APTS Design of Experiments - Practical 2

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In this practical session, you will use the `acebayes` package to investigate some aspects of (Bayesian) optimal design. Feel free to also use the practical sessions to understand and reproduce results from the notes.

1. **Bayesian D-optimal design using `optim`**
Adapt the code from the notes to find locally D-optimal designs with $n = 2$ runs for the Poisson regression model with linear predictor $\eta(x) = \beta_0 + \beta_1 x$ with the log link (log-linear regression).

Fix $\beta_0 = 0$ and find designs for $\beta_1 = 1, \ldots, 6$, restricting to $-1 \leq x \leq 1$. For each design, plot the design points along with the expected response. There is a pattern in the optimal designs, can you identify it?

(Hint: there is a relationship between the first design point and $1/\beta_1$).

The `acebayes` package has a function `aceglm` that can find Bayesian optimal designs for generalised linear models. Read the help file, and the use the function to find a Bayesian D-optimal design when we assume $\beta_0 = 0$ and a uniform prior distribution for β_1 with support $[1, 6]$.

Important: when using `aceglm` set $N2 = 0$ (otherwise it will result in a nasty crash!).

2. **Bayesian D-optimal design using `acebayes`**
In the notes, we examine a compartmental model. A simplified version of this model is given by

$$y_i \sim N\left(\mu(x_i\,;\boldsymbol{\theta}), \sigma^2\right)\,,$$

with

$$\mu(x_i\,;\boldsymbol{\theta}) = \theta_3\left[\exp(-\theta_1 x) - \exp(-\theta_2 x)\right]\,.$$

Use the `acenlm` function in `acebayes` to find a Bayesian D-optimal design with $n = 18$ runs, assuming $x \in [0, 24]$. Assume uniform prior distributions for θ_1, θ_2:

$$\theta_1 \sim U(0.01884, 0.09884); \qquad \theta_2 \sim U(0.298, 8.298)$$

with $\theta_3 = 21.8$; see Gotwalt, Jones, and Steinberg (2009) and Overstall, Woods, and Adamou (2017). Plot the optimal design and realisations from the model.

References

Gotwalt, C. M., B. A. Jones, and D. M. Steinberg. 2009. “Fast Computation of Designs Robust to Parameter Uncertainty for Nonlinear Settings.” *Technometrics* 51: 88–95.

Overstall, A. M., D. C. Woods, and M. Adamou. 2017. “acebayes: An R Package for Bayesian Optimal Design of Experiments via Approximate Coordinate Exchange.” *arXiv:1705.08096*.