APTS: Prediction

Prediction in Bayesian Inference

- Model is

$$y \sim f(y|\boldsymbol{\theta}, \mathbf{x}),$$

where

$$\begin{aligned} \boldsymbol{\theta} &\quad - \quad \text{parameters;} \\ \mathbf{x} &\quad - \quad \text{covariates.} \end{aligned}$$

- Posterior predictive distribution for future response y_0 at covariates $\mathbf{x}_0$:

$$\pi(y_0|\mathbf{y}, \mathbf{x}_0) = \int f(y_0|\boldsymbol{\theta}, \mathbf{x}_0)\pi(\boldsymbol{\theta}|\mathbf{y})\mathrm{d}\boldsymbol{\theta}.$$

- Given an MCMC sample $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_M$, a sample from this is produced as follows

$$y_{0j} \sim f(y_0|\boldsymbol{\theta}_j, \mathbf{x}_0).$$

Prediction for LMM

- Prediction from within a subject i with covariate $\mathbf{x}_0$

$$y_0 \sim \mathrm{N}\left(\mathbf{x}_0^T\boldsymbol{\beta} + b_i, \sigma^2\right).$$

- Doing this with MCMC sample $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_M$, $b_{i1}, \ldots, b_{iM}$ and $\sigma_1^2, \ldots, \sigma_M^2$:

$$y_{0j} \sim \mathrm{N}\left(\mathbf{x}_0^T\boldsymbol{\beta}_j + b_{ij}, \sigma_j^2\right).$$

Prediction for LMM

- Prediction from new subject with covariate $\mathbf{x}_0$

$$
\begin{aligned}
y_0 &\sim \mathrm{N}\left(\mathbf{x}^T\boldsymbol{\beta} + b_0, \sigma^2\right), \\
b_0 &\sim \mathrm{N}\left(0, \sigma_b^2\right).
\end{aligned}
$$

- Doing this with MCMC sample $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_M$, $\sigma_1^2, \ldots, \sigma_M^2$ and $\sigma_{b1}^2, \ldots, \sigma_{bM}^2$:

$$
\begin{aligned}
y_{0j} &\sim \mathrm{N}\left(\mathbf{x}_0^T\boldsymbol{\beta}_j + b_{0j}, \sigma_j^2\right), \\
b_{0j} &\sim \mathrm{N}\left(0, \sigma_{bj}^2\right)
\end{aligned}
$$