.56 ± 0.45 ± 0.43 ± 0.15</td>
<td>+44 ± 10 ± 4 ± 13</td>
<td>3.7σ</td>
</tr>
<tr>
<td>( f_0(980)K^+ ); ( f_0(980) \rightarrow \pi^+ \pi^- )</td>
<td>18.9 ± 0.9 ± 1.7 ± 2.8</td>
<td>10.3 ± 0.5 ± 1.3 ± 1.5</td>
<td>-10.6 ± 5.0 ± 1.1 ± 3.4</td>
<td>1.8σ</td>
</tr>
<tr>
<td>( \chi_{c0}K^+ ); ( \chi_{c0} \rightarrow \pi^+ \pi^- )</td>
<td>1.29 ± 0.19 ± 0.15 ± 0.03</td>
<td>0.70 ± 0.10 ± 0.10 ± 0.06</td>
<td>-14 ± 15 ± 3 ± 5</td>
<td>0.5σ</td>
</tr>
<tr>
<td>( K^+ \pi^- \pi^+ ) nonresonant</td>
<td>4.5 ± 0.9 ± 2.4 ± 0.6</td>
<td>2.4 ± 0.5 ± 1.3 ± 0.3</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( K_2^{0}(1430)\pi^+ ); ( K_2^{0}(1430) \rightarrow K^+ \pi^- )</td>
<td>3.40 ± 0.75 ± 0.42 ± 0.09</td>
<td>1.85 ± 0.41 ± 0.28 ± 0.08</td>
<td>+5 ± 23 ± 4 ± 18</td>
<td>0.2σ</td>
</tr>
<tr>
<td>( \omega(782)K^+ ); ( \omega(782) \rightarrow \pi^+ \pi^- )</td>
<td>0.17 ± 0.24 ± 0.03 ± 0.05</td>
<td>0.09 ± 0.13 ± 0.02 ± 0.03</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( f_2(1270)K^+ ); ( f_2(1270) \rightarrow \pi^+ \pi^- )</td>
<td>0.91 ± 0.27 ± 0.11 ± 0.24</td>
<td>0.50 ± 0.15 ± 0.07 ± 0.13</td>
<td>-85 ± 22 ± 13 ± 22</td>
<td>3.5σ</td>
</tr>
<tr>
<td>( f_X(1300)K^+ ); ( f_X(1300) \rightarrow \pi^+ \pi^- )</td>
<td>1.33 ± 0.38 ± 0.86 ± 0.04</td>
<td>0.73 ± 0.21 ± 0.47 ± 0.02</td>
<td>+28 ± 26 ± 13 ± 7</td>
<td>0.6σ</td>
</tr>
</tbody>
</table>

- \( f_X(1300)K^+ \), phase-space nonresonant

**Evidence for direct CP violation**

But significant model dependence
$B^+ \rightarrow K^+ \pi^+ \pi^-$

Evidence for direct CP violation
But significant model dependence

![Graphs showing invariant mass distribution](image)

### Table

<table>
<thead>
<tr>
<th>$\mathcal{B}(B^+ \rightarrow \text{Mode})$ (10^{-6})</th>
<th>$A_{CP}$ (%)</th>
<th>DCPV sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$54.4 \pm 1.1 \pm 4.5 \pm 0.7$</td>
<td>$2.8 \pm 2.0 \pm 2.0 \pm 1.2$</td>
<td></td>
</tr>
<tr>
<td>$7.2 \pm 0.4 \pm 0.7 \pm 0.3 \pm 0.5$</td>
<td>$+3.2 \pm 5.2 \pm 1.1 \pm 1.1 \pm 1.1$</td>
<td>$+0.7 \pm 1.2 \pm 0.7$</td>
</tr>
<tr>
<td>$24.5 \pm 0.9 \pm 2.1 \pm 0.9 \pm 1.1$</td>
<td>$+3.2 \pm 3.5 \pm 2.0 \pm 1.9 \pm 1.1$</td>
<td>$+2.7 \pm 2.7 \pm 2.7 \pm 2.7$</td>
</tr>
<tr>
<td>$3.56 \pm 0.45 \pm 0.43 \pm 0.38 \pm 0.15$</td>
<td>$+44 \pm 10 \pm 4.5 \pm 4.5 \pm 4.5$</td>
<td>$+5 \pm 5 \pm 5 \pm 5 \pm 5$</td>
</tr>
<tr>
<td>$10.3 \pm 0.5 \pm 1.3 \pm 1.3 \pm 1.5 \pm 1.5$</td>
<td>$-106 \pm 5.0 \pm 1.1 \pm 1.1 \pm 1.1 \pm 1.1$</td>
<td>$-13 \pm 13 \pm 13 \pm 13 \pm 13$</td>
</tr>
</tbody>
</table>

### Notes

- Evidence for direct CP violation
- But significant model dependence

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Weak Phases from Dalitz Plots

*BaBar PRD 78 (2008) 012004*

See also Belle PRL 96 (2006) 251803
γ from $B^0 \to K\pi\pi$

- Use $B_d \to K^{*+}\pi^-$ and $B_d \to K^{*0}\pi^0$
  - form isospin triangles
  - $A_{ij} = A(B^0 \to K^{*i}\pi^j)$

- Both contribute to $B_d \to K^+\pi^−\pi^0$
  - determine $\phi = \arg(\frac{A_{00}}{A_{+-}})$

- Need relative phase between $B_d$ and $\bar{B}_d$
  - determine $\Delta\phi = \arg(\frac{A_{+-}}{\bar{A}_{+-}})$ from time-dependent analysis of $B_d \to K_S\pi^+\pi^-$

- Can now extract $\Phi_{3/2} \approx \gamma$ (with corrections due to EW penguins)
$\gamma$ from $B^0 \to K\pi\pi$ – $B$ factory results

- $B_d \to K^+\pi^-\pi^0$ results
  - multiple solutions reduce precision
  - improvement expected with updated analysis (arXiv:0807.4567)

- $B_d \to K_S\pi^+\pi^-$ results

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Weak Phases from Dalitz Plots

BaBar PRD 78 (2008) 052005
(two solutions)
\[ \bar{\eta} = \tan \Phi_{3/2} [\bar{\rho} - 0.24 \pm 0.03] \]

- **$B_d \rightarrow K^+ \pi^- \pi^0$ results**
  - multiple solutions reduce precision
  - improvement expected with updated analysis (arXiv:0807.4567)

- **$B_d \rightarrow K^0 \pi^+ \pi^-$ results**
  - does not include all latest data

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Weak Phases from Dalitz Plots

Does not include all latest data
Other Related Ideas

- Exactly the same thing but with $B_s \rightarrow K^{*+}\pi^-$ and $B_s \rightarrow K^{*0}\pi^0$
  - analysis then similar to the Snyder-Quinn method for $\alpha$

- A similar idea with $B_s \rightarrow K^{*+}K^-$ and $B_s \rightarrow K^{*0}K_S$
  - some complications since $K^+\pi^-K_S$ not flavour-specific

- Use isospin to relate $B^+ \rightarrow K^+\pi^+\pi^-$ to $B_d \rightarrow K_S\pi^+\pi^-$
  - don't need $\pi^0$ – good for LHCb
  - untagged analysis also possible

Ciuchini et al., PLB 645 (2007) 201
Ciuchini et al., hep-ph/0602207v1
Bediaga et al., PRD 76 (2007) 073011

All good opportunities for LHCb
B $\rightarrow$ KKK

**BaBar** $B \rightarrow K^+K^-K^+$
PRD 74 (2006) 032003
(also Belle PRD 71 (2005) 092003)

**BaBar** $B \rightarrow K^+K^-K_S$
arXiv:0808.0700
(also Belle PRD 82 (2010) 073011)

**BaBar** $B \rightarrow K_SK_SK_S$
CKM2008 preliminary

Large nonresonant components
Poorly understood scalar (?) contributions
\( \gamma \) from \( B \to DK \) type decays involving Dalitz plots
How To Measure $\gamma$

- Focus on theoretically pristine measurement
  - Interference between

$\propto V_{cb} V_{us}^*$

- colour allowed
- final state contains $D^0$

$\propto V_{ub} V_{cs}^*$

- colour suppressed
- final state contains $\bar{D}^0$

Relative magnitude of suppressed amplitude is $r_B$

Relative weak phase is $-\gamma$, relative strong phase is $\delta_B$
Use of Dalitz plots in γ measurement

- Problems of conventional (GLW/ADS) B → DK analyses
  - ambiguities
  - unknown hadronic parameters
  - lack of statistics
- Good way to address all these: study D decay Dalitz plot (typically $K_s \pi^+ \pi^-$)
  - Dalitz plot analysis can disentangle relative amounts, and relative phases, of $D^0$ and $\bar{D}^0$ contributions
  - Very successfully applied by the B factories
    - Giri et al., PRD 68 (2003) 054018; Bondar @ BINP Belle Dalitz plot workshop
  - Similar idea: $B \to D h^0$ (time-dependent) → determination of $\cos(2\beta)$

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Weak Phases from Dalitz Plots
**B → DK, D → K_S π^+ π^- results**

**BaBar PRL 105 (2010) 121801**

Model: NR + 8 resonant component + LASS (Kπ)_0 + K matrix (ππ)_0

\[ \gamma = (68 \pm 14 \pm 4 \pm 3)^\circ \]

**Belle PRD 81 (2010) 112002**

Model: NR + 18 resonant components

\[ \gamma = (78.4^{+10.8}_{-11.6} \pm 3.6 \pm 8.9)^\circ \]

**Model dependence**

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How to resolve model dependence

- Use CP-tagged D mesons to provide model-independent information about phase variation

\[
A(D_{CP} \to f) = \frac{1}{\sqrt{2}} \left| A(D^0 \to f) \pm A(D^0 \to f) \right|
\]

\[
|A(D_{CP} \to f)|^2 = \frac{1}{2} \left( |A(D^0 \to f)|^2 + |A(D^0 \to f)|^2 \pm 2 |A(D^0 \to f)||A(D^0 \to f)| \cos(\delta) \right)
\]

- Can be done by charm factory ($\Psi(3770) \to DD$)
  - CLEOc / BES / SuperB
- First measurements from CLEOc
  - $D \to K\pi$ PRL 100 (2008) 221801, PRD 78 (2008) 012001
  - $D \to K\pi\pi, K3\pi$ PRD 80 (2009) 031105
  - $D \to K_S\pi\pi, K_SKK$ PRD 80 (2009) 032002, PRD 82 (2010) 112006
Model-independent results on $D \to K_S \pi \pi$

$(C_i, S_i) \equiv$ weighted average of $(\cos \delta, \sin \delta)$ across bin $i$
y from $B^0 \rightarrow DK^{*0}$

- $B^0 \rightarrow DK^{*0}$ has many attractive features
  - large CP violation expected $\rightarrow$ good sensitivity to $y$
  - flavour specific $\rightarrow$ time-dependent analysis not required
  - all charged final state

- But finite width of $K^{*0}$ could be a problem
  - other contributions dilute sensitivity

- Turn problem into advantage:
  - Dalitz plot analysis
  - Exploit interference with $D_2^{*}K$, where D flavour is fixed

- But now suffer $B \rightarrow DK\pi$ model dependence
  - Can be removed in $B \rightarrow DK\pi$, $D \rightarrow K_S\pi\pi$ double Dalitz plot analysis


T.G and A. Poluektov, PRD 81 (2010) 014025

Very exciting prospects for LHCb
Other possibilities

- I have of course mentioned only a small subset of the interesting B decay Dalitz plot analyses.

- In the unlikely event that I have any time left by now, I would like to mention:
  
  - \( B_{d,s} \rightarrow J/\psi \pi^+ \pi^- \)
  
  - \( B_{d,s} \rightarrow J/\psi K^+ \pi^- \)
  
  - \( B_{d,s} \rightarrow J/\psi K^+ K^- \)
Summary

• Dalitz plot analyses provide promising methods to measure weak phases and CP violation
• Many attractive features …
• … but significant complications due to model dependence
• Need progress on several fronts
  – Understand better \((\pi\pi), (K\pi), (KK), (D\pi), (DK)\) systems
  – “Nonresonant” contributions and 3-body unitarity
  – Methods to combat model-dependence
  – Nabis initiative set up to try to address this
• Many new possibilities opening up with LHCb
Reminder – CP violation formalism

• Consider decay of neutral particle to a CP eigenstate

\[ \lambda_{CP} = \frac{q}{p} \frac{\bar{A}}{A} \]

- \( |\frac{q}{p}| \neq 1 \)  
- \( |\frac{\bar{A}}{A}| \neq 1 \)
- \( \Im \left( \frac{q}{p} \frac{\bar{A}}{A} \right) \neq 0 \)

CP violation in mixing

CP violation in decay (direct CPV)

CP violation in interference between mixing and decay
LHCb yields in $B^\pm \rightarrow D \pi^\pm$ & $B^\pm \rightarrow D K^\pm$

**B$^\pm \rightarrow D \pi^\pm$ with $D \rightarrow \pi K$**

- $m_B = 5275.16 \pm 0.30$ MeV
- $\sigma_B = 40.661 \pm 3.169$ MeV
- $\sigma_A = 19.461 \pm 0.624$ MeV
- $N_{signal} = 7786 \pm 98$

**B$^\pm \rightarrow D K^\pm$ with $D \rightarrow \pi K$**

- $m_B = 5278.15 \pm 1.57$ MeV
- $\sigma = 21.866 \pm 1.52$ MeV
- $N_{signal} = 444 \pm 30$

LHCb yield with $\sim 34/pb : 444 \pm 30$
c.f. CDF with $1/fb : 516 \pm 37$

**B$^\pm \rightarrow D \pi^\pm$ with $D \rightarrow K K$**

- $m_B = 5275.23 \pm 0.90$ MeV
- $\sigma_B = 17.317 \pm 1.683$ MeV
- $\sigma_A = 53.839 \pm 13.227$ MeV
- $N_{signal} = 1035 \pm 54$

LHCb yield with $\sim 34/pb : 1035 \pm 54$
c.f. CDF with $1/fb : 780 \pm 36$

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Weak Phases from Dalitz Plots
Prospects for $\gamma$ measurement from $B_s \to K^+K^-$

- $B_s \to K^+K^-$
  - LHCb yields in $\sim 35$ pb: $254 \pm 20$ $B_s \to K^+K^-$ & $229 \pm 23$ $B_d \to \pi^+\pi^-$
  - c.f. CDF in $1$ fb: $1307 \pm 64$ $B_s \to K^+K^-$ & $1121 \pm 63$ $B_d \to \pi^+\pi^-$
  - Expect first time-dependent measurements in 2011
    - (including measurement of $B_s$ lifetime in CP-even $K^+K^-$ final state)

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Weak Phases from Dalitz Plots
Prospects for direct CP violation in $B_{d/s} \to K^{\pm} \pi^{\mp}$

- Raw asymmetries clearly visible in existing data
- Central values consistent with expectations & previous measurements
- Calibration and evaluation of systematic uncertainties in progress

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Weak Phases from Dalitz Plots
Toy model for $B \to \pi^+\pi^-\pi^0$ Dalitz plot

Contributions only from $\rho^+\pi^−$, $\rho^−\pi^+$ and $\rho^0\pi^0$

<table>
<thead>
<tr>
<th>Time dependence</th>
<th>Kinematic form</th>
<th>Amplitude measured</th>
<th>$\alpha$ dependence (all $P_\perp = 0$)</th>
</tr>
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<tbody>
<tr>
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<td>$f^+f^+*$</td>
<td>$S_3S_3^* + S_4S_4^*$</td>
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<td>Im($gS_3S_3^*$)</td>
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Note: physical observables depend on either $\sin(2\alpha)$ or $\cos(2\alpha)$ – never "directly" on $\alpha$

f terms contain hadronic physics (lineshape, spin)

$S_3 = A(\rho^+\pi^-)$, $S_4 = A(\rho^-\pi^+)$, $S_5 = A(\rho^0\pi^0)$,
\( B \rightarrow \pi^+\pi^-\pi^0 \) – B factory results

- Results from
  - BaBar, 375 M BB pairs: PRD 76 (2007) 012004
Resolving the sin(2\(\beta\)) ambiguity

- The Dunwoodie method
  - \(B_d \rightarrow J/\psi K_S \pi^0\) (\(b \rightarrow ccs\) transition)
  - Exploit interference between \(K^*(892)\) and \(K^*_0(1430)\)

- NB. ambiguity otherwise unbroken in \(B \rightarrow VV\) analysis

**FIG. 9:** Comparison of the variation of \(\gamma = \delta_S - \delta_0\) with \(m_{K_S}\) for the \(J/\psi K^+ \pi^-\) events, for “Solution I” (open points, Eq. (29)) and “Solution II” (full points, Eq. (30)), with that measured by the LASS experiment [22, 39, 40] (diamond markers).

**FIG. 11:** The distribution of \(\Delta t\) for events in the signal region, for (a) \(B^0\) and (b) \(B^0\) tags with the fit result (full curve) overlaid. In (c) we show the raw asymmetry in the number of \(B^0\) and \(B^0\) tags in the signal region, \((N_{B^0} - N_{B^0})/(N_{B^0} + N_{B^0})\) for data, with the fit result (full curve) overlaid. Note that above distributions are not sensitive to cos2\(\beta\) since this dependence vanishes when integrated over the angular variables.
Prospects for LHCb

- Dalitz plot analyses are not something that we will do with first data
  - and, be warned, they are hard work
- But longer term there are many possibilities
  - many channels that are well suited for LHCb
    - all charged (or nearly all charged) final states
  - some have been looked at before
  - many have not
  - still room for new ideas