

Pasta equations

The Objective

When cooking filled pasta like ravioli or tortellini, the process has two objectives:

1. To cook the pasta
2. To heat up the filling.

The equations

If we start with a pan of boiling water at 100 Celsius (100 °C) we take the pasta from the fridge at a temperature of about 5C. The moment it hits the water, the water cools down and the pasta starts to warm up and cook. The heat spreads from the outside in and is governed to a good approximation by the heat loss equation:

$$M_p C_p \frac{dT_p}{dt} = -\kappa A_p (T_p - T_w) \quad (1)$$

At the same time the water is cooled by the pasta but is still being heated up by the cooker. After a while it reaches boiling point again. The temperature of the water is a combination of the heat added by the cooker, the heat lost to the air, the energy lost as water is boiled and the energy that goes into the pasta. We can model that with two equations: before and after the pasta is added.

Boiling steadily at 100C:

$$M_w C_w \frac{dT_w}{dt} = -\kappa_{pan} A_{pan} (T_w - T_a) + P_{in} + L_w M_s = 0 \quad (2)$$

Pasta added, no longer boiling:

$$M_w C_w \frac{dT_w}{dt} = -\kappa_{pan} A_{pan} (T_w - T_a) + P_{in} - M_p C_p \frac{dT_p}{dt} \quad (3)$$

Combining (1), (2) and (3) gives the critical formula for the pasta heating up and the water cooling down:

$$M_p C_p \frac{dT_p}{dt} = -\kappa A_p (T_p - T_w) = -M_w C_w \frac{dT_w}{dt} \quad (4)$$

The combined equations gives the rise in temperature of the pasta (LHS) from the heat going in (centre expression) and the resulting drop in water temperature on the RHS. This allows us to calculate how quickly the pasta cooks and the filling heats up. It also determines how much the water temperature drops depending on the amounts of water and pasta.

The symbols

T_p, T_w, T_a are the all-important pasta, water and ambient (room) temperatures.

M_p, M_w the weights of pasta and water 250g is typical for pasta for two and 1.5 to 2 kg for the water.

C_p, C_w Are the heat capacities of the pasta and water. It is a measure of how much heat is required to raise the temperature one degree. For water it will be 4.2 k J per kg per degree. Pasta will be a bit lower, but very similar.

A_p is the surface area of the pasta, that will be a few cm squared for each pasta piece depending upon the pasta size and shape. For the formula, the total area is required.

κ is the heat conductivity of the pasta. J per square cm per degree per second.

P_{in} is the power given out by the cooker that goes into the pan, measured in Watts

$L_w M_s$ is the term accounting for the energy needed to boil off a certain mass of water per second .

L_w is the latent heat of vaporisation of water (c 2 kJ per gram)to steam

Conclusions

With a few further assumptions and approximations this gives us some simple conclusions:

A) At a very gentle simmer the terms $-\kappa_{pan} A_{pan} (T_w - T_a)$ and P_{in} cancel each other out and the amount of water boiling is negligible so $L_w M_s$ is also zero the temperature is steady at 100C as you would expect.

B) If heat is turned down to give the gentlest of simmers then adding the pasta and waiting gives pasta and water at the same temperature of:

$$T_{equil} \sim \frac{M_w 100 + M_p 10}{M_w + M_p}$$

C) The larger the volume of water the smaller the temperature drop: so for 1.75ltr of water and 250g of pasta it will drop to 88C

D) With more heat from the hob, there will be a battle between the heat in from the cooker and the heat being absorbed by the pasta and the filling. The temperature will still drop the moment the pasta is added but not drop quite as much as in (B) above and will then rise up to 100C.

E) Once out of the pan two effects cool the pasta: it will lose heat to the surroundings and the filling will warm up a bit more at the expense of the pasta casing.

F) The pasta will heat up quicker if the area A_p is larger compared with the total mass of pasta and filling M_p . The volume to surface area ratio. But heating up quicker means the water will drop in temperature faster when it is first added.

G) If the pasta is thinner, then it will cook faster and the heat will transfer faster to the filling: κ will be larger.

- H) Cooking tortellini for one minute to get perfect al dente pasta is difficult. With 1.75 ltr of water, and the pan simmering, the water drops to 95C and takes over one minute to get back up to 100C. When do you start timing the cooking? Well it is still cooking at 95C – a little bit slower than at 100C, but still cooking. So 1 ¼ to 1 ½ minutes after adding the pasta and it is ready.
- I) With al dente pasta there is not much time in the pan to heat the filling. In the example in (H) the tortellini filling was measured at 50C

Tips and tricks

Here are some tips to get the perfect pasta onto your plate. We are aiming for pasta cooked al dente so that it is firm and does not fall apart. We also want the filling to be hot. The tips are not critical, but all help a bit.

- 1) This is England and its winter so get ready: heat the plates before you start – you won't have time once cooking starts:
- 2) Melt some butter or get the olive oil ready, again do this before cooking.
- 3) Take the pasta out of the fridge about an hour before cooking. The filling is fresh, so you must store it in the fridge, but if it starts at room temperature, 20C, rather than fridge, 5C, it all helps.
- 4) Use a large volume of water: this is the most critical tip 1.5 to 2ltr should be fine for 250g of pasta.
- 5) Add a bit of salt, this is for flavour. Technically it will increase the water temperature of boiling water a bit, but you won't notice that with normal amounts of salt.
- 6) Add the pasta and time it from when the pasta hits the water (allow just over one minute for the tortellini). It hardly matters what your cooker setting is because of the short cooking time, but turning it right up will help a bit.
- 7) Avoid vigorous boiling water before or during the cooking because the pasta is delicate – it will fall apart water will fill the pasta and the filling will disappear into the water.
- 8) For larger pasta shapes with longer cooking times: still time the cooking from when the pasta hits the water, but control the cooker settings to get the pan back up to a simmer without vigorous boiling.
- 9) Get the pasta quickly from the pan to the hot plates. Guests and wine should be ready long before this.

Filling and Sauce

A plate of spaghetti bolognese may have equal amounts of pasta and sauce, a 50:50 mixture. That could be sauce on top of spaghetti or mixed in Italian style, Either way 50:50 is quite generous. Ratios down to 30:70 sauce : pasta would be quite reasonable.

The Giovanni Rani filled pasta has the bolognese in the filling but the total amount is close to the generous 50:50 ratio.

Adding Olive oil or melted butter is all that is recommended. A ratio of 50:50:10 gives 25 g oil per 250g of pasta, just over two teaspoons per helping. That is enough to give a thin layer of 200 microns ($1/5^{\text{th}}$ of a mm) over the surface of all the pasta – enough to make it glisten.