Katie
Graham
27/10/14

RQM 9: Coulomb Scattering of Charged Spin-½ Particles

1) Show that the interaction of spin-½ particles proportional to $\bar{u}_f \gamma^\mu u_i$ can be decomposed as

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f \left\{ (p_f + p_i)^\mu - \frac{1}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)(p_f - p_i)_\nu \right\} u_i .$$

[3]

Start with the term $\frac{1}{2} \bar{u}_f (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)(p_f - p_i)_\nu u_i$.

Use $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$:

$$\frac{1}{2}\bar{u}_f(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_f - p_i)_\nu u_i = \frac{1}{2}\bar{u}_f\left\{(2\gamma^\mu\gamma^\nu - 2g^{\mu\nu})p_{f\nu} - (2g^{\mu\nu} - 2\gamma^\nu\gamma^\mu)p_{i\nu}\right\}u_i$$

$$= \bar{u}_f \left\{ \gamma^\mu (i\gamma^\nu \partial_\nu) - p_f^\mu - p_i^\mu + (i\gamma^\nu \partial_\nu)\gamma^\mu \right\} u_i \quad (*)$$

(using $p_\nu = i\partial_\nu$). Now use the Dirac equations for u_i and $\bar{u}_f$:

$$(i\gamma^\mu \partial_\mu - m)u_i = 0 \quad \text{and} \quad \bar{u}_f(i\gamma^\mu\partial_\mu - m) = 0$$

$$\therefore (*) = \bar{u}_f \left\{ 2m\gamma^\mu - (p_f + p_i)^\mu \right\} u_i$$

$\therefore$ dividing by 2m and rearranging,

$$\bar{u}_f \gamma^\mu u_i = \frac{1}{2m} \bar{u}_f \left\{ (p_f + p_i)^\mu + \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_f - p_i)_\nu \right\} u_i$$

wrong sign?

end not too clear, but basically there

3