Relativisitc Quantum Mechanics

Problem Sheet 1

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Special relativity, Lorentz covariance and the Klein-Gordon equation

1. Show that length
$$A^2 = (A^0)^2 - (A^1)^2 - (A^2)^2 - (A^3)^2$$
is invariant under the Lorentz transformation.

[2]

2. Show that $g_{\mu\nu}g^{\mu\nu} = 4$.

[2]

3. Using Schrödinger's equation and the definition of particle density, $\rho = \psi^*\psi$, show that the system satisfies the continuity equation with a current defined as
$$\vec{j} = \frac{1}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right).$$

[4]

4. Show that plane waves
$$Ne^{-i(wt-\vec{k}\cdot\vec{x})}$$
are solutions of the Klein-Gordon equation. Obtain expressions for the energies of the solutions.

[2]