Quark flavour physics

Michal Kreps

Plan

- Kaon physics and SM construction (bit of history)
- Establishing SM experimentally
- Looking for breakdown of SM

Hard to cover everything in details in three lectures, some details are offloaded to exercises
Outline – lecture 1

- What is flavour physics and what is not
- Kaon physics – understand its importance for development of standard model of particle physics
  - Weak decays – quark mixing
  - FCNC kaon decays – GIM mechanism
  - Neutral kaon mixing
  - CP violation in neutral kaons
  - How to accommodate CP violation to model
Content of standard model

Fermions ("matter")

\[
\begin{align*}
\text{Quarks} & : \{ uuu, ccc, ttt, ddd, sss, bbb \} \\
\text{Leptons} & : \{ e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau \}
\end{align*}
\]

\[
\times \left\{ \begin{array}{c}
\text{MATTER} \\
\text{ANTIMATTER}
\end{array} \right\}
\]

Bosons ("forces")

\[
\begin{align*}
gg & \\
\gamma & \\
W^+ & \\
W^- & \\
Z & \\
H &
\end{align*}
\]
Parameters of standard model

- 3 gauge couplings
- 2 Higgs parameters
- 6 quark masses
- 3 quark mixing angles + 1 phase
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase)

Flavour parameters

Why 3 generations (are we sure about it)?
Why hierarchy in mass?
Why hierarchy in mixing?
Why do we have only matter in current Universe?
What is not flavour physics

- QCD: Strong interactions
  - Any details of QCD including studies of different “exciting” states
- Electroweak physics
  - There is some relation, but questions do not overlap
- Energy frontier
  - Search for new particles in production (on-shell)
Going to history
Kaon discovery

- 1947, G. D. Rochester and C. C. Butler
- Using fancy detector called cloud chamber

$K^0 \rightarrow \pi^+ \pi^-$

- Produced in strong interaction
- Decay rather slow, lifetime of $10^{-8} - 10^{-10}$ s
Quark mixing - Cabibbo

- Quark content of kaon is $u\bar{s}$ ($K^+$) and $d\bar{s}$ ($K^0$)
- Main decays are
  - $K^+\rightarrow\mu^+\nu_\mu$, $K^+\rightarrow\pi^0\bar{e}^+\nu_e$, $K^0\rightarrow\pi\pi$
  - Weak interaction has to allow transition $s\rightarrow u$
- There are good reasons why $W$ (weak interaction) couples only to left-handed doublets
  - How to construct doublets to allow $s\rightarrow u$ and $d\rightarrow u$?
- Cabibbo provided solution in terms of quark mixing
  - Doublet of weak interaction is $(u,d')=(u,d\cos(\theta)+s\sin(\theta))$
  - $\theta$ is quark mixing angle, which was determined experimentally
- Actually solved difference in $G$ between nuclear and muon decay
Next piece of puzzle comes from FCNC kaon decays

Cabibbo fixed one issue (s→u transition), but introduced another one

If doublet of weak interaction is (u,d'), than also $Z^0$ can couple to d'd'

What does it mean in terms of original quarks?

$$u\bar{u} + d\bar{d} \cos^2 \theta + s\bar{s} \sin^2 \theta + (s\bar{d} + \bar{s}d) \sin \theta \cos \theta$$

First three terms are fine, but last part causes problem

It would allow flavour changing neutral current decays at tree-level

- $K^+ \rightarrow \pi^+ e^+ e^-$ would be approximately 5% of $K^+ \rightarrow \pi^0 e^+ \nu_e$
- But in experiment it was known to be < $10^{-5}$
Is Cabibbo wrong, or can we find some way to fix it?

Not quite with three quarks known at the time, but

In 1970 Glashow, Iliopoulos, Maiani suggested way

To existing doublet \((u,d')=(u,d\cos(\theta)+s\sin(\theta))\) add second one \((c,s')=(c,d\cos(\theta)-s\sin(\theta))\)

Now \(Z^0\) would also couple to \(s's'\) which would give us term like 
\(-(sd+sd)\sin\theta\cos\theta\) which approximately cancel contribution from other doublet

Approximately comes from other factors contributing

So in 1970 charm quark was predicted, despite that not everybody accepted existence of the quarks
Neutral Kaon Oscillation

- Now we come to next puzzle about neutral kaon lifetimes

- Originally two particles were seen with same mass, but very different lifetimes $9 \cdot 10^{-11}$ s and $5 \cdot 10^{-8}$

- They not only have very different lifetimes, but first one decays to $2\pi$ while other to $3\pi$

- Both were produced in same type of interaction in association with other strange particles

- Is this just strange coincidence or is there something more behind it?

- Different decays suggest that lifetimes have something to do with CP symmetry
Neutral kaon mixing

- To explain different lifetimes, let's look to CP properties
  - \( \text{CP} \mid 2\pi \rangle = + \mid 2\pi \rangle \)
  - \( \text{CP} \mid 3\pi \rangle = - \mid 3\pi \rangle \)
- Shorter lived kaon (call it \( |K_1\rangle \)) decays to \( 2\pi \) and is CP-even
- Longer lived kaon (call it \( |K_2\rangle \)) decays to \( 3\pi \) and is CP-odd
- Difference in the lifetimes come from different phase space available in two decays
  - \( m(2\pi) = 279 \text{ MeV}, m(3\pi) = 419 \text{ MeV} \) and \( M(K^0) = 497 \text{ MeV} \)

\[
\frac{d\Gamma}{d^3p_i} = \frac{(2\pi)^4}{2M} |M|^2 \delta^4(P - \sum p_i) \prod \frac{d^3p_i}{(2\pi)^3 2E_i}
\]
Neutral kaon mixing

Now we have to put together fact that in strong interaction we produce $K^0$ or $\bar{K}^0$, while in weak interaction (decay) we have $K_1$ and $K_2$

But we already know from quark mixing, that quarks entering strong and weak interaction are not exactly same

So we can connect kaons from strong interaction to those in weak interaction via mixing

We can define those as

- $K_1 = \frac{1}{2}(K^0 + \bar{K}^0)$
- $K_2 = \frac{1}{2}(K^0 - \bar{K}^0)$
- With CP $|K^0> = + \bar{K}^0$ and CP $|\bar{K}^0> = + K^0$ all fits together
Time evolution

- Now we can start to look at time evolution
- Definitely one set of states will behave like $e^{(\Gamma/2+im)t}$
- Question is which one?
- After deciding above question, it is easy to calculate what to expect at given time for all four states
- There is interesting effect called kaon regeneration which we will discuss during exercises

Blackboard is my friend here
$K^0 - \bar{K}^0$ oscillation

$|K^+_1\rangle = |K^+_1(0)\rangle e^{(-\frac{m_1^2}{2} + i m_1) t}$

$|K^+_2\rangle = |K^+_2(0)\rangle e^{(-\frac{m_2^2}{2} + i m_2) t}$

$|K^-\rangle = \frac{1}{\sqrt{2}} (|K^+_1\rangle + |K^+_2\rangle)$

$\langle K^0(0) | K^0(0) \rangle$

$\frac{i}{2} \left( \langle \bar{K}^- | K^- \rangle + \langle \bar{K}^+_1 | K^+_1 \rangle + \langle \bar{K}^+_2 | K^+_2 \rangle + \langle \bar{K}^+_1 | K^+_2 \rangle \right)$

$\left[ e^{-\frac{m_1^2}{2} t} + e^{-\frac{m_2^2}{2} t} + \frac{e^{(-\frac{m_1^2}{2} + i m_1) t} e^{(-\frac{m_2^2}{2} + i m_2) t}}{e^{-\frac{m_1^2}{2} t} + e^{-\frac{m_2^2}{2} t}} \right]$
CPLEAR experiment at CERN

$\bar{p}p \rightarrow K^+ \pi^- K^0$

$K^- \pi^+ K^0$

200 MeV/c

$10^8 \bar{p}$/s

Particle ID

TOF, dE/dx, Čerenkov detector

Hydrogen target

Drift chambers

Electromagnetic calorimeter

6 drift chambers

3 proportional chambers

Čerenkov scintillator

$\sigma(M_{K^0}) \approx 13$ MeV/$c^2$

$\tau \approx (5 - 10)$ ps
Kaon mixing in experiment

- Gives $\Delta M = 529 \cdot 10^{-7} \text{ s}^{-1}$

Explain details on blackboard
CP Violation

- We defined $K_1$ to be CP-even and $K_2$ CP-odd
- If CP is conserved, than
  - $K_1$ decays only to $2\pi$
  - $K_2$ decays only to $3\pi$
- What happens when CP is violated?
- Could we experimentally test CP violation?
CPV Discovery

Christenson, Cronin, Fitch, Turley
If CP is violated, should see $K_2 \rightarrow 2\pi$

Angle between sum of the momenta of $2\pi$ and beam should be zero

Experiment measured

\[ R = \frac{N(K_2 \rightarrow \pi^+\pi^-)}{N(K_2 \rightarrow \text{all charged})} = (2 \pm 0.4)10^{-3} \]
How to make sense out of it?

\[ k_s = \frac{1}{\sqrt{1+\varepsilon^2}} \left( k_1 + \varepsilon k_2 \right) = \frac{1}{\sqrt{2(1+\varepsilon^2)}} \left( 1 - \varepsilon \right) k_0 + \left( 1 + \varepsilon \right) k_L \]

\[ k_L = \frac{1}{\sqrt{1+\varepsilon^2}} \left( k_2 + \varepsilon k_1 \right) = k_L \]

\[ \varepsilon = \frac{R_\tau}{\tau_2} \]

\[ R_\tau = \frac{2}{3} R \]

\[ \varepsilon = 2.3 \times 10^{-3} \]
Example of $K_L \rightarrow e^+ \pi^- \nu$

$$\Delta = \frac{\left( N(K_L \rightarrow e^+ \pi^- \nu) - N(K_L \rightarrow e^- \pi^+ \nu) \right)}{\left( N(K_L \rightarrow e^+ \pi^- \nu) + N(K_L \rightarrow e^- \pi^+ \nu) \right)}$$

$$\text{Num} = (1+\epsilon)[K^0 \rightarrow \pi^+ e^- \nu] + (1-\epsilon)[\bar{K}^- \rightarrow \pi^- e^+ \nu] - (1+\epsilon)[K^0 \rightarrow \pi^+ e^+ \nu] - (1-\epsilon)[\bar{K}^- \rightarrow \pi^- e^- \nu]$$

Assume $\bar{K}^0 \rightarrow \pi^+ e^- \nu = K^0 \rightarrow \pi^- e^+ \nu$.

$$\text{Num} = -2\epsilon R(K^0 \rightarrow \pi^- e^+ \nu)$$

$$\Delta = 2 \text{ Re} (\epsilon)$$

Experiment

$$\Delta e = 3.39 \times 10^{-3}$$

$$\Delta \mu = 3.04 \times 10^{-3}$$
Indirect CPV

\[ |\psi\rangle = p |K^0\rangle + q |\bar{K}\rangle \]

\[ p, q \text{ - complex numbers} \]

\[ |p|^2 + |q|^2 = 1 \]

\[ \begin{pmatrix} p \\ q \end{pmatrix} \]

\[ i \frac{d}{dt} |\psi\rangle = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{bmatrix} - \frac{i}{2} \begin{bmatrix} \Gamma_{11}^* & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{bmatrix} |\psi\rangle \]

\[ (\frac{q}{p})^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \]

\[ \Gamma_{12}^* \text{ are Hermitian matrices} \]

\[ \text{mass matrix} \Rightarrow \text{oscillation} \]

\[ \text{decay matrix} \Rightarrow \text{decay} \]

\[ \text{eigenstates, eigenvalues} \]

\[ |q| = |p| \Rightarrow M_{12}^* - \frac{i}{2} \Gamma_{12} = M_{12} - \frac{i}{2} \Gamma_{12} \Rightarrow \text{zero phase between } \pi^0 \text{ and } \pi^+ \]
Standard model

\[
\begin{pmatrix}
1 & 2/2 & \lambda \\
-2 & 1 & A \lambda^2 \\
A \lambda^2 (1-p-i\eta) & -A \lambda^2 & 1
\end{pmatrix} = V_{\text{mix}}
\]

\[
\Rightarrow \text{parametrized by rotations and 1 complex phase}
\]

\[
\begin{pmatrix}
d' \\
\bar{s}' \\
b'
\end{pmatrix} =
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d \\
\bar{s} \\
b
\end{pmatrix}
\]

\[
KoJaxu\text{ishi, } \text{Rakawa}
\]

proposed third generation of quarks
Note on CKM matrix

- CKM matrix is unitary matrix
- It has only four parameters
- Product of any two rows or two columns is equal to zero
  - It can be visualized as triangle in complex plane (called unitarity triangle)
- All unitarity triangles have same area given by Jarlskog invariant
- Jarlskog invariant is measure of CP violation in quark sector
- The CKM matrix is hierarchical
Some questions for thinking

- What are implications of observation of CP violation?
- What you would do to confirm Kobayashi-Maskawa mechanism
- If you have answers in terms of experiment, what capabilities experiment has to have?
- How would you determine CKM matrix elements?