Relativisitc Quantum Mechanics

Problem Sheet 2

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Transition amplitudes and particle scattering

1. Show that solution of the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = \left[H_0(\vec{x}) + \kappa V(t,\vec{x})\right]\psi$$

written in form of linear superposition of stationary states $\phi_n(\vec{x})$

$$\psi = \sum_m a_m(t)\phi_m(\vec{x})e^{-iE_m t}$$

yields a system of differential equations

$$\frac{\mathrm{d}a_f(t)}{\mathrm{d}t} = -i\kappa\sum_m a_m(t)e^{i(E_f-E_m)t}\int \phi_f^* V(t,\vec{x})\phi_m(\vec{x})\mathrm{d}^3\vec{x}$$

.

[3]

2. Show that the Lorentz invariant phase-space for $A+B\to C+D$ scaterring

$$\mathrm{d}Q = (2\pi)^4\delta^4(p_C+p_D-p_A-p_B)\frac{\mathrm{d}^3\vec{p}_C}{(2\pi)^3 2E_C}\frac{\mathrm{d}^3\vec{p}_D}{(2\pi)^3 2E_D}$$

can be written in polar coordinates as

$$\mathrm{d}Q = \frac{1}{4\pi^2}\frac{|\vec{p}_C|}{4\sqrt{s}}\mathrm{d}\Omega\ ,$$

where $\mathrm{d}\Omega$ is the element of solid angle and $s=(p_A+p_B)^2$. Hence, show that the differential cross-section for the process is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_C|}{|\vec{p}_A|}|\mathcal{M}|^2\ .$$

[4]

3. In the very high-energy limit ($E \gg m$), show that the differential cross section for spinless electron-muon scattering in the centre of mass system becomes

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{CM}} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2 ,$$

where $\alpha = e^2/4\pi$ and θ is the scattering angle.

[**3**]