VISUALISING UNCERTAINTY A SHORT INTRODUCTION

AU4DM
Network

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A SHORT INTRODUCTION

The **Analysis Under Uncertainty for Decision Makers Network** (AU4DM) is a community of researchers and professionals from policy, academia, and industry, who are seeking to develop a better understanding of decision-making to build capacity and improve the way decisions are made across diverse sectors and domains. For further details about the network, visit http://www.au4dmnetworks.co.uk/.

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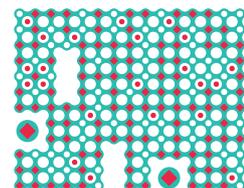
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VISUALISING UNCERTAINTY

1 | SUMMARY
VISUALISING UNCERTAINTY

We must distinguish the different 'Types of Uncertainty' that we believe to be present.

See chapter 5, 'Types of Uncertainty'

The basis of this short introduction is a review of the literature on communicating uncertainty, with a particular focus on visualising uncertainty. From our review, one thing emerges very clearly: **there is no 'optimal' format or framework for visualising uncertainty.** Instead, the implementation of visualisation techniques must be studied on a case-by-case basis, and supported by empirical testing.

Developing solutions for uncertainty visualisation thus requires interdisciplinary expertise. The effectiveness of different techniques is highly context-sensitive, and current understanding of how to differentiate relevant contextual factors remains patchy. For this reason, **communication formats should ideally be developed through close collaboration among researchers, designers, and end-users.** The building blocks brought together here provide a starting point for these kinds of dialogues.

In order to develop an uncertainty visualisation format for a case study, **we must distinguish the different types of uncertainty** that we believe to be present (see '*Types of Uncertainty*'). It is important to make these distinctions as clearly and as early as possible. Definitions and understandings of uncertainty should also be regularly reviewed as the design and testing process unfolds.

Without reliable evaluation methods, there is the danger of developing dazzling, seductive visualisations that fail to deliver appropriate decision support, or even subvert decision-making by slowing it down and/or introducing biases. Self-reporting by users is not a reliable way to assess the effectiveness of a visualisation format, so **evaluation should be performed by objective and reproducible methodologies.**

Users of uncertainty information have diverse capacities and needs, and there is as yet no deep theory which formalises which differences

Key ideas

case-by-case basis	*review & testing*
development through collaboration	*reliable evaluation*
different types of uncertainty	*reproducible methodologies*

There is no 'optimal' format or framework for visualising uncertainty.

are relevant in a given case. This is important, because **how we visualise uncertainty is not easily separable from how we interpret and reason about that uncertainty.** The diverse identities of decision-makers—our various cultural, political, social, linguistic, institutional, and individual characteristics—shape our practices in dealing with uncertainty. This means that effective uncertainty visualisation must ideally do more than encode all the relevant information: it must also invite, reinforce, and sometimes even teach appropriate modes of interpretation and reasoning. This catalogue offers illustrative discussion of selected heuristics and biases, and how they can interact with visualisation techniques. This gives a flavour of a vast area of research, and helps to illuminate some key features of the existing evidence base.

Although there are many challenges associated with visualising uncertainty for decision-making, there are also many potential benefits. In fact, the horizons of the possible are continually growing. In the longer term, expanding the repertoire of uncertainty visualisation formats, using robust methodology on a case-by-case basis, will improve the analysis and communication of uncertainty across a broad spectrum of decision-making contexts.

Finally, we must remember that **visualisation is not always the most appropriate tool.** Sometimes words or numbers do a better job of conveying a particular type of uncertainty, to a particular audience, in a particular context, for particular purposes. At the same time, we should be conscious that there is almost always some visual dimension involved in how we analyse and share information, how we design policies and processes, and how we imagine and plan for the future. Even when visualising uncertainty is not the main focus, being aware of the visual dimensions of a decision may be helpful in understanding and managing its uncertainties.
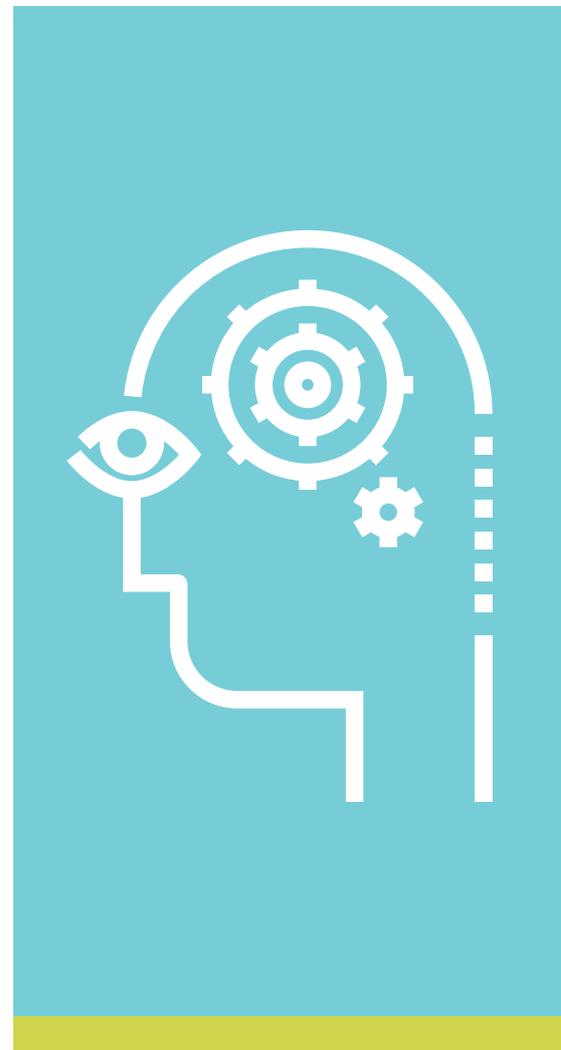
2 | INTRODUCTION
THE INFLUENCE OF VISUALISATION

Decision theory is the study of how choices are made. It is primarily grounded in economics, statistics, and psychology, but also benefits from the insights of sociology, computer science, environmental science, design theory, the arts and humanities, and other expertise. The prescriptive side of decision theory—often called decision analysis—provides many conceptual resources to specify decisions formally and to create recommended courses of action. What counts as a 'rational' decision may vary from context to context, but decision analysis can help decision-makers to better fulfil their chosen standards of rationality. For example, if the future circumstances around a decision can be divided into scenarios, each of which can be assigned a probability, then a decision-maker can choose a course of action which is robust across all those scenarios—instead of just optimistically hoping for the best case scenario, or pessimistically steeling themselves against the worst. Decision analysis also allows us to anatomise a problem into its more tractable constituents, separating the technical from the value aspects. Then, it helps us to identify the role played by 'preferences' that must be generated by stakeholders through political processes, under conditions of more or less ethical legitimacy (or inferred by other means, e.g. in the case of future generations or non-human stakeholders).

Classifying, quantifying, and reasoning about uncertainty is central to decision analysis. So too is communicating about uncertainty. It has long been clear that not just what, but how we choose to communicate has considerable influence on the interpretations and actions that follow. Uncertainty can be communicated in different formats, including verbal descriptors ("high confidence"), numerical ranges, statistical graphics (e.g. probability density functions), pictograms, infographics, or various combinations. How uncertainty is communicated can be fundamental to the effective transfer of information between individuals, agencies, and organisations, and is thus crucial to decision-making. Visualisation may therefore play an especially significant role in analysis and decision-making involving multiple stakeholders. But even when only one person is responsible for every aspect of analysis and decision-making, visualisation may still play a significant role in how uncertainty is understood—i.e. in how the decision-maker 'communicates uncertainty to themselves.'

In many domains, there are already tried-and-true methods for classifying, quantifying, and propagating uncertainty in statistically robust ways. However, presenting uncertainty to decision-makers requires careful design and testing on a case-by-case basis. Visualisation can potentially influence decision-making in many ways, including:

- Whether or not a decision is made
- Which stakeholders input into the decision
- How evidence is prioritised
- Whether additional evidence is sought
- What forms of reasoning and analysis are used
- Whether biases are active and the extent of their influence
- How goals are formulated and success evaluated
- How much workload decision-making causes
- How long a decision takes
- Correctness of a decision
- Confidence in a decision
- Kinds of errors made
- How decision outcomes are interpreted

To support a decision, multi-dimensional uncertainty requires a representation that humans (acknowledging the variation in our capabilities) find natural to understand and operate. Decision-makers often use heuristics to assess uncertainty, and

Frequently, the challenge is to present visualisations in ways which allow the user intuitively to perceive the uncertainty probabilistically and arrive at effective decisions.

such assessments are often incompatible with probability theory, i.e. 'rational' treatment of uncertainty. Frequently, the challenge is to present visualisations in ways which allow the user intuitively to perceive the uncertainty probabilistically and arrive at effective decisions.

Visualisation can be used to counteract known biases, such as anecdotal evidence bias (Fagerlin et al., 2005), side effect aversion (Waters et al., 2006, Waters et al., 2007), and risk aversion (Schirillo & Stone, 2005). The effects of visualising uncertainty are frequently positive—but not always. Riveiro et al. (2014) write that 'even if the effects of visualizing uncertainty and its influence on reasoning are not fully understood, it has been shown that the graphical display of uncertainty has positive effects on performance.' Kinkeldey et al. (2015) observe: 'Overall, based on studies reviewed, uncertainty visualization has tended to result in a positive effect on decision accuracy. The evidence is less clear for decision speed, but it could be observed that usually, uncertainty visualization does not slow down decision-making and in one case it even decreased the decision time.'

Negative effects of visualisation reported in the literature point to interactions with cognitive biases and human psychology more generally, resulting in delays in decision-making when extra uncertainty-related information needs to be processed; irrational attitudes to risk such as focusing on the worst-case scenarios, or focusing on the mean rather than variance; confusion between risk and uncertainty; etc. Several studies explore how visualisation affects cognitive strategies for dealing with uncertainty, and find that visualisations might lead to less effort being expended by the decision-maker on acquiring and processing information relevant to uncertainty. This can be an undesirable consequence: Riveiro et al. (2014) report a study where visualisation of uncertainty in a military scenario led to significantly fewer attempts to identify a target, as well as higher threat values assigned to uncertain targets. This is sometimes explained by visualisations triggering an 'availability heuristic,' which puts undue emphasis on events that are readily imagined or easy to recall—such as the worst-case scenario—in disproportion to the chance of occurrence.

Training and experience of decision-makers plays a role in determining the impact of a particular visualisation. Kinkeldey et al. (2015) write that 'in order to use data and related uncertainty effectively, users must know how to interpret data together with related uncertainty.' Boone et al. (2018) conducted experiments on whether additional training on how to interpret specific graphical conventions for uncertainty visualisations could reduce flaws in decision-making. Using visualisation to express a hurricane's current location and projected path, together with uncertainty, they found that training can reduce misconceptions. However, one experiment also revealed an unexpected side-effect: reduced incidence of misinterpretation was correlated with lowered risk perception (decreased estimates of hurricane damage) relative to the group that did not receive training. Such evidence from past experiments reinforces the imperative to test new visualisation formats whenever possible.

Key ideas

decision theory	*stakeholders' preferences*
decision analysis	*multi-dimensional uncertainty*
robustness	*reduction of misconceptions through training*

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3 | THE CHALLENGES
THE STATE OF VISUALISATION RESEARCH

While there is solid evidence that visualisation influences decision-making, this influence is not uniformly positive, and theoretical models do not provide sufficient basis to predict case-specific impacts. Reviewing the state of knowledge in 2005, MacEachren et al. concluded that 'we do not have a comprehensive understanding of the parameters that influence successful uncertainty visualization, nor is it easy to determine how close we are to achieving such an understanding.' In a follow-up to that review in 2016, Riveiro asserted that this conclusion is still valid. This is not a reflection of the paucity of studies, as visualisation is a burgeoning research topic across diverse fields.

Indeed, a major difficulty in developing a broadly applicable theory is the sheer variety of methodological approaches and theoretical frameworks. Other common problems in the literature include small sample sizes for audience testing (statistical significance); inappropriate subjects (students rather than relevant decision-makers); lack of reproducibility;

small effect sizes when comparing different visualisation approaches; biased self-perception (with little correspondence between self-reported impact and actual impact of visualisations); and transferability issues, confounded by the difficulty in controlling for differences in individual interpretation. Transferability is an especially salient problem since it implies that studies done with one group of people may not be applicable for another, or that the results are reproducible in general. Even on an individual level, responses to a visualisation may vary with factors such as time or stress-inducing constraints; the same visualisation may therefore influence decision-making in one way under a particular set of circumstances and in a contrary way under another.

When it comes to testing visualisation formats, there are broadly two types of investigations: *objective* (which rely on measured outcomes such as decision speed and decision accuracy) and *subjective* (which rely on self-reporting). Several studies

offer evidence that self-reporting is unreliable, i.e. the user of a visualisation is not necessarily a good judge of how well or poorly the visualisation has supported their decision-making. This finding goes against the grain of a typical designer-client relationship, in which the designer has done a good job if the client is satisfied. In fact, client satisfaction may have little to do with the efficacy of the product: decisions can be improved by visualisations the client does not favour, or can be impaired by the visualisations the client happens to prefer. As often happens with good design, the benefits may not be noticeable to users. Several studies have reported that the users of visualisations may become better at decision-making without realising it. Kinkeldey et al. (2015) mention one study that revealed a striking lack of correlation between independently measured performance and self-reported confidence in making decisions: 'decision accuracy was significantly higher with uncertainty depicted, meaning that user performance and confidence did

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Known problems

small sample sizes	*small effect sizes*
inappropriate subjects	*biased self–perception*
lack of reproducability	*transferability*

We do not have a comprehensive understanding of the parameters that influence successful uncertainty visualization, nor is it easy to determine how close we are to achieving such an understanding. (MacEachren et al., 2005)

not correspond.' The evidence pointing to users' inability to assess their own performance is broader than just research on visualisation, as Hullman et al. (2008) point out: 'evidence from other disciplines suggests that people are not very good at making accurate judgments about their own ability to make judgments under uncertainty.'

Even the level of expertise interacts with visualisation in ways which are difficult to anticipate. Greater expertise has been associated both positively and negatively with the effectiveness of visualisation. For example, in some studies a higher level of expertise enabled decision-makers to make better use of visualisation, arriving at decisions faster and with greater accuracy. In others, the level of expertise was associated with undesirable effects of visualisations; experiments 'showed that participants with a high level of experience had the strongest bias towards selecting areas of low uncertainty' (Kinkeldey et al., 2015).

Currently, much of the research on uncertainty visualisation relates to spatial reasoning. This research addresses the potential of visualisation to improve probabilistic spatial reasoning which, like all probabilistic reasoning, is plagued by cognitive biases. Some researchers, like Pugh et al. (2018), are optimistic: 'Visualizations have the potential to influence how people make spatial predictions in the presence of uncertainty. Properly designed and implemented visualizations may help mitigate the cognitive biases related to such prediction.'

Of course, creating visualisations that communicate uncertainty well is not necessarily a guarantee for effective decision support; even when understood correctly, uncertainty may be eschewed by decision-makers. Uncertainty is not something people are comfortable with no matter how well it is communicated. Resistance towards incorporating uncertainty into decision-making is widely reported in the literature. For example, Riveiro et al. (2014) report that 'participants' greater uncertainty awareness was associated with lower confidence.' Lower confidence, however, may be a desired outcome of visualisation in contexts where overconfidence is a known problem.

Kinkeldey et al. (2015) recommend that decision-makers are supported with more than just well-crafted visualisations: 'decision makers needed additional information for interpreting and coping with uncertainty (e.g. when a high degree of uncertainty is a problem and when not).' They point out that 'decision-makers often have little time to explore uncertainty in the data.' In order to ensure that decision-makers prioritise uncertainty information appropriately, the development of visualisation formats must be placed in a much wider context. This wider context includes the graphic literacy of decision-makers, the available ensemble of decision support tools, and the surrounding technical, social, and institutional infrastructures.

In short, what is lacking currently is the ability to predict how specific visualisations will impact particular decision-making processes. Uncertainty visualisation can be rewarding, but it is also a challenging and unpredictable territory. This is not to say that experience and existing research offer no guidance whatsoever, but to emphasise that any new visualisation needs to be tested and evaluated in its intended context and with the relevant audience, using robust methodology with an objective component (not just subjective appraisal). With these caveats in mind, the following sections present some basic approaches and practical examples of visualising uncertainty for decision support.

4 THE FRAMEWORK
DEVELOPING VISUAL SOLUTIONS

Before we begin discussing the foundational elements that can be called upon in the visualisation task, let's emphasise that selecting a visualisation method is not the first step. The process itself should begin with the identification of uncertainty, understanding of the various components that contribute to uncertainty, and discussing the aims of visualisation. We recommend considering the framework below or a close equivalent.

As emphasised in the previous section, self-reporting is an unreliable method for testing uncertainty visualisations. Decisions may be improved by visualisations the user does not favour, or may be impaired by the visualisations the user regards as helpful. This can create additional challenges within the design thinking process. On the one hand, as Deitrick and Wentz (2015) warn, visualisation research has often been 'normative in nature, reflecting what researchers think decision makers need to know about uncertainty,' instead of setting aside preconceptions and building empathy. On the

other hand, although designers and researchers must cultivate a deep understanding of decision-makers' lived experience, they must also work with decision-makers to explore how this experience may be misleading, once information from objective testing has been incorporated. The literature on bounded rationality, heuristics, and cognitive biases offers useful concepts in this regard (see section 08 "The User").

The following twelve-step guide demonstrates how design thinking can be implemented in the domain of uncertainty visualisation. It is largely based on A. Lapinsky's 'Uncertainty Visualization Development Strategy (UVDS).'

Step 1 is to classify the nature of the uncertainty. We explore possible approaches in the next section. Whatever the approach chosen, it will likely reveal that only a portion of uncertainty lends itself to visualisation. Deitrick and Wentz (2015) list common assumptions about the conditions under which uncertainty can be effectively visualised:

'First, it is assumed that uncertainty, or at least uncertainty of interest, is both knowable and identifiable. Similarly, to be visualized, uncertainty must be quantifiable, such as through statistical estimates, quantitative ranges, or qualitative statements (e.g. less or more uncertain). Moreover, evaluations define effectiveness as an ability to identify specific uncertainty values, which assumes that identifying specific uncertainty values is useful to decision-makers and that the values of interest can be quantified. Lastly, there is an assumption that the quantification of uncertainty is beneficial, applicable to the decision task, and usable by the decision maker, even if users do not currently work with uncertainty in that way.'

These pervasive assumptions mean that deep uncertainty, i.e. uncertainty that cannot be quantified given available resources, poses special challenges for visualisation.

Following Step 1, Steps 2 to 9 are the research phase, subdivided into 'Understand,' 'Decide,' and 'Determine.' Step 2 ensures that the data

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Key ideas

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self-reporting is unreliable

visualisation research shout set aside preconceptions and build empathy

literature on bounded rationality, heuristics, cognitive biases offers useful concepts

...although designers and researchers must cultivate a deep understanding of decision-makers' lived experience, they must also work with decision-makers to explore how this experience may be misleading

Figure 01.
12–Step Strategy for Uncertainty Visualisation. Based on the Uncertainty Visualization Development Strategy (UVDS) by Anna–Liesa S. Lapinsky (2009). Created by Jana Kleineberg.

itself is understood, and takes into account such things as the origin and precision of the data, whether it has been modified (e.g. processed or aggregated), its format and the format's limitations, as well as other factors. Steps 3 and 4 look at the intended target audience: who will use the uncertainty construct? How is it used? How will the visualisation help? It is important to determine the purpose for which presentation is required. Is it to explore the problem? To facilitate understanding between different stakeholders, roles, or forms of expertise? To analyse a situation and make decisions? Some other purpose(s)? Step 5 helps decide on an information hierarchy, if multiple uncertainties are associated with the inputs. Step 6 formalises a definition of the uncertainty. Step 7 looks at the specific cause of uncertainty. Step 8 determines causal categories. Step

Decisions may be improved by visualisations the user does not favour, or may be impaired by the visualisations the user regards as helpful.

9 determines visualisation requirements, or the needs of the visualisation—e.g. what should the dominant features be, what level of understanding does the user have, and how will this influence the necessary level of detail? What tasks does the user need to perform, and what information is relevant to these tasks?

Step 10 leads into the actual design process by preparing the data for visualisation: sorting and organising measurements, converting them if necessary, converting uncertainty from collected data, and/or combining multiple uncertainties. Only in Step 11 does the creative part of the visualisation start. The principle of appropriate knowledge and the semantic principle provide guidance here. Different techniques can be used to create visualisation formats, encoding information in ways likely to support effectively attention and analysis. For example, a saliency algorithm (Padilla, Quinan, et al., 2017) can optionally be used to identify elements likely to attract viewers' attention. Once candidate visualisation formats have been developed, these are then tested in Step 12, ideally with the intended end-users themselves, employing evaluation methodologies that are transparent and reproducible. Steps 10 to 12 are connected, as they may be repeated multiple times to refine and improve on the visualisation.

Following general semantic and other design principles is no guarantee that visualisations will be correctly interpreted. Rather, a holistic approach to uncertainty visualisation includes principles of design, the testing of visualisation methods, the graphic literacy of end-users, as well as the overall decision-making context.

DESIGN THINKING

In graphic design the 'design thinking' approach has been widely adopted. It refers to the cognitive, strategic, and practical processes designers use to tackle complex problems. It is a flexible approach, focused on collaboration between designers and users, with an emphasis on bringing ideas to life based on how intended users think, feel, and behave.

The process is carried out in a non-linear fashion: the five stages are not always sequential and can often occur in parallel and/or iteratively. However, the design thinking model identifies and systematises the five stages one would expect to carry out in a design project.

Empathy: Understanding human needs; learning about the user who will interact with the design.

Definition: Framing and defining the problem; shaping a point of view based on user needs and insights.

Ideation: Creating ideas and creative solutions in ideation sessions, e.g. through brainstorming.

Prototyping: Adopting a hands-on approach by building (a) representation(s) to show to others.

Testing: Developing a solution to the problem and returning to the original user group for testing and feedback. Results are used to review the empathy stage, redefine problems, and refine the design.

DEEP UNCERTAINTY

Deep uncertainty may refer to uncertainty which we cannot quantify, which we cannot quantify given our available resources, or whose quantification is on balance undesirable.

So the distinction between uncertainty and deep uncertainty is not rigid, but rather is a function of the methods, resources, and choices we bring to bear on classifying and quantifying uncertainty. Identifying some uncertainties as deep uncertainties does not place them beyond quantification once and for all, and does not rule out the possibility that they may be reclassified at a later stage in the process.

Furthermore, adopting the designation of deep uncertainty does not mean that decision-makers are justified in excluding these uncertainties from their reasoning, or that developers of decision support tools can safely set them to one side.

Indeed, it is even theoretically possible to visually depict deep uncertainty: many artworks (e.g. the abstract impressionist works of Rothko) might be considered representations of deep uncertainties that are perceived intuitively. However, in line with the majority of research done to date, **this primer focuses on uncertainties which can be quantified.**

5 TYPES OF UNCERTAINTIES
CATEGORISING UNCERTAINTY

WHY CATEGORISE?

Decision-making can be improved by understanding the uncertainty in the data being used. Categorising uncertainty is a preliminary step towards recognising and dealing with uncertainty in the decision-making process.

Uncertainty can come in many forms, and with many different qualities. Each type of uncertainty applies to different types of information, and may be quantified, and thus represented, in different ways. However, there is no one-to-one correlation of uncertainty types and visualisation techniques. As Chung and Wark (2016) confirm, often the same technique, e.g. colour coding, has been variously applied to depict distinct types of uncertainty, as well as subsets of combined/compounded uncertainties from different categories.

Uncertainty has been decomposed in a variety of ways by a multitude of theoretical treatments, and each classification scheme carries its own set of assumptions and motives. In particular, sometimes uncertainty is broken up into different types simply to help us spot where uncertainty lies, and to organise how we investigate and manage uncertainty, but on the understanding that these distinctions don't ultimately prevent integration into a single analysis and decision procedure. On other occasions, the purpose of a classification scheme is to draw attention to more fundamental differences between uncertainty types, which may be difficult or impossible to reconcile.
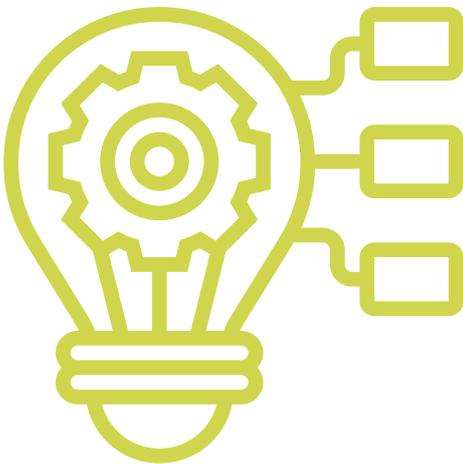
Key ideas

categorizing is key

uncertainty comes in many different forms & qualities

understanding is tailored to context

Uncertainty can come in many forms, and with many different qualities.

SOME PROMPTS

French et al. (2016) list the following types of uncertainty: **stochastic** uncertainties (i.e. physical randomness), **epistemological** uncertainties (lack of scientific knowledge), **endpoint** uncertainties (when the required endpoint is ill-defined), **judgemental** uncertainties (e.g. setting of parameter values in codes), **computational** uncertainties (i.e. inaccurate calculations), and **modelling errors** (i.e. however good the model is, it will not fit the real world perfectly, or if it seems to, it is likely to have little predictive power). There are further uncertainties that relate to **ambiguities** (ill-defined meaning) and partially formed **value judgements**; and then there are **social** and **ethical** uncertainties (e.g. how expert recommendations are formulated and implemented in society, what the ultimate ethical value of a decision and all its consequences will be). Some uncertainties may be deep uncertainties; that is, within the time and data available to support the decision process, there may be little chance of getting agreement on their evaluation or quantification.

Chung and Wark (2016) list these categories in their review of uncertainty visualisation literature:

- **Accuracy** *the difference between observation and reality*
- **Precision** *the quality of the estimate or measurement*
- **Completeness** *the extent to which information is comprehensive*
- **Consistency** *the extent to which information elements agree*
- **Lineage** *the pathway through which information has been passed*
- **Currency** *the time span from occurrence to information presentation*
- **Credibility** *the reliability of the information source*
- **Subjectivity** *the extent to which the observer influences the observation*
- **Interrelatedness** *the dependence on other information*
- **Experimental** *the width of a random distribution of observations*
- **Geometric** *the region within which a spatial observation lies*

The two categorisations presented illustrate the breadth of approaches to uncertainty, and suggest that the choice of a schema for understanding uncertainty might need to be tailored to the context.
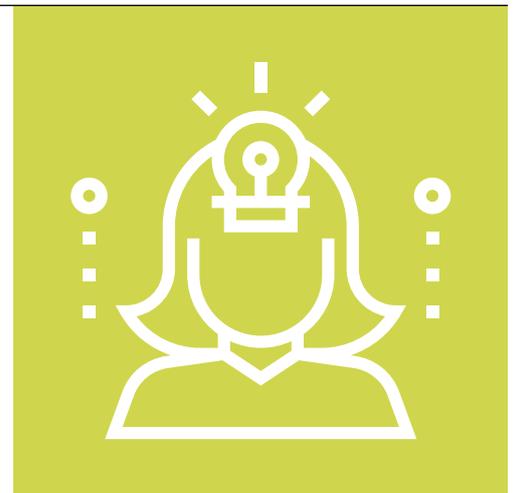
6 | CYNEFIN

Another approach to uncertainty called Cynefin — a Welsh word for habitat, and used here to describe the context for a decision — categorises our knowledge relative to a specific decision. Cynefin roughly divides decision contexts into four spaces (see figure 02). Note that placing a decision in one of these four spaces does not preclude certain aspects of that decision being associated with a different space. It may also occasionally be appropriate to situate a decision in a particular space for one set of purposes, and in a different space for another set of purposes. Acquiring more information and/or conducting analysis may also shift a decision from one space into another.

In the **Known Space**, also called *Simple,* or the *realm of Scientific Knowledge,* relationships between cause and effect are well understood, so we will know what will happen if we take a specific action. All systems and behaviours can be fully modelled. The consequences of any course of action can be predicted with near certainty. In such contexts, decision-making tends to take the form of recognising patterns and responding to them with well-rehearsed actions, i.e. recognition-primed decision-making. Such knowledge of cause and effect will have come from familiarity. We will regularly have experienced similar situations. That means we will not only have some certainty about what will happen as a result of any action, we will also have thought through our values as they apply in this context. Thus, there will be little ambiguity or value uncertainty in such contexts

In the **Knowable Space**, also called *Complicated,* or the *realm of Scientific Inquiry,* cause and effect relationships are generally understood, but for any specific decision further data is needed before the consequences of any action can be predicted with certainty. The decision-makers will face epistemological uncertainties and probably stochastic and analytical ones too. Decision analysis and support will include the fitting and use of models to forecast the potential outcomes of actions with appropriate levels of uncertainty. Moreover, although the decision-makers will have experienced such situations before they may be less sure of how their values apply and will need to reflect on these in making the final decision.

REPEATABILITY AND INCREASED FAMILIARITY

Simple/Known space

The realm of Scientific Knowledge, also called the 'known knowns.' Rules are in place, the situation is stable. Cause and effect relationships are understood; they are predictable and repeatable.

Complicated/ Knowable space

The realm of Scientific Inquiry, or domain of 'known unknowns.' Cause and effect relationships exist. They are not self-evident but can be determined with sufficient data.

'MESSY' DECISIONS, MANY UNCERTAINTIES ARE DEEP

Complex space

The realm of Social Systems, or domain of 'unknown unknowns.' Cause and effect are only obvious in hindsight and have unpredictable, emergent outcomes.

Chaotic space

No cause and effect relationships can be determined.

Cynefin: Welsh, without direct translation into English, but akin to a place to stand, usual abode, and habitat. It is pronounced /ˈkʌnɪvɪn/KUN-iv-in.

In the **Complex Space**, also called the *realm of Social Systems,* decision-making faces many poorly understood, interacting causes and effects. Knowledge is at best qualitative: there are simply too many potential interactions to disentangle particular causes and effects. There are no precise quantitative models to predict system behaviours such as in the Known and Knowable spaces. Decision analysis is still possible, but its style will be broader, with less emphasis on details, and more focus on exploring judgement and issues, and on developing broad strategies that are flexible enough to accommodate changes as the situation evolves. Analysis may begin and, perhaps, end with much more informal qualitative models, sometimes known under the general heading of soft modelling or problem structuring methods. Decision-makers will also be less clear on their values and they will need to strive to avoid motherhood-and-apple-pie objectives, such as minimise cost, improve well-being, or maximise safety.

Contexts in the **Chaotic Space** involve events and behaviours beyond our current experience and there are no obvious candidates for cause and effect. Decision-making cannot be based upon analysis because there are no concepts of how to separate

COMPLEX
Probe — Sense — Respond
Emergent

COMPLICATED
Sense — Analyse — Respond
Good Practice

disorder

CHAOTIC
Act — Sense — Respond
Novel

SIMPLE
Sense — categorize — Respond
Best Practice
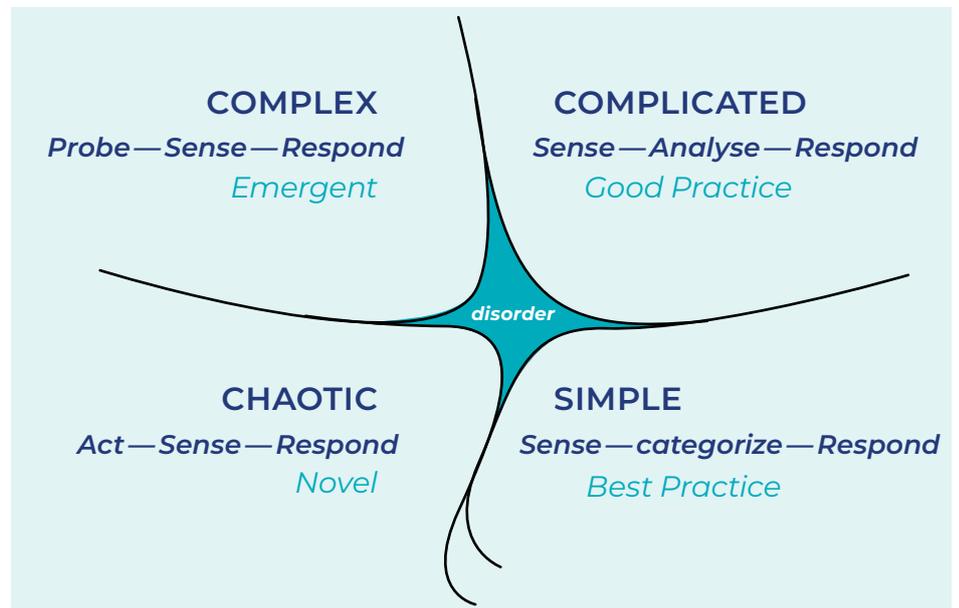
entities and predict their interactions. The situation is entirely novel to us. Decision-makers will need to take probing actions and see what happens, until they can make some sort of sense of the situation, gradually drawing the context back into one of the other spaces.

The **central blob** in figure 02 is sometimes called the Disordered Space. It simply refers to those contexts that we have not had time to categorise. The Disordered Space and the Chaotic Space are far from the same. Contexts in the former may well lie in the Known, Knowable, or Complex Spaces; we just need to recognise that they do. Those in the latter will be completely novel.

Figure 02. Uncertainty Cynefin

For more information on categories of uncertainty and Cynefin, please see the AU4DM Uncertainty catalogue "Decision Support Tools for Complex Decisions under Uncertainty" edited by Simon French from contributions from many in the AU4DM network.

7 VISUALISING UNCERTAINTY
THE TOOLKIT

As noted, developing an uncertainty visualisation solution begins with discussing the needs of the user, and understanding the various components that contribute to uncertainty. Only once this has been done can assessment of appropriate visualisation techniques begin. Furthermore, since there is no general theory of uncertainty visualisation, whichever techniques we select will still need to be tested, potentially through multiple iterations. Testing should ideally be undertaken with the actual end-users of the uncertainty visualisation. Given well-attested problems with self-reporting, testing should also not rely solely on users' subjective experience. In summary, any toolbox sits within a larger process of preparation and testing. There are various credible approaches to managing this process; this catalogue proposes an approach described in Section 4.

CLASSIFYING VISUALISATION TECHNIQUES

There are a variety of classification schemes for visualisation techniques used in uncertainty visualisation. Deitrick (2012) distinguishes between implicit and explicit visualisation of uncertainty information. Uncertainty is implicitly visualised when encoded in the image in such a way that uncertainty cannot be separated out as a feature: that is, no design element represents uncertainty by itself without also signifying some other value. 'Explicit visualization refers to methods where uncertainty is extracted, modeled and quantified separately from the underlying information' (Deitrick, 2012). As an example of implicit visualisation, Deitrick offers a scenario in which a decision-maker is reviewing three graphics associated with three policy options. In each graphic, the vertical and horizontal axes represent two variables whose future state is not known, and the graph space is shaded to represent how successful the policy will be for any given combination of values. In the explicit version, by contrast, there is an underlying model to predict the outcome of each policy, and each visualisation encodes the model's uncertainty in the transparency/opacity dimension. Experiments suggest that visualising uncertainty implicitly versus explicitly can impact decision-making in divergent ways (Deitrick, 2012). In this catalogue, we generally focus on explicit visualisations.

Kinkeldey et al. (2014) review an array of studies in terms of how uncertainty is visualised, classifying approaches to visualisation according to three theoretical dichotomies. (i) Coincident/adjacent distinguishes information represented together with its uncertainty (coincident) from information and associated uncertainty that are visualised separately (adjacent). (ii) Intrinsic/extrinsic distinguishes visualisations achieved through manipulation of existing graphical elements (intrinsic), from

Key ideas

coincident/adjacent	*selective attention & visual salience*
intrinsic/extrinsic	*using colour to visualise uncertainty*
static/dynamic	*(introducing some examples)*
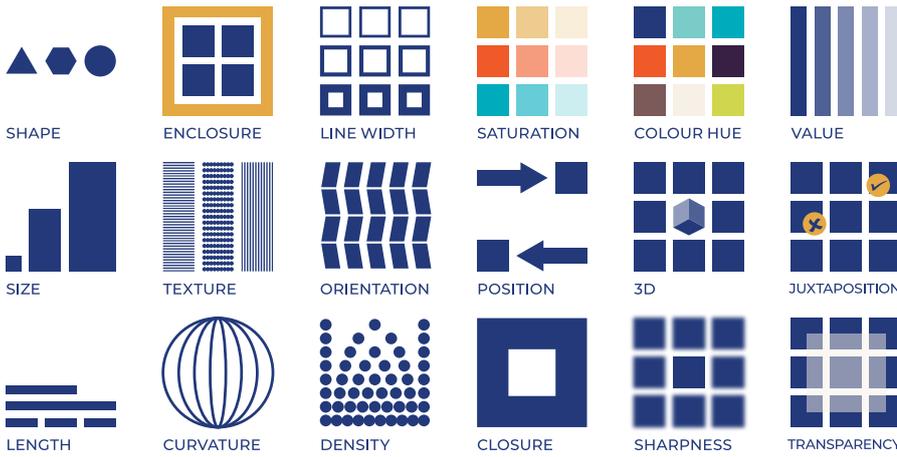
Figure 03. Attributes of Graphics. Crated by Jana Kleineberg.

SHAPE	ENCLOSURE	LINE WIDTH
SATURATION	COLOUR HUE	VALUE
SIZE	TEXTURE	ORIENTATION
POSITION	3D	JUXTAPOSITION
LENGTH	CURVATURE	DENSITY
CLOSURE	SHARPNESS	TRANSPARENCY

those derived through addition of elements such as grids, glyphs, and icons (extrinsic). (iii) Finally, static/dynamic refers to the potential of a visualisation to change through time, as with animation or interactive techniques. Pragmatic approaches by Meredith et al. (2008) and Matthews et al. (2008) are motivated by the question 'what can be done to visualise uncertainty?' In their review of the approaches to the visualisation of uncertainty, Meredith et al. (2008) mention the following:

• adding glyphs*

• adding icons**

• adding geometry

• modifying geometry

• modifying attributes

• animation

• sonification

• fluid flow

• surface interpolants

• volumetric rendering

• differences in tree structures

glyphs: In typography, a glyph is an elemental symbol within an agreed set of symbols, i.e. an individual mark of a typeface, such as a letter, a punctuation mark, an alternate for a letter. A glyph is usually a mark that represents something else. For example, the @ sign is a glyph that commonly represents the word 'at'. Thus, e.g. in flow fields, one would speak of glyphs (usually pointing arrows) that show a trend or a direction, as there cannot be a direct pictorial representation of "flow".

**icons:* An icon is a more direct representation of something else; a pictogram or ideogram that shows a simplified, comprehensible symbol of the function or thing it represents. Icons are often recognisable depictions of familiar objects, such as fish, cars, or trees. The sections below will provide examples of glyphs and icons.
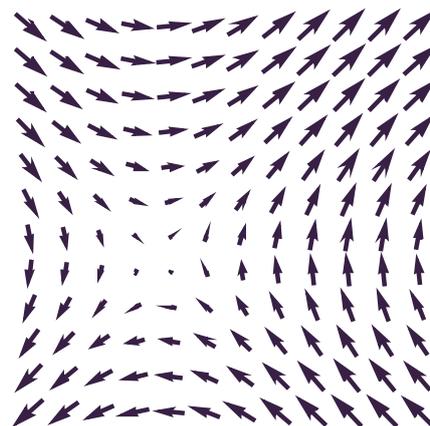
Figure 04. Vector field, or flow field. Source : Wikipedia.

Matthews et al. (2008) offer four nested categories of techniques: alteration (1), addition (2), animation (3), and interaction (4):

Free graphical variables (e.g. colour, size, position, focus, clarity, fuzziness, saturation, transparency and edge crispness) can be used to alter aspects of the visualizations to communicate uncertainty.

1. Additional static objects (e.g. labels, images, or glyphs) can be added to the visualizations to communicate uncertainty.

2. Animation can be incorporated into the visualizations, where uncertainty is mapped to animation parameters (e.g. speed, duration, motion blur, range or extent of motion).

3. Uncertainty can be discovered by mouse interaction (e.g. mouse-over).'

There are only so many graphical attributes that can be manipulated (figure 03). Various combinations of graphical parameters have been explored in the context of visualising uncertainty. Individual studies and reviews of existing research offer valuable insight into their usefulness.
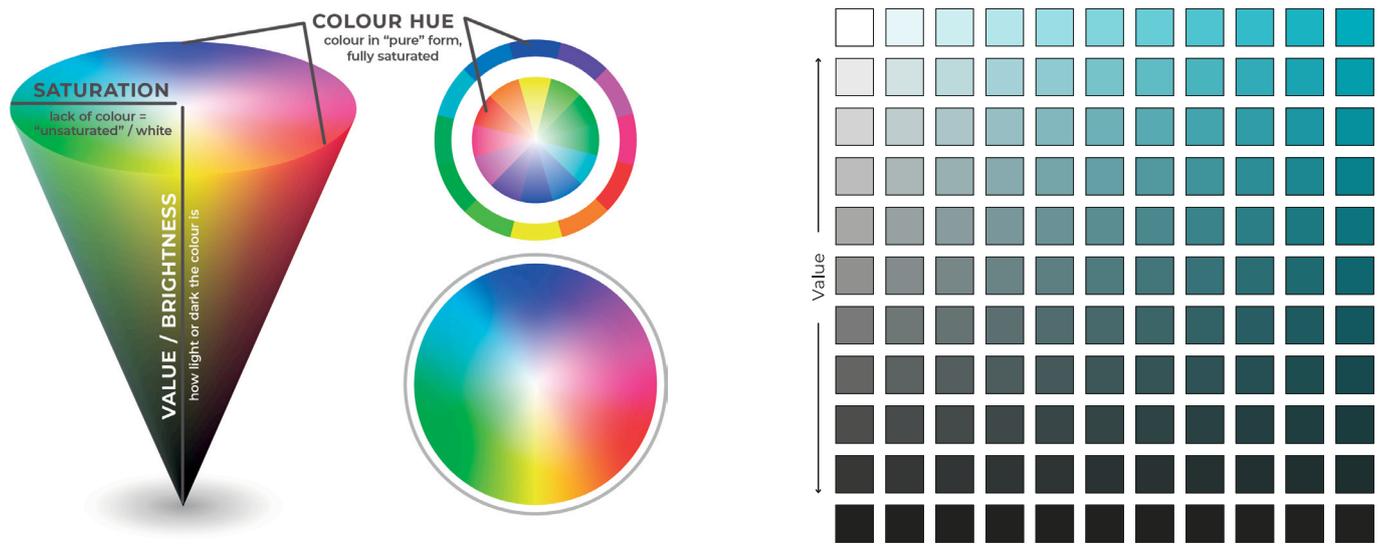
Figure 05. Hue, Saturation, Value. Created by Jana Kleineberg

SELECTIVE ATTENTION AND VISUAL SALIENCE

Certain visual attributes can have a significant impact on our so-called 'preattentive' perception. For example, certain stimuli tend to 'jump out' at us, regardless of our top-down preferences, strategies, and goals in processing visual information. Influences on preattentive perception:

- **Colour** hue, saturation, and value

- **Size** the surface area of an element

- **Position** where an element sits within a visualisation's overall space and/or various subspaces

- **Predictability** whether the element is in its expected position

- **Set size** the total number of elements in a visualisation

- **Emotional connotations** e.g. smiling or angry faces, 'cute' imagery of animals or babies

- **Contrast** more broadly, how an element compares

to other elements in the visualisation as regards these attributes and others

- **Visual salience** more broadly still, the conspicuousness of an element relative to its environment, influenced by all of the above as well as other factors, e.g. motion, sharpness of edges, orientation

Manipulation of these features may impact the allocation of attention, e.g. the likelihood that the decision-maker notices an element at all, and their likelihood of fixating on it. The term visual salience refers to the conspicuousness of a visual element relative to the visual surroundings in which it appears (Itti and Koch, 2000). A salience model takes as input any visual scene and produces a topographical map of the most conspicuous locations, i.e. those locations that are brighter, have sharper edges, or different colours than their surroundings. The saliency map (Koch & Ullman, 1985) usually considers three channels: colour,

intensity, and orientation — drawn from a variety of different spatial scales. The map itself represents the visually most important regions in the image. Under normal viewing conditions, there is believed to be a positive association between the location of fixation and salience of the stimulus at those locations. In other words, salience exerts a small but significant effect on fixation likelihood, so that decision-makers are more likely to fixate on objects with a greater level of salience (Milosavljevic, Navalpakkam, Koch, and Rangel, 2012). for instance, Milosavljevic et al. (2012) demonstrated that, during rapid decision-making tasks, visual saliency influences choices more than personal preferences (Kahneman et al., 1982). Such salience bias increases with cognitive load and is particularly strong when individuals do not have strong preferences related to different options. Salience has also been shown to influence the fixation order, with more salient objects being fixated on earlier (Peschel and Orquin, 2013).

ANY COLOUR CAN BE DEFINED ACCORDING TO ITS POSITION IN THREE DIMENSIONS: HUE, VALUE, AND SATURATION

We should take care to note some terminological inconsistency, especially around descriptions of colour. for example, other terms for saturation include intensity and purity. Other terms for value include brightness, lightness, and luminosity. All these terms risk being confused with transparency, which is not really a perceptual dimension of the colour itself, but the degree to which the colour(s) underneath are allowed to show through. Sometimes the terms colour and hue are used interchangeably. for example, in a review of visualisation of uncertainty by Aerts et al. (2003) we read: 'Bertin (1983) describes an extended set of visual variables to portray information, such as position, size, value, texture, color, orientation, and shape. Among these variables, "the strongest acuity in human visual discriminatory power relates to varying size, value and color".' Colour in this context may refer to hue, or to a combination of hue and saturation. As well as terminological inconsistency, there is not infrequently some conceptual confusion associated with these dimensions of colour.

USING COLOUR TO VISUALISE UNCERTAINTY

Any colour can be defined according to its position in three dimensions: hue, value, and saturation. Figure 05 illustrates the relationship between these three perceptual dimensions. Studies such as Seipel and Lim (2017) explore the use of hue, saturation, and value in communicating uncertainties. However, Kinkeldey et al. (2014) report that 'from current knowledge, colour saturation cannot be recommended to represent uncertainty. Instead, colour hue and value as well as transparency are better alternatives.'

There is another mention of saturation as an unsuitable graphical parameter for the visualisation of uncertainty in Cheong's 2016 review, although the same studies are considered in both reviews and so there is a risk of double counting and drawing stronger conclusions than the evidence supports. Cheong (2016) further notes inconsistencies across various inquiries into the use of hue and value, with some experiments confirming effectiveness while others dispute it; these apparent conflicts are likely due to studies not being directly comparable, e.g. in terms of experimental subjects. Where value is effective, the literature suggests that

people tend to associate darker values with more certainty, and lighter values with less (e.g. MacEachren, 1992).

Hue is determined by the dominant wavelength, and is the term that describes the dimension of colour we first experience when we look at a colour ("yellow," "blue," etc.). When we speak of hue, we are generally referring to the colour in its "pure" and fully saturated form. Saturation refers to how pale or strong the colour is. To simplify just a little, it can be said that the more white you add, the less saturated the colour becomes. Value refers to how light or dark a colour is. Adding grey or black will
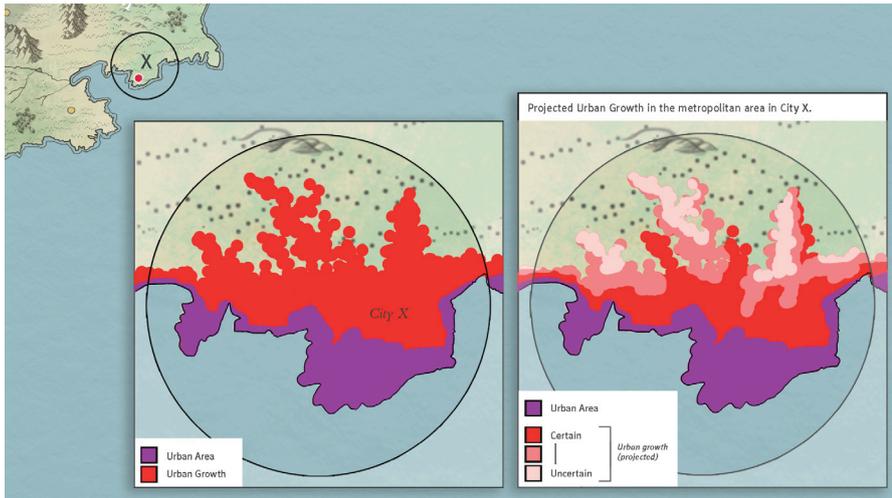
Figure 06 (left). Illustrating value and uncertainty in separate representations, arranged side–by–side.

Figure 07 (bottom). Using hue to directly illustrate uncertainty.

Figure 08 (page 21). Comparing changing different attributes to communicate uncertainty. Based on Cheong et al. (2016).

All created by Jana Kleineberg

change the value: a low value is dark grey or black, and a high value is light grey or white.

A distinction is also made between pigment primaries (e.g. print) and light primaries (e.g. pixels). Pigment primaries use subtractive colour mixing: dyes, inks, paints, pigments absorb some wavelengths of light and not others. Typical 'backgrounds,' such as fabric fibres, paint base, and paper without pigments, are usually made of particles that scatter all colours in all directions, meaning they look white. When a pigment or ink is added, specific wavelengths are absorbed (subtracted) from white light, so light of another colour reaches the eye (the colour we see). The primary colours of this colour model are cyan, magenta, and yellow (CMY); combining all three pigment primary colours yields black. By contrast, light on a monitor display, projector, etc. uses additive colouring: mixing together light of two or more different colours. The primary colours of this colour model are usually red, green, and blue (RGB). All three primary colours together yields white.

VISUALISING UNCERTAINTY: SOME EXAMPLES

Figures 06 to 11 give some examples to illustrate these techniques. Figure 07 demonstrates the use of hue to convey uncertainty about spatial values. This fictitious example shows the projected territorial range of an invasive species. The key or legend, an essential feature of graphical displays, explains how two hues convey the probabilities that a species will spread to respective areas.
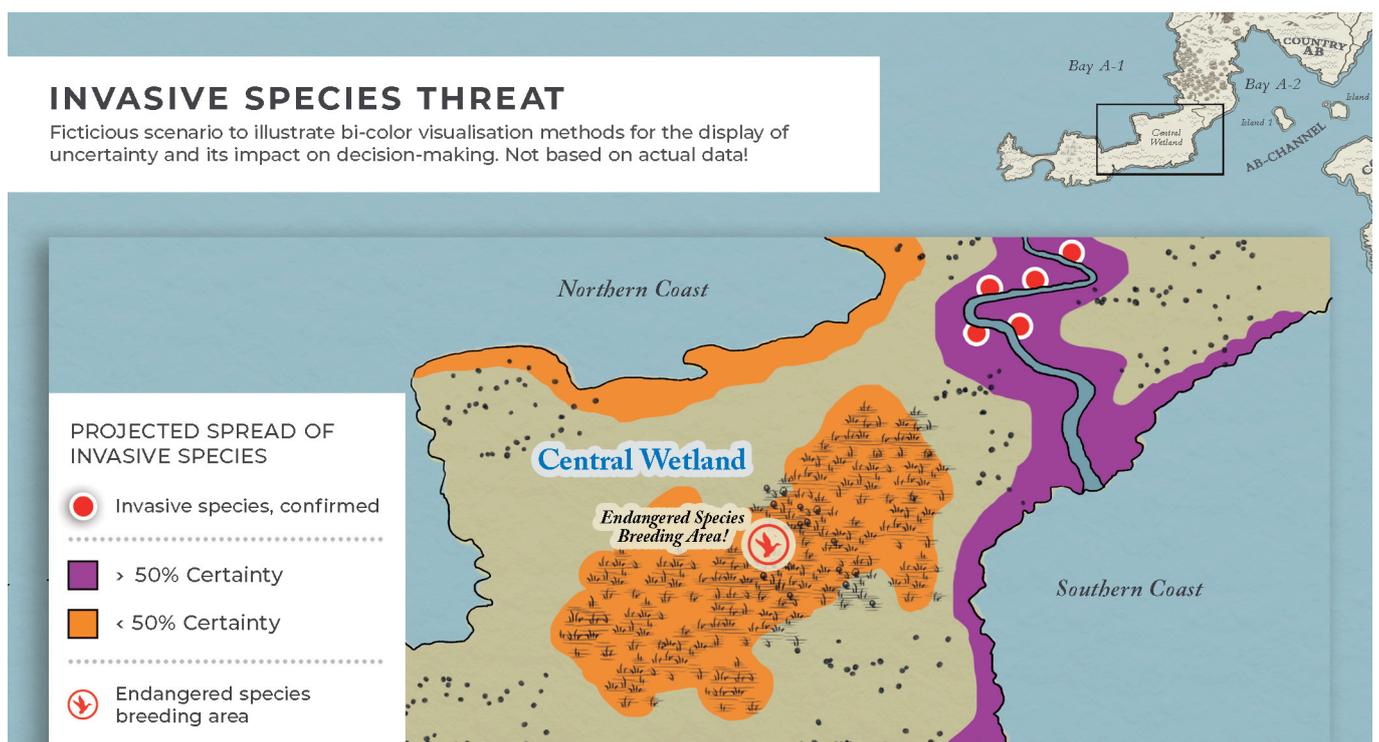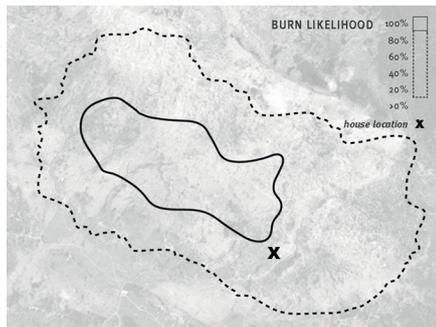
a. Boundary
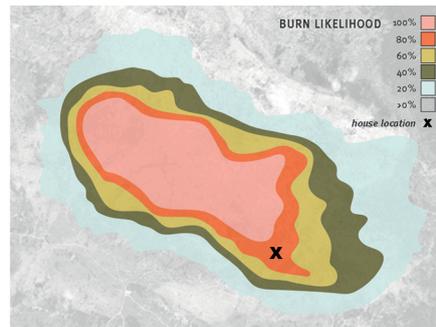
b. Colour hue
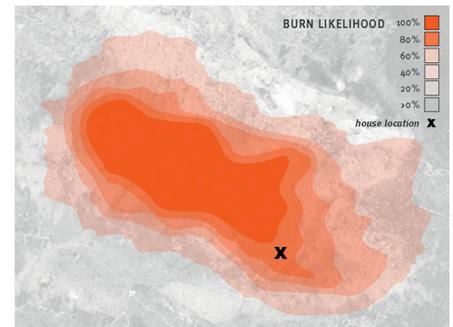
c. Colour value
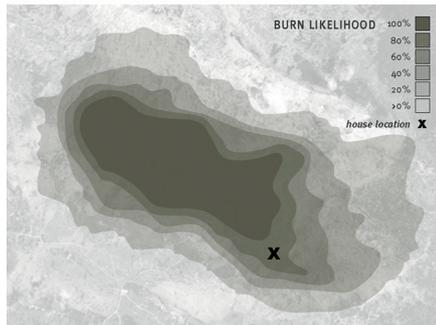
d. Transparency
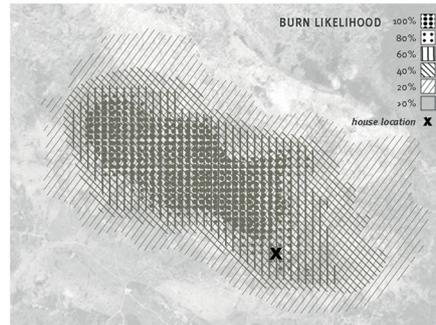
e. Texture

f. Text

Some studies have suggested using side-by-side representations of the variable and uncertainty relating to the variable, e.g. Deitrick and Wentz (2015). For illustration purposes, consider Figure 06, where the left panel depicts projected future size of a metropolitan area of a fictional city and the right panel shows uncertainty in these projections.

An alternative to a side-by-side display is an interactive display, enabling users to switch between representations of a variable and uncertainty. Various studies, involving high stakes, high uncertainty, and time pressure, have shown that in simulations the ability to switch between alternative representations of uncertainty is helpful. Finger and Bisantz (2000) explored communicating uncertainty in radar contacts by degrading or blurring the icons used to represent them. Bisantz et al. (2011) expanded this research, as Riveiro et al. (2014) summarise: 'several display methods were used in a missile defense game: icons represented the most likely object classification (with solid icons), the most

likely object classification (with icons whose transparency represented the level of uncertainty), the probability that the icon was a missile (with transparency) and, in a fourth condition, participants could choose among the representations. Task performance was highest when participants could toggle the displays, with little effect of numeric annotations. As such, the authors once more support the use of graphical uncertainty representations, even when numerical presentations of probability are present.'

Research suggests that representations involving hue (b), value (c) and transparency (d) worked best (Figure 07). In addition to transparency, value, and hue, graphical attributes that were found useful in representing uncertainty include resolution, fuzziness, and blurring (Kinkeldey et al., 2014).

Riveiro et al. (2014) concur, citing a couple of visualisation of uncertainty evaluations where 'fuzziness and location seem to work particularly well, and both size and transparency are potentially usable.'

These approaches make use of metaphors. Kinkeldey et al. 2014 write: 'The contention is that fog and blur are metaphors for lack of clarity or focus (as in a camera) and thus directly signify uncertainty. These metaphors have been suggested to have the potential to enable a better understanding of uncertainty (Gershon, 1998) and we make the assumption that the use of metaphors can lead to more intuitive approaches.' Metaphors can be both useful and misleading: for example, particular colour hues carry connotations which might interfere with intended signification. For example, discussing climate change modelling visualisations, Harold et al. 2017 warn against the use of blue that may be misinterpreted as representing water (Figure 11).

Metaphors are not universal, and associations might differ depending on the culture and experiences of users. Further, as Kinkeldey et al. (2015) show, the choice of colour hue can have an impact on the perception of risk, and hence on decision-making
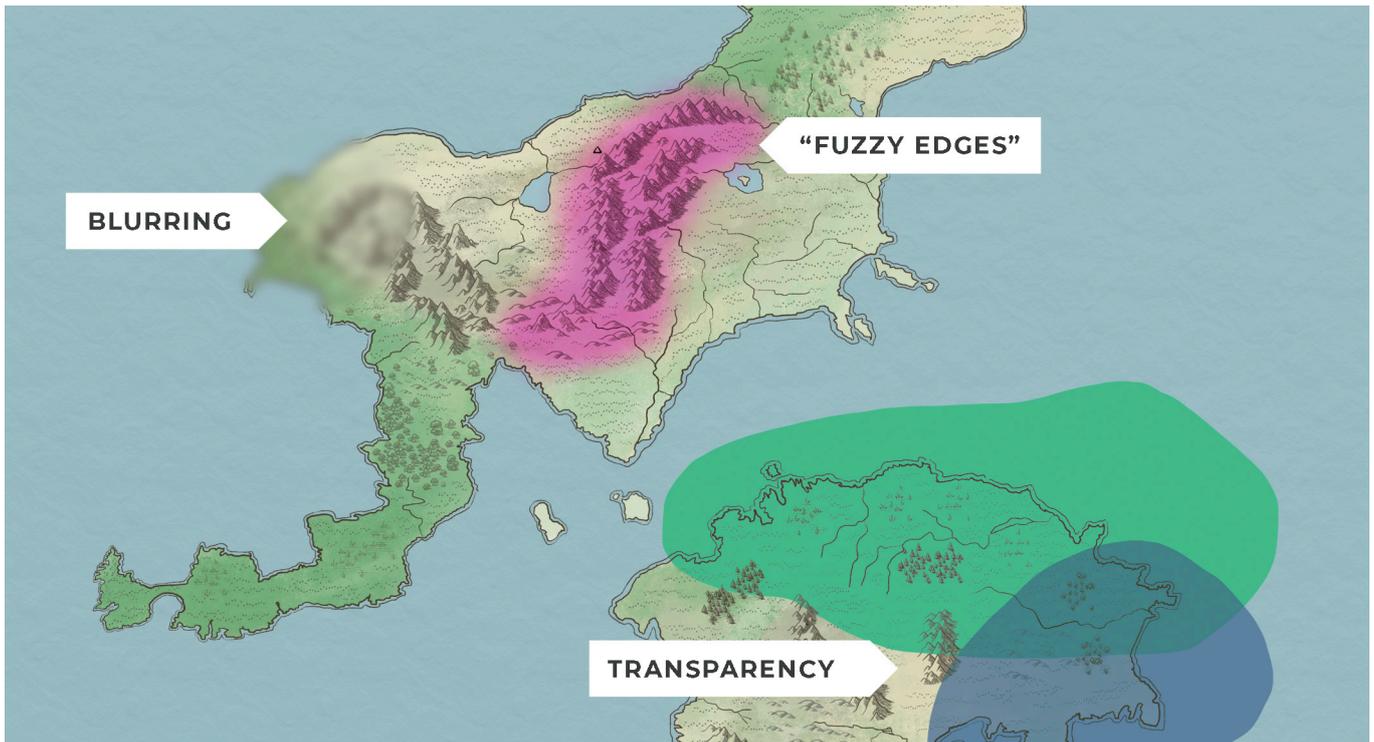
under uncertainty, such as the decision about whether or not to follow an evacuation order. To understand the impact of colour hue we must understand the emotional significance for users, which might be different for different users based on individual and group-linked factors. Further, the impact of the same colour hue on a person's decision-making might depend on the circumstances, for instance triggering a different set of heuristics when the person is under pressure. The choice of colour hue on a map could translate into a number of

lives lost if design influences people's decision not to follow an evacuation order, where some other hue would have better conveyed an appropriate level of urgency. In situations where an audience for the visualisation is diverse but it is impossible to tailor visualisations to distinct groups (as with hurricane warnings), it may not be trivial to make choices regarding visualisation formats, as trade-offs between the overall efficacy and group specific impacts might need to be considered.

Figure 09 (above).
Using blurring, fuzziness, and transparency to communicate uncertainty.

Figure 10 (bottom).
Using pixelation to represent uncertainty.

Figure 11 (page 23).
Colour and metaphor. Based on Harold et al. 2017. The colour blue is traditionally used to represent water; thus data might not be read correctly.

All created by Jana Kleineberg.
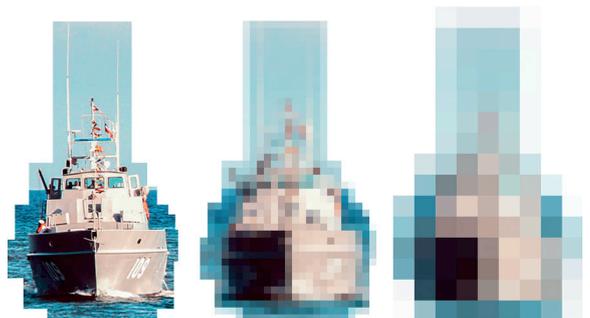
EFFECTIVE GRAPHIC DESIGN PRINCIPLES
Boone et al. (2018) list the following principles of effective graphic design:

'Effective graphic design takes account of

- the **specific task** at hand (Hegarty, 2011)

- **expressiveness** of the display (Kosslyn, 2006)

- data-ink **ratio** (Tufte, 2001)

- issues of **perception** (Kosslyn, 2006; Tversky, Morrison, & Betrancourt, 2002; Wickens & Hollands, 2000)

- **pragmatics** of the display, including making the most relevant information salient (Bertin, 1983; Dent, 1999; Kosslyn, 2006)

It also takes account of semantics:

- **compatibility** between the form of the graphic and its meaning (Bertin, 1983; Kosslyn, 2006; Zhang, 1996)

- **usability** of the display, such as including appropriate knowledge (Kosslyn, 2006)'

Ignoring these principles, or failing to implement them effectively, may lead to misunderstandings, or other forms of suboptimal decision support.
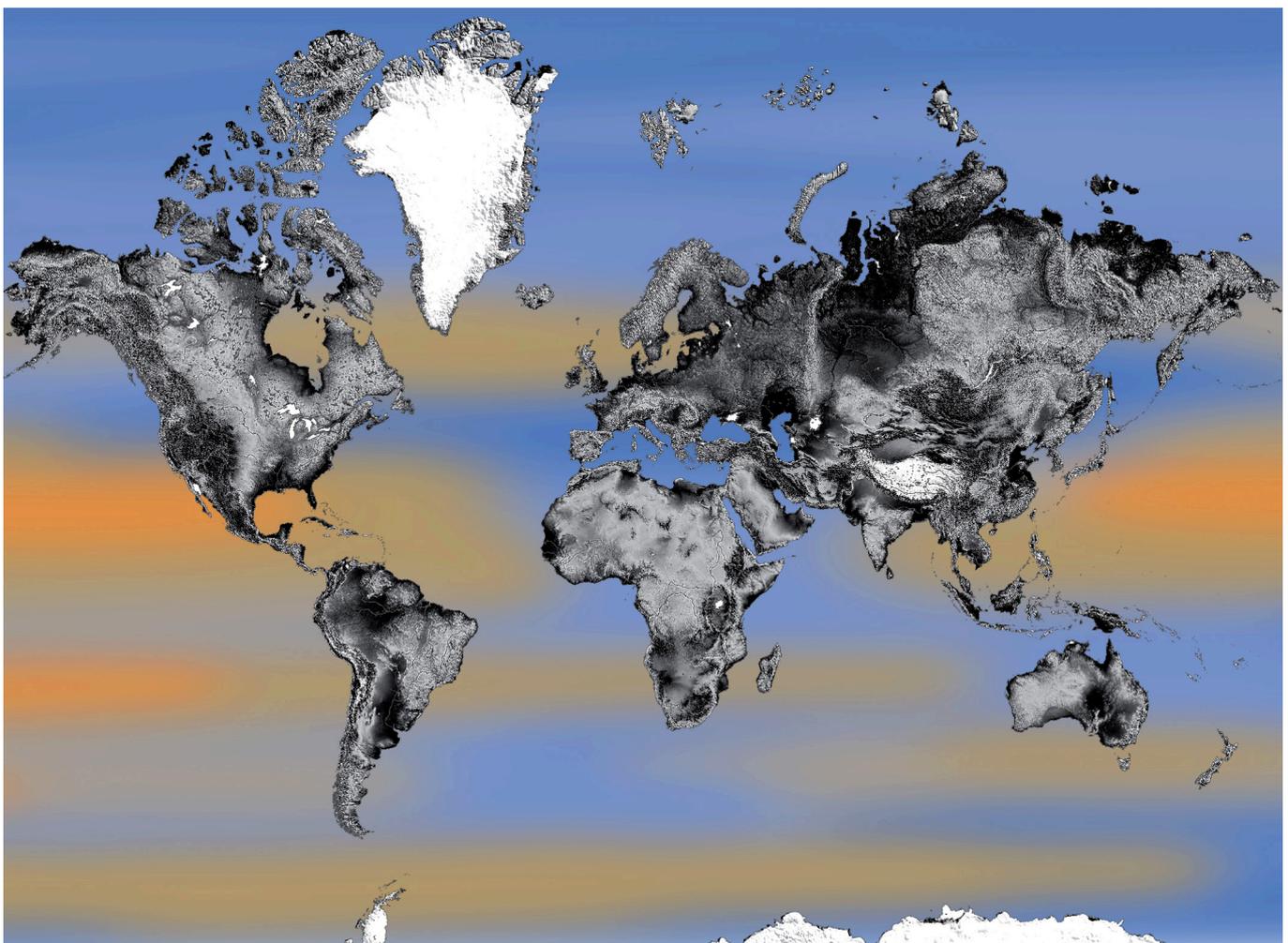
data

8 | THE USER
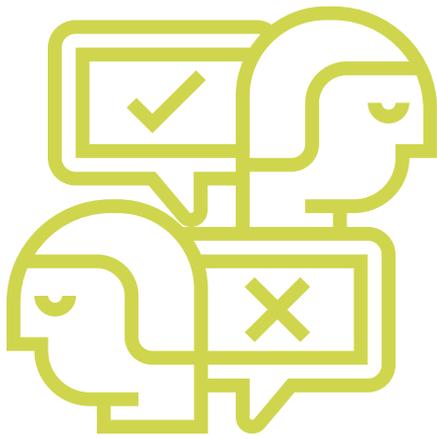
In order to effectively visualise uncertainty, it is necessary to understand how mistakes can occur in reception. As emphasised throughout this catalogue, such understandings should be developed through empirical testing with users. However, two design principles do offer some guidance: the principle of appropriate knowledge (which relates to the user's familiarity with conventions) and the semantic principle (which relates to 'natural' mappings between visualisations and visualised information).

Furthermore, it is useful to appreciate that a visualisation format may have several different kinds of user, as well as stakeholders who rely on it in a more indirect fashion. In particular, it is useful to be aware of how the persuasive power of visualisation can play out in distributed decision-making settings, and how it may interact with asymmetries of information, expertise, experience, authority, and accountability among analysts, decision-makers, and other stakeholders.

Finally, the process of developing visualisation formats for decision support can be informed by a range of different models of cognition and decision-making, including an understanding of the decision-maker as a boundedly rational agent, an understanding of common heuristics and biases related to perceiving and reasoning about uncertainty, and an understanding of the counterintuitive quirks of visual perception (i.e. those that underlie optical illusions). In this section we briefly touch on these topics.

Keywords

principle of appropriate knowledge	*clustering illusion*	*framing effect*
semantic principle	*priming*	*confirmation bias*
visualisation & persuasion	*anchoring effects*	*set size effect*
heuristics & cognitive biases	*Weber's law*	*surface size effect*
confounding mean and variance	*Ostrich effect & risk compensation bias*	*position effects & predictable locations*
colour contrast phenomena	*availability bias*	*emotional stimuli*
		inter–individual variability

The user of a visualisation must have knowledge of the conventions necessary to extract its relevant information.

THE PRINCIPLE OF APPROPRIATE KNOWLEDGE AND THE SEMANTIC PRINCIPLE

Heuristics are short-cut procedures or rules of thumb that may generate good-enough results under certain conditions but are also implicated in generating biased decisions. Cognitive biases are systematic errors in one's thinking relative to either social norms for reasoning and/or formal logic. Without testing, it is not possible to know how a particular visualisation format will interact with a user's heuristics and biases. However, some principles of good design practice exist, and are applicable to a wide range of visualisation formats and their users. Two such principles, which are related to each other, are the principle of appropriate knowledge and the semantic principle.

The principle of appropriate knowledge simply states that the user of a visualisation must have knowledge of the conventions necessary to extract its relevant information. The conventions of the display are often encoded in a legend expressing the correspondence between visual variables and their meaning. There may also be additional instructions,

cautions, or recommendations to help with interpretation. The principle of appropriate knowledge also invites us to think about the risk that users will interpret a visualisation based on inappropriate conventions. If the user does apply a different convention to the one intended, will this produce dissonance, causing the user to feel that something is wrong? Or will the visualisation apparently accommodate the incorrect convention, allowing the user to incorrectly interpret the visualisation indefinitely? Furthermore, the principle of appropriate knowledge invites us to think carefully about the conventions we rely on in encoding and interpreting information visually, since practices that feel obvious or inevitable may actually rely on norms that have been learned at some point, and that are not necessarily shared by all users.

In practice, it may not be possible to know the user's familiarity with various conventions. This is where the semantic principle comes in. In the context of creating visualisations of uncertainty, Boone et al. (2018) describe 'the semantic principle of natural mappings between variables in the graphic and what they represent.' As examples, they offer 'classic metaphors such as "larger is more" and "up is good" (Tversky, 2011).' According to the semantic principle, visual attributes should be mapped to underlying data in 'common sense' ways, so that most users would correctly guess how to interpret it, even without knowing the conventions used. 'An example of a match [between visualisation and underlying data] is using the length of a line to denote length of time. An example of a mismatch would be using higher values on a graph to show negative numbers' (Boone et al., 2018). Another example of a match would be data values that are related to one another being physically proximate. Padilla et al. (2018) recommend that we '[a]im to create visualizations that most closely align with a viewer's mental schema and task demands' and '[w]ork to reduce the number of transformations required in the decision-making process.'

The semantic principle raises some interesting theoretical questions (see sidebar). However, practically speaking, the chief drawback of the semantic principle is that it does not always provide a reliable or sufficiently detailed guide to support the visualisation of complex information such as uncertainty information. It is therefore usually

DIGGING DOWN INTO THE SEMANTIC PRINCIPLE

As a principle of design, the semantic principle teaches that adhering to 'natural,' 'classic,' 'common sense,' 'straightforward,' 'efficient,' 'self-evident,' 'self-explanatory' mappings can help to **reduce misinterpretation and/or miscommunication**. However, it is easier to spot violations of the semantic principle than to give a full account of how and why they are violations, or to articulate criteria by which to judge borderline cases. One interpretation of the semantic principle is as an implicit appeal to resemblance, e.g. the use of colours on a map that roughly resemble an aerial photograph (blue for water, green for grasslands, etc.). However, it is unlikely that every implementation of the semantic principle can be easily explained in this way, and it does leave significant space for culturally acquired meanings (e.g. blue may be a more intuitive representation even of water that is black, brown, or green). In their brief account, Boone et al. (2018) also cite Zhang (1996); Zhang suggests that no visualisation format is universally optimal for all the cognitive tasks users may want to perform on the underlying data, but that 'there does exist a general principle that can identify correct or incorrect mappings between representations and tasks'. This principle is that the representation should contain no extraneous information, and should contain all the necessary information (e.g. external information about interpretive conventions should be unnecessary). This may suggest we understand the semantic principle as embodying a preference for efficient and straightforward visual encoding of data. But again, this interpretation is not unproblematic: highly compressed information may be efficiently encoded, but involve several conceptual steps to decode. Identifying the semantic principle with Zhang's principle would also make it difficult to accommodate desirable redundancy. The semantic principle might be understood to favour 'self-evident' or 'self-explanatory' visualisations, perhaps in the sense that it is not possible to recognise that data has been encoded without also recognising how it has been encoded, or in the sense that incorrect interpretation of the visualisation eventually generates the knowledge necessary to interpret it correctly. These would however be quite narrow and demanding interpretations. Other useful perspectives might be drawn from phenomenological philosophy, insofar as 'spatial perceptions and values that are grounded on common traits in human biology [...] transcend the arbitrariness of culture' (Tuan, 1979), and/or from the field of biosemiotics, which theorises how meaning may be produced and interpreted by non-humans. Overall, the semantic principle is a useful tool for research and design, but we should be a little cautious of taking for granted the 'naturalness' of the mappings that Boone et al. allude to. Describing these relationships as 'natural' may obscure how our intuitions about them are cultivated and reinforced by social, cultural, institutional, and other factors—even when those intuitions are deep-seated and widely-held.

necessary to supplement the operation of the semantic principle with clearly-indicated conventions. Thus, when designing visualisation formats, the principle of appropriate knowledge and the semantic principle are complementary. Users should be equipped with knowledge of appropriate conventions, and the conventions that are chosen should align as closely as possible with what most users would expect anyway.

VISUALISATION AND PERSUASION

Often the agent best qualified to analyse a decision-context does not have the authority to personally make the decision. Visualisation may be one way that such experts communicate their analysis to relevant decision-makers. In this sense, visualisation can be used to overcome gaps between different levels of expertise. Likewise, visualisation may be used to communicate an analysis to a diverse set of stakeholders, so that they can all come to decisions in accordance with their diverse preferences and spheres of authority. Visualisations may thus become reference points to facilitate conversations bringing together multiple perspectives, interests, and forms of expertise.

In all these communication activities, the questions arise of what counts as legitimate influence, and when and how such influence should be deliberately wielded. The choice of visualisation can influence how decision-makers and other stakeholders treat uncertainty, as well as other aspects of decision-making, through mechanisms such as emotional priming, allocation of attention, and selection of heuristics. In particular, '[w]hen incorporated into visualization design, saliency can guide bottom-up attention to task-relevant information, thereby improving performance' (Padilla et al., 2018). More broadly, the process of classifying uncertainty, developing

a visualisation format, and fostering visual literacy, may create opportunities to steer decision-makers and other stakeholders toward or away from particular goals, assumptions, evidence, procedures, and methodologies. These difficulties can be compounded by the so-called 'curse of knowledge,' the cognitive bias which means that experts frequently mistake how non-experts perceive and reason about information related to their area of expertise. The question is then, how do we distinguish valid decision support from improperly manipulative formats and practices? There is no easy answer. The question's tractability will vary from context to context, and may call upon value judgements, and legal, political, and ethical considerations.

For example, in a setting where decisions are relatively standardised and fungible, and there is strong consensus about what comprises sound decision-making and good outcomes, decision quality may be a fairly unproblematic guide to the legitimacy of the decision support mechanisms used. In more complicated settings, however, there is the risk that decision quality is improved at the expense of undesirable distributions of knowledge production and validation, e.g. certain stakeholders being institutionally accountable for knowledge which they cannot actually account for, insofar as it is actually understood only by other stakeholders, and/or understood by nobody within the decision-making centre, because it has been so thoroughly cognitively offloaded into decision support systems.

Dual process theory does offer some helpful perspectives. It is widely acknowledged that visualisation can push certain information to the forefront, while hiding other information in plain sight. But how does this actually occur? One mechanism is the use of visual salience to influence the likelihood that information is noticed and

taken seriously. According to dual process theory, the thought processes which underlie decision-making can be attributed to either **System 1 or System 2** (also sometimes called Type 1 or Type 2 cognition). Briefly, System 1 thinking refers to rapid, instinctive thinking on the fringes of consciousness, with a relatively heavy reliance on heuristics. System 2 refers to more conscious, explicit, analytic patterns of thought. These two systems compose a spectrum rather than a strict dichotomy. Sound visualisation will take into consideration how System 1 and System 2 can work together effectively (Padilla et al., 2018).

How we allocate attention is heavily influenced by what presents itself as salient to System 1. Research on preattentive processing is concerned with 'What visual properties draw our eyes, and therefore our focus of attention, to a particular object in a scene?' (Haeley, 2007). Those visual properties that draw our eyes — the 'bottom-up' properties of a visualisation — can play a critical role in what the decision-maker perceives and fixates on. Because visual information is processed by our low-level visual system, with relatively little cognitive effort, the system can be harnessed by well-designed visualisation, in order to draw attention to task-relevant information, and/or to minimise and mitigate biases, i.e. 'using vision to think.' However, we should be careful not to construe System 1 and System 2 as operating in a simplistic sequential fashion:
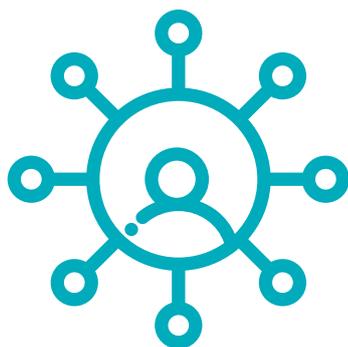
System 1 does not conduct a triage and then pass on what it determines to be important to System 2. Rather, each system dynamically influences the other's activities. In this sense visual perception and reasoning can also be understood as a 'middle-out' process, rather than a bottom-up process that hands over to a top-down process at a certain point.

All this is especially important for providing decision support under uncertainty, insofar as there are many well-attested cognitive biases related to uncertainty information. However, structuring the decision-maker's attention for the purpose of de-biasing cannot be done ad hoc (Orquin et al., 2018); it requires a robust design and validation of a decision-making environment, with sensitivity to how asymmetries in visual literacy map onto accountability structures. The concept of '**choice architecture**,' from behavioural economics and cognitive psychology, may prove somewhat useful here.

Choice architects are people in a position to design the environment in which people make decisions. In the same way that traditional architects design the buildings that people inhabit, choice architects design the way choices are presented to decision-makers. The theory usually assumes that such choice architecture is ubiquitous and cannot be avoided. We can therefore either allow it to develop haphazardly, or we can design it in ways which protect decision-makers from cognitive biases and guide them toward more rational decisions. For example, if a visualisation fails to make appropriate use of System 1 to focus the user's attention on task-relevant information, there is a risk the user will instead focus on distractors, reducing decision quality (Padilla et al., 2018). In the context of visualisation for decision support, choice architecture might be understood as structure that connects (i) the diverse cognitive resources of the boundedly rational agent with (ii) the wider decision-making environment in which such an agent acts. This includes but is not limited to the visualisation itself.

'**Nudging**' is one of the tools in the choice architect's toolbox. According to Thaler & Sunstein (2008), for a technique to count as nudging — as opposed to mandating or forcing — it must be a technique that allows altering people's behaviour without closing off any options or imposing significant costs on them. Thinking of visualisations as part of choice architecture allows us to more vividly describe the pitfalls of improper decision support. On the one hand, a given choice architecture may be too 'open,' offering the decision-maker plenty of information, without giving them sufficient nudges to sift through this information and to understand it. In this case, a decision-maker may fail to apply a useful heuristic, and/or apply one associated with damaging bias, without being alerted to it or receiving the opportunity to reflect critically on how they are forming their judgment. On the other hand, a choice architecture may nudge too hard, potentially exploit biases to funnel the decision-maker toward a particular course of action, making them nominally accountable for something that has essentially already been decided. That is, when a visualisation format is designed to be foolproof, there can be a danger that it impinges on the decision-maker's legitimate freedom of interpretation.

A related challenge — again attesting to the importance of robust testing on a case-by-case basis — arises because the precise graphical layout that faces the user is often procedurally generated, at least in part, by underlying data. In other words, aesthetic features that may impact visual salience (such as a sense of proportion, harmony, balance, 'rhythm,' and so on) will vary somewhat according to what data is inputted. To the extent that these features do impact visual salience, there is the possibility that they also alter the details of the choice architecture. When the visualisation format is static and only ever needs to encode one information set, then the designer has substantial scope to assess and to modify its overall aesthetics. However, when the visualisation format is dynamic, and/or

Figure 12. Illustrations of selected aesthetic features. Created by Jana Kleineberg.

Figure 13. The cognitive Bias Codex. Source: Wikipedia, by John Manoogian III.

when it is used with multiple data sets over time, then these features cannot be micromanaged. Instead to some extent these features may be determined as emergent 'side effects' of the information that is represented.

HEURISTICS AND COGNITIVE BIASES

In the 1970s, psychologists began to explore the mental tools, called heuristics, which humans use to assess probability and make decisions (Tversky and Kahneman, 1973). In general, a heuristic (or a heuristic technique or heuristic rule) is a rough-and-ready mental process that is used for problem-solving. Heuristics generate answers that might not be perfectly accurate, but that might be adequate for everyday life or for a certain purpose. Because of their crude and approximate nature, however, heuristics can also lead to systematic errors and behaviour that is irrational from the perspective of decision theory.

People can apply heuristics without even thinking about or knowing that they are doing so. The term heuristic often refers specifically to these reflex patterns of reasoning that are widely attested and apparently 'deep-seated' features of human psychology. Sometimes heuristic refers more broadly to any kind of mental shortcut, including those that are acquired through domain-specific study or experience. In the latter sense, a heuristic can be associated with the development of expertise. The distinction between a heuristic and a cognitive bias is not completely clear-cut—if you are lucky enough to wind up in just the right context, a bias may lead to effective action—but in general we can say that a heuristic is "a simple procedure that helps find adequate, though often imperfect, answers to difficult questions" (Kahmeman, 2011), whereas a bias is a systematic distortion of judgment, often a result of the limitations of the heuristic(s) used.

Figure 14. Hurricane Michael's path 2018 leads to landfall across the Florida panhandle on Wednesday as a Category 1 storm, according to the Sunday 10 a.m. update from the National Hurricane Center. Image: NOAA.


Figure 15. The impact of visualisation of uncertainty on subject's understanding on spatial and temporal uncertainty in a hurricane forecast. Based on visualisations used by Ruginski et al. (2016) to illustrate their findings. Created by Jana Kleineberg.

A. cone w. centerline
B. centerline
C. ensemble
D. fuzzy cone
E. cone only

Dimara et al. (2018) suggest that there are currently over 154 known cognitive biases, including biases or systematic errors related to making causal attributions, recalling information, testing and assessing a hypothesis, conducting estimations, opinion reporting, etc. In visualisation research, only a handful of these biases have been scientifically assessed (for a review, see Valdez et al., 2018a). Overall, findings suggest a moderate to strong impact of biases on selective attention, performance speed, memory recall and retention, as well as decision-making process and quality (Wall et al., 2017).

Decision-makers can be considered boundedly rational agents whose 'selection' of heuristics (even when this is not a conscious selection) may be influenced by the specific uncertainty visualisation format used. One possible objective in developing uncertainty visualisation formats, therefore, is the promotion of context-appropriate heuristics and deactivation of inappropriate ones, recognising that what is 'appropriate' must vary with the user and decision environment. In particular, as Correll & Gleicher (2014) put it, 'how we visually encode uncertainty and probability can work to "de-bias" data which would ordinarily fall prey to an otherwise inaccurate set of heuristics (by comparison to an outcome maximizing classical statistical view).'

On the next few pages, we mention a selection of the kinds of biases which may become relevant in the development of an effective visualisation format. As these are intended to be illustrative, we make no attempt to categorise them in any detail.

However, Valdez et al. (2018a) suggest that within visualisation research we can divide cognitive biases into three kinds—perceptual, action, and social—while also acknowledging a lack of hard boundaries. Perceptual biases, such as Weber's Law and the clustering illusion, occur on the 'lowest' level of cognitive processing, the sensory-motor-sensory loop, and thus are mostly irresponsive to training. That is, one cannot 'unsee' the effects of such a bias, even if one knows that it is there. Action biases, such as the availability bias and the ostrich effect, are more related to interpretation and thus more responsive to training: what is perceived may be an adequate representation of what can be objectively known, but it is analysed in systematically incorrect ways. Finally, social biases, such as the curse of knowledge and framing effects, are those biases which can only be understood in relation to socio-cultural institutions and norms, and the deeply engrained capabilities and dispositions that we acquire through lived experience.

Visualisation can alleviate the effects of bias, but visualisation can also exacerbate these effects, or create opportunities for biases to manifest when they otherwise would not. The study of cognitive biases within visualisation is challenging. 'Some biases might counteract each other, and experiments have to be meticulously planned to isolate the desired effect from other effects' (Valdez et al., 2018a).

CONFOUNDING MEAN AND VARIANCE

Evidence suggests that current visualisation formats are more effective at communicating uncertainty about the mean of a set of sample values than uncertainty about the variance. Reducing overconfidence is particularly hard. Pugh et al. (2018) draw

Bar chart with error bars
length of bar = proportional to the values they represent.
whiskers represent error margin (here ±5)

Modified box plot
whiskers represent error margin
center line = median (50% percentile)
dots = outliers (extreme datapoints)

Gradient plot
Point estimate with probability density function shown as a gradient

Violin plot
symmetrical probability density

Figure 16. Alternative visualisations of the confidence interval. Created by Jana Kleineberg.

attention to this fact, which is likely a general feature of human psychology: 'Even when presented with an uncertainty visualization, people still exhibited greater attentional focus on the mean and overconfidence in their understanding of the variance. These results have implications for decision makers and the consequences related to not considering alternatives to the most likely outcome.' However, Pugh et al. (2018) also note that some designs perform better than others. In the context of visualising the predicted path of a hurricane, they write that it is possible that 'different forms of visualizations may better enhance the understanding of variance, improve transfer effects, and reduce related overconfidence.' They point in particular to Ruginski et al. (2016), who found that 'the addition of a centreline, fuzzy shading, and/or ensemble paths to a cone visualization decreased the perception of variance.'

Another common problem is the misinterpretation of increase in variance over time as if it were an increase in the mean. Here the semantic principle of larger size meaning something is bigger is adhered to, but leads to confusion about what gets bigger. In this case, it is the spatial

uncertainty that grows, rather than the strength of the hurricane (which is not depicted at all). Ruginski et al. (2016) point to yet another misinterpretation that the cone represents an area of impact rather that an area made up of possible hurricane trajectories. One solution proposed was to represent trajectories as individual lines, as a set of possible realisations, denser around the most likely path. This, however, was found to make people less afraid of the hurricane—the path ensemble visualisation reduced their estimates of risk, potentially leading some to ignore evacuation orders, as opposed to alternative visualisations based on the same predictions. Ruginski et al. (2016) note that 'the various visualizations caused participants to notice different visual properties of the displays and to base their judgments on different heuristics'; for instance they recount that, 'The fuzzy-cone and the cone-only visualizations (both without the centerline) resulted in lower damage judgments than the cone-centerline. It is possible that the presence of the salient center forecast track leads users to cognitively assess the intensity of the hurricane to be greater.'
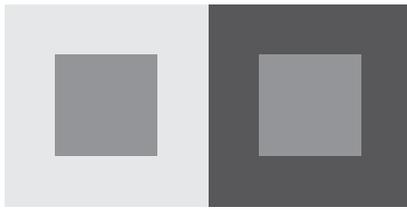
Surround a colour with a lighter color and it will appear darker; surround a color with a darker color and it will appear lighter.
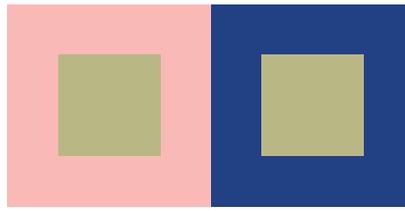
Surround a colour with different hues and it will shift in appearance towards the complementary hue of the surrounding color.
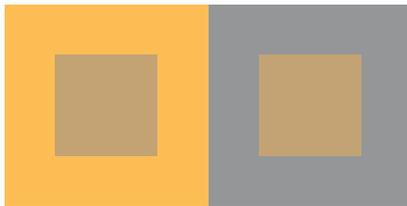
Surround a colour with a less saturated color and it will appear more saturated; with a more saturated color it will appear less saturated.
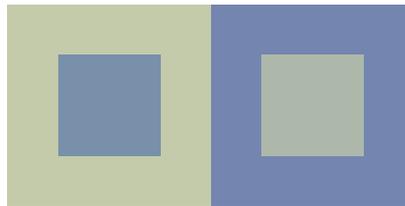
You can surround two different colours with two other colours to make them appear more similar.

Figure 17. Colour contrast phenomena.

Misunderstanding variability is a common problem even among experts. When error bars are used to depict confidence intervals for any distribution (e.g. continuous, non-uniform), one frequent mistake is to interpret the uncertainty distribution as uniform, bounded, and discrete, i.e. to assume that every value inside the error bars is equally probable, and the probability of a value falling outside of that range falls sharply to zero. Correll & Gleicher (2014) also reported another bias, where a bar chart was used to indicate the mean and error bars to indicate the confidence interval. Participants treated the bottom part of the margin of error, which overlapped with the bar, as more probable, incorrectly interpreting symmetrical distributions as skewed.

Traditional error bars, despite being the most common visualisation of uncertainty, are arguably in violation of both the semantic principle and the principle of appropriate knowledge. Visualisations using these error bars violate the semantic principle, insofar as they are interpreted to represent what may be an infinite domain of a function by a fixed interval, and 'emphasize an "all or nothing" approach to interpretation—values are either within the bar or they are not' (Correll & Gleicher, 2014).

They may also often violate the principle of appropriate knowledge if, as Boone et al. (2018) describe, they fail to state 'what measure of error is represented by the bars (e.g. whether they show the standard error, standard deviation, or 95% confidence interval),' especially since 'even scientists do not always appreciate the differences in the inferences that can be made in each of these instances (Belia, Fidler, Williams, & Cumming, 2005).'

Correll & Gleicher (2014) also proposed and tested alternative visualisations. Participants were shown a data point representing a potential outcome, and asked to judge how likely this outcome was given the sample mean and the margin of error, testing across different visualisation formats. It was found that gradated visualisations — such as colour gradients or tapering, violin-like shapes — can ameliorate misinterpretations associated with traditional error bars, at least in some cases.

COLOUR CONTRAST PHENOMENA

Our perception of a colour is influenced by the colour(s) adjacent. Colour contrast phenomena may be considered a classic instance of a perceptual bias, in that one cannot 'unsee' the effect, despite knowing that it is there.

CLUSTERING ILLUSION

The clustering illusion is a part of a perceptual bias which causes respondents to see patterns in small sets of randomly generated data. For example, apparently significant clusters or streaks are perceived in low-density scatter-plots (Blanco et al., 2017). The clustering illusion leads to irrelevant, inaccurate inferences of causal relationships (also called causal illusion or illusory correlation), and has been shown to decrease the accuracy and quality of decision-making (Blanco et al., 2017). Three principles are responsible for the perception of a potential cause-effect relationship. The first two are priority and contiguity, which are represented in the temporal ordering (i.e. priority) and the proximity (i.e. contiguity) of the stimuli. If two events occur close in time and space, this may create a belief that one caused the other. The third principle, the principle of contingency, refers to the fact that causes and their effects must be correlated, and it is this principle that is believed to create a clustering illusion bias In our perception of contingency, we are prone to systematically overestimating causal linkage. One suggested reason for respondents' tendency to overestimate data as correlated is that it provides a feeling of control over the situation, and reduces anxiety that might occur in the context of risk and uncertainty (Blanco et al., 2017).

PRIMING

Priming refers to the phenomenon by which response to a stimulus can be influenced by prior exposure to some related stimulus (Kristjansson, 2006). for example, when asked to complete the word 'SO_P,' respondents who have been shown a picture of a shower are relatively more likely to choose 'SOAP,' and respondents who have been shown a picture of bread and butter are more likely to choose 'SOUP' (Valdez et al., 2018b). In the context of visualisation, priming effects challenge the assumption that '[g]iven the same stimulus and the same person, one should "see" the same thing every time' (Valdez et al., 2018b). When it comes to perceptual processing for visual search targets, primed behaviour can be facilitated by means of visual 'cues,' e.g. in certain uses of colour to represent certain types of content, or by repeating cues in expected positions or locations. Such techniques have the potential to enhance performance speed, accuracy and recognition of identified search item, as well as to decrease response latencies. Priming appears to be of particular value when the task contains ambiguity. As a consequence, in tasks where standardised responses are critical, it has been proposed that visual aids can help to de-bias the decision-making process, reducing variability in search identification activities, and controlling for effects that cause response latencies. However, more research in this area is still needed (Valdez et al., 2018b).

ANCHORING EFFECTS

Anchoring consists of the use of a previous stimulus as some sort of reference or anchor, which is used to help make judgments about the current stimulus, even if the stimuli are unrelated and the anchor is completely random. Anchoring effects are related to priming; priming appears to be one of several mechanisms that underlie anchoring (Valdez et al., 2018b; Wilson et al., 1996). The anchor provides an initial reference point which the decision-maker then adjusts by incorporating relevant beliefs (the 'anchor-and-adjust' heuristic), but since these adjustments are often insufficient, the initial choice of anchor tends to carry undue weight. In relation to decision-making generally (Langeborg and Eriksson, 2016), studies have found that anchoring effects take place predominantly automatically and unconsciously, and in situations of uncertainty, where users are likely to hold on to a narrative that is readily available, anchoring is associated with making more conservative decisions (Ellis and Dix, 2015). for example, when a previous estimate or decision has been shown to be inaccurate, an anchoring effect may prevent the decision-maker from making sufficient adjustments on the next iteration. Anchoring can in effect act as a stability bias, meaning that it is one of those biases may make one cling to the status quo (Kahneman, 2011).
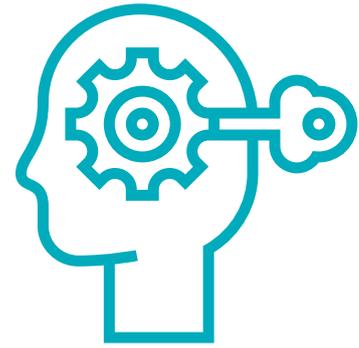
In visualisation research, anchoring has been studied in terms of search tasks within naturalistic visual scenes (Boettcher et al., 2018).
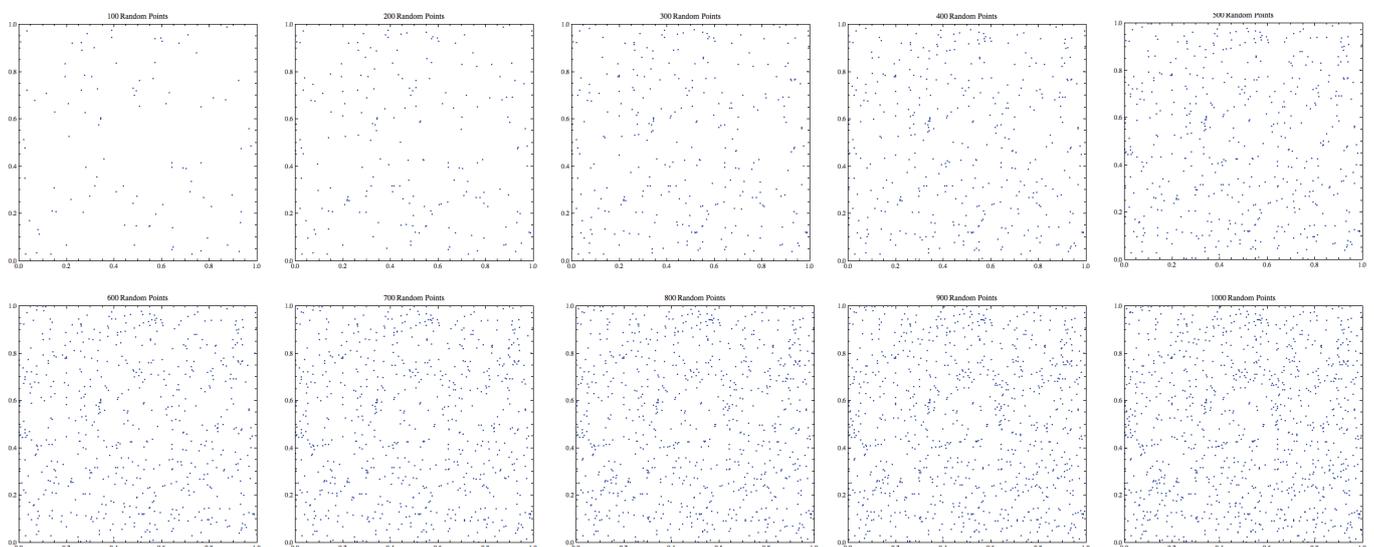
Figure 18. Clustering Illusion. Source: CaitlinJo, Wikipedia.
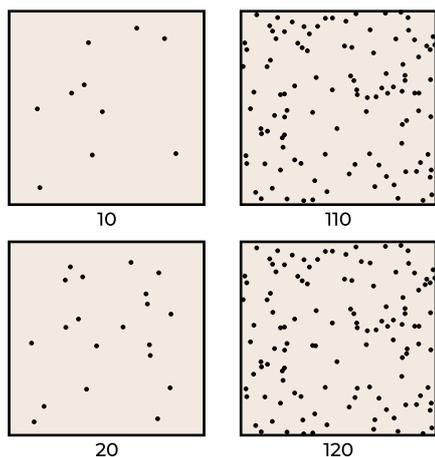
Figure 19. Weber–Fechner Law 01, source Wikipedia, MrPomidor.

On each side, the lower square contains 10 more dots than the upper one. However the perception is different: On the left side, the difference between upper and lower square is clearly visible. On the right side, the two squares look almost the same.
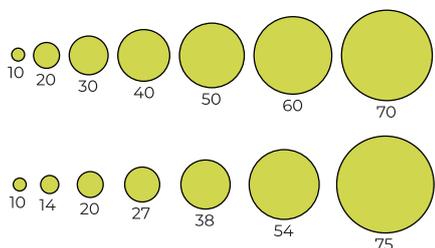
Figure 20. Weber–Fechner Law 02, source Wikipedia, MrPomidor.

Circles in the upper row grow in arithmetic progression; they make an impression of growing initially fast and then slower.

Circles in the lower row grow in geometric progression: each one is larger by 40% than previous one. They make an impression of growing by the same amount at each step.

Anchor objects are objects that hold a relatively high amount of predictive information about objects with which they frequently co-occur in spatially consistent arrangement. For example, across many different scenes, a bathroom sink may hold information about the likely locations of plughole, taps, toothbrush, toothpaste, soap, mirror, and so on. By tracking eye movements, it has been demonstrated that the presence of relevant anchor objects within scenes alters search strategy, resulting in less scene coverage overall. Relevant anchor objects can somewhat enhance perceptual processing by faster reaction times, and less time between fixating the anchor and the target (Boettcher et al., 2018). Anchoring has also been studied in visualisation research in terms of how anchoring during training may influence how analysts use an exploratory visual analysis system (Wesslen et al., 2019).

Anchoring may become problematic when decision-makers use a 'worst case' scenario as an anchor. Nadav-Greenberg et al. (2008) asked participants to forecast wind speeds based on visualisation of median wind speeds as wall as various representations of uncertainty information. They found that where uncertainty information was displayed as an upper boundary of projected wind speeds, forecasts showed a bias toward higher wind speeds. Kinkeldey et al. (2015) note in their review: 'Thus, the authors give the warning that in a real-world setting providing worst-case maps could lead to more false alarms. They explain this effect with evidence from past research that anchors (in this case the depiction of the worst case) unconsciously influence people's judgments (Chapman and Johnson, 2002). Similar results were provided by Riveiro et al. (2014) in a target identification experiment, where an expert group

aided by uncertainty visualization selected higher priority values and more hostile and suspect identities. This suggests that, when safety was an issue, the experts put themselves in the "worst-case scenario" in the presence of uncertainty.'

WEBER'S LAW

How we judge and categorise sensory magnitude is affected by a number of biases (Poulton 1979). As an illustration, the smallest perceptible change in the brightness of a light source will be different depending on the initial intensity of the light. Weber's Law is a mathematical formula that states that the minimum difference in intensity needed to perceive a change between two given stimuli is proportional to the stimuli (Carr, 1927; Harrison et al., 2014; Kay and Heer, 2016; Valdez et al., 2018a). Using Weber's Law, we can make predictions about whether a given change in a stimulus intensity will be perceived, and about what magnitude of change will be perceived. The formula requires an empirically derived constant, Weber's fraction (k). The law has been shown not to hold for extremes, e.g. a very dim or very bright light.

The law implies the risk of a response bias when judging and categorising sensory magnitudes (Poulton, 1979). As an illustration, such an effect could be modelled in correlated data representations (such as scatterplots), with first evidence showing that the just-noticeable difference in correlation strength is indeed different in different parts of the correlation spectrum (Harrison et al., 2014). The findings suggest that a user's ability to identify correlation in visualisation formats can be modelled using Weber's Law, and the authors interpret this to mean that the underlying information-bearing visual features also follow Weber's Law. They therefore suggest that Weber's Law, and

other perceptual laws from psychology and cognitive science, can be used to model and rank the precision afforded by different visualisation formats. For example, the median just-noticeable-difference of the scatterplot format can be compared with the median just-noticeable-difference of other visualisation formats. This might mitigate the need for extensive empirical experimentation, and support more targeted testing of visualisation formats. It could do so by excluding certain poorly-performing formats in advance, and/or by helping to isolate specific design features that are responsible for differences in performance.

OSTRICH EFFECT AND RISK COMPENSATION BIAS

The Ostrich effect is represented in the tendency to neglect information that generates a feeling of discomfort, and leads respondents to overlook information or to downplay information that would be considered negative (Valdez et al., 2018a; Dimara et al., 2018). Similarly, risk compensation bias is the tendency to adjust behaviour in response to risk, being more cautious during cases of greater perceived risk and less cautious in situations of protection and security (Dulisse, 1997). for example, drivers wearing seat-belts have been shown to drive somewhat less cautiously (Janssen, 1994). Both

biases are not well-researched in visualisation research; however, it has been suggested that in the context of uncertainty, in order to mitigate poor decision-making, automated systems should be put in place, highlighting critical, uncomfortable information to counteract the occurrence of such potential biases (Dimara et al. 2018).

AVAILABILITY BIASES

People are influenced by the availability of examples of an event, even when the ease with which they can think of examples bears no relation to actual frequency. This bias is called availability bias (Tversky & Kahneman, 1973), and can exert a considerable influence on reasoning and decision quality. In visualisations, for example, an availability bias can be created by having to assemble a set of documents on the screen before undergoing the analysis stage; information in these available documents may be given undue weight in comparison to information that is sought out elsewhere during analysis (Ellis & Dix, 2015).

FRAMING EFFECT

First mentioned by Goffman in 1974, the framing effect identifies how the presentation or 'framing' of information can influence the decision-making process. If options are presented through positive or negative semantics, settings, or situations, decision-makers are likely to incorporate these features into their judgements, even when they have no bearing on the factual circumstances. For example, a policy option may be prove attractive when it is presented in terms of how much it would save rather than how much it would cost, even if both description contain mathematically identical information.

One special case of the framing effect is the attraction effect (Mansoor & Harrison, 2017). The attraction effect (or decoy effect) is

primarily used in market research and assumes that 'if people are deciding between two products ("target" and "competitor"), a third product ("decoy") that is close to the target but objectively suboptimal to both attributes, can make the target look more attractive' (Dimara et al., 2018). Expressed in more game theoretic terms, the attraction effect 'suggests that any decision involving a set of points that belongs to the Pareto front is influenced by the dominated datapoints below it' (Dimara et al., 2018). Within visualisation research, Dimara et al. (2018) embedded the attraction effect within a scatterpoint-based task, and invited respondents to choose between different optimal points on the diagram. Their results confirmed that respondents are more attracted to optimal options situated near decoys. It was suggested that dynamic visualisation formats could help to de-bias such decisions, including 'computational aids that highlight optimal decisions based on objective criteria, but more counterintuitively, a system where users can systematically delete information as they make comparative decisions at a more local level of analysis' (Dimara et al., 2018).

CONFIRMATION BIAS

The confirmation bias describes the tendency to accept information or evidence that confirms preexisting hypotheses, and which results in the
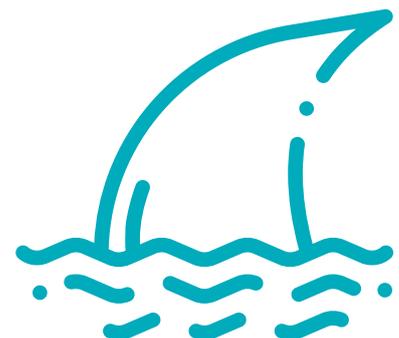
denial or dismissal of information that functions contrarily to those beliefs (Rajsic et al., 2015). In visualisation research, this bias has been shown to guide respondents' attention, and to lead to a visual selection of information that has been initially prioritised, even when the strategy is not the most optimal for the task at hand (Rajsic et al., 2015; Padilla et al., 2018). As a consequence, suggestions have been made to enable mitigation by analysing a range of competing hypotheses that require careful consideration before reaching a conclusion (Dimara et al., 2018; cf. a confirmation bias mitigation software [Wright et al., 2006]).

SET SIZE EFFECT

Since attention is a limited resource, increasing the set size (i.e. the number of elements in a data set or a visualisation) typically leads decision-makers to fixate on a smaller proportion of a set of data, and to an increase response latency in visual search tasks (Spinks and Mortimer, 2015; Palmer, 1993). This effect is a common heuristic and can lead to attribute non-attendance, whereby respondents ignore one or more attributes when making decisions. Tasks involving increasing complexity of decision problems have demonstrated that the set size is one of the strongest predictors of attribute non-attendance. Other factors include time pressure and prior experience with the decision problem. It is believed that increasing the set size does tend to impede visual selection, even when it allows for desirable enhancement of local feature contrasts (Becker and Ansorge, 2013).

SURFACE SIZE EFFECT

The surface size is the area the object occupies within an environment (Peschel & Orquin, 2013), and may best be explained as an 'increase in

object signal strength which depends on object size, number of objects in the visual scene, and object distance to the centre of the scene.' (Peschel & Orquin, 2013). The surface size effect has shown to exert a robust and medium to strong effect on fixation likelihood (Peschel & Orquin, 2013). Larger objects are likely to diminish the attention toward smaller objects (i.e. the decision-maker is more likely to fixate on larger objects). The surface size effect does not only exist in visualisations, but has also been found in text-based information, with an increase in the surface size related to the text significantly and positively impacting selective attention and attention span (Rik & Wedel, 2004).

POSITION EFFECTS AND PREDICTABLE LOCATIONS

When information is structured in a one-dimensional array, viewer have a strong tendency to begin reading from the top of a column towards the bottom (Chen & Pu, 2010; Simola et al., 2011) or from the left of a row towards the right (Navalpakkam et al., 2012). for example, Navalpakkam et al. (2012) found, contrary to their hypothesis that semantic factors such as user interest in the content would matter most for sustaining attention, that the position effect remained dominant, with significantly higher dwells at top/left positions than other positions. Similarly, in two-dimensional arrays, the viewer tends to fixate on the middle of the array, whereas the corners often go unnoticed (Meißner, Musalem, & Huber, 2016). However, these findings only account for Western societies where the reading direction is from left to right. Furthermore, attention can also be guided by controlling the predictability of object locations (Orquin et al., 2018). Studies show that participants are more likely to fixate
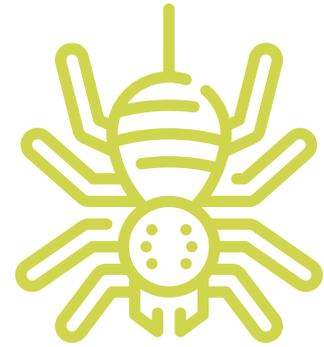
a high-relevance label in a predictable location as opposed to low- and medium-relevance objects in predictable locations and unpredictable locations, respectively (Orquin et al., 2018). In other words, 'predictable locations enhance top-down control by allowing decision makers to attend or ignore information that they perceive to be important or irrelevant' (Orquin et al., 2018).

EMOTIONAL STIMULI

Emotional stimuli of typically negative valence (e.g. angry faces or spiders) and those of typically positive valence (baby faces or mini pigs) both attract attention faster and with a higher likelihood than emotionally neutral stimuli (Calvo & Lang, 2004). When comparing negative versus positive stimuli alone, negative pictures tend to create a greater impact on our visual attention than positive stimuli (Nummenmaa, Hyönä, and Calvo, 2006; for a detailed review, see Pessoa and Ungerleider, 2015). In visual search tasks, this is especially the case during the first 500ms, suggesting that emotional stimuli are detected within preattentive processing (Calvo and Lang, 2004). As such, Calvo and Lang (2004) conclude that 'this early allocation of attention to pleasant and unpleasant stimuli [is] a central cognitive mechanism in the service of prompt detection of important events and activation of motivational resources for approach or avoidance.'

*Graphical literacy, just like literacy, needs to be taught and developed.*

INTER-INDIVIDUAL VARIABILITY

Systematic errors can also arise when visualisation formats fail to take into account inter-individual differences, including cognitive characteristics, cultural background, and types and levels of expertise (Conati et al., 2014; Lallé et al., 2015; Green & Fischer, 2010). In these cases, adapting visualisations accordingly and correcting for human factors is key, especially when the task increases in complexity and cognitive workload required (Valdez et al., 2018a; Micallef et al., 2017; Parson, 2018). Ensuring stakeholders are empowered and motivated to share their beliefs and knowledge is critical to enabling a shared mental understanding of the task (Birch & Bloom, 2007).

The use of a visualisation as a common reference point offers no guarantee that stakeholders will use it in similar or compatible ways. Even robustly designed visualisations may sometimes accommodate conflicting understandings, which may go unnoticed if there is insufficient validation and/or insufficient opportunities for reflection and dialogue. Moreover, communication about the development of a visualisation format, or the meaning of a completed visualisation, can be impeded by the 'curse of knowledge.' This refers to the difficulty that experts experience when trying to put themselves in the position of non-experts. To be an expert in a given domain does not necessarily entail good awareness of when or how a particular judgment draws on that domain knowledge, let alone the skill of helping others to join one in expert perspectives and reasoning. Xiong et al. (2019) show that 'when people are primed to see one pattern in the data as visually salient, they believe that naive viewers will experience the same visual salience.'

Boone et al. (2018) point out the visualisation of complex information, such as uncertainty information, is constrained by a general lack of knowledge and experience with reading a graphical language. 'Another factor that can greatly influence the effectiveness of a graphic is the knowledge the viewer has about the conventions of the graphic type in question.' Graphical literacy, just like literacy, needs to be taught and developed; such knowledge according to the authors is often insufficient. See also 'The semantic principle and the principle of appropriate knowledge,' discussed earlier in this section.

Lastly, Stacey and Eckert (2001) warn about another potential source of misinterpretation that stems from the fact that the user is drawing conclusions not just from what is shown but from a negative space, i.e. from what is not depicted: 'Understanding how much of what is not shown is fixed, and what can be varied, is as essential as understanding the explicit content of a representation. Alternative interpretations of the omitted elements of a design are made possible by uncertainty or misunderstanding about the interpretive conventions to be applied to a representation, as well as the context in which it is embedded and the assumptions the generator makes about how the gaps will be filled in. Thus it can be ambiguous by omission. In other words, what is implicit in any representation depends on the interpretive skills of the recipient and the extent of the shared understanding of context established between the sender and the recipient.'
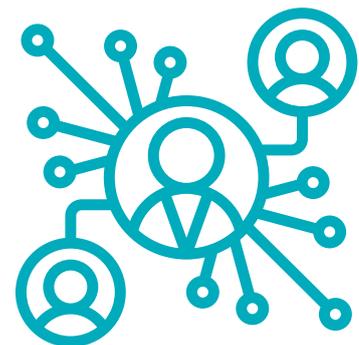
9 CONCLUDING REMARKS
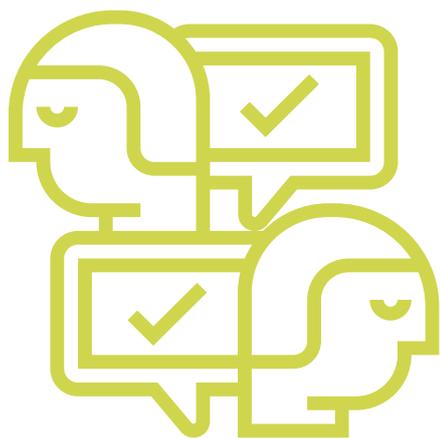
The literature on communicating uncertainty is vast and mixed. This catalogue offers a very brief selection, guided by several criteria: assessment of the quality of the research; of its relevance to the decision-making context; and of its capacity to highlight the art of the possible in visualisation of uncertainty.

There is no 'optimal' format for communicating uncertainty, nor is it easy to determine if and how a format influences decision-making. Identifying instances in which the communication format has a significant impact is key to improving the communication of uncertainty. It is desirable to develop communication formats through close dialogue between designers and end-users. Self-reporting is unreliable for assessing the effectiveness of a communication format; evaluations should be performed by methodologies which bear hallmarks of reproducible research.

Across the spectrum of uncertainty types, a common set of visualisation techniques are potentially applicable. There is no consistent mapping between categories of uncertainty and methods for visualising uncertainty, and it is an open question whether such a mapping is either possible or desirable. The users of uncertainty information have diverse capacities and needs. There is not yet any deep theory to formalise which differences are relevant in any given case. Uncertainty representation is not easily separable from interpretations of value, thus individual and group differences (cultural, political, social, linguistic, etc) play an important role.

One clear recommendations that emerges from this multidisciplinary literature review is that the implementation of visualisation techniques must be studied on a case-by-case basis, and ideally supported by empirical testing. The success of different techniques for visualising uncertainty is highly context-sensitive, and current understanding of how to differentiate relevant contextual factors appears patchy. In short, current research offers an insufficient basis for robust generalisations about the visualisation of uncertainty.

Complex decision-making inevitably involves visual information to some degree. From the organisation of text on a page or screen, to the internal mental representations evoked by verbal reasoning, there is a potential visual aspect to all practices of analysis, communication, and decision-making. Thus even if we attempt to simply opt out of the visualisation of uncertainty, we still risk allowing visuals to shape our processes in ways which go unregistered and unstudied.

NOVEL APPROACHES TO UNCERTAINTY VISUALISATION

There is currently a paucity of visual signifiers of uncertainty that can be consistently interpreted. The available graphical language has too few elements, and interpretative communities are too diverse and fragmented to interpret it consistently. In this catalogue, we provide some basic building blocks, as well as perspectives, concepts, and methods that may be useful in developing and testing visualisation formats. Throughout this catalogue we emphasise the importance of developing these on a case-by-case basis, using objective and reproducible testing, ideally with the relevant end-users. However, there are also contexts in which, for one reason or another, more informal and exploratory approaches are appropriate. of course, tailored evidence-based solutions are vital to advance the field. But so too are those spaces where designers, artists, data journalists, and researchers, among others, can explore the potential of visualisation in free and open ways, or in response to constraints other than those associated with decision support. These wider visual cultures are important in that they nourish the visual imagination and are an informal testing ground for novel visualisation techniques.

For example, visual and conceptual artists frequently engage with data, sometimes exploring new ways of representing data and/or new affordances for interacting with it. Experimental musicians such as Cathy Berberien, Hans-Christoph Steiner, and many more have also developed numerous types of visual notation and an associated body of theory. Moreover, data journalists constantly seeking bold, striking, and/or beautiful ways of using data visually, and of conveying complexity in ways that are clear and persuasive. Likewise communications professionals working in campaign organisations and NGOs have a strong and longstanding interest in the cognitive and emotional effects of visualisation formats, especially in relation to environmental crisis and economic inequality. Similarly,

professionals working in marketing and advertising agencies, and the creative industries more generally, respond to commercial incentives to find new and distinctive ways of communicating visually. Within popular culture, creators of science fiction often depict imaginary user interfaces, and imagine how such interfaces might be woven into their characters' everyday working lives.

Decision analysis can also benefit from interdisciplinary input, and not just from the most obvious candidate disciplines, such as user experience design. for example, in the digital humanities—across literary studies, history, heritage studies, and other discplines—researchers have long been exploring new ways of curating texts, artefacts, and other objects of study, and managing and augmenting human attention and analysis. To give just one small example, the classic open source application Voyant Tools offers basic analytics for textual corpora and a suite of dozens of visualisation options.

Perhaps most significantly of all, visualisation and decision making intersect in an extremely rich way in computer games. Computer games often need to represent relatively complex information in ways that feel intuitive and immersive for players. The conventions that are widely understood within gaming communities, and are adopted by major games developers on big budget projects, are of interest; so too are the potentially more eccentric visualisation formats invented by indie game developers. Game design studies offers conceptual frameworks around the use of visual hierarchy, a visual language, visual themes, calls to action, and so on, and the games themselves offer specimens that test and perhaps exceed such concepts. While game development has its own distinct goals, and playtesting is not conducted according to the same evidentiary standards as most empirical science, game development does tend to be heavily iterative, and incorporates a great deal of user experience

and feedback. In this respect, it may also be useful to take a media-archaeological approach and examine the historically evolving conventions of visual representation within computer games. In addition to computer games, other digital applications, as well as analog games such as tabletop wargames and 'eurogames,' may offer starting points for thinking innovatively about the visualisation of uncertainty. Finally, there is also now a strong and ever-evolving visual dimension to everyday online communication, from emoticons and emojis, to reaction GIFs and memes, to filters and editing tools for user-produced visual content.

Of course, many of these wider visual cultures are not be specifically concerned with communicating uncertainty information (although they are sometimes interested in communicating fairly complex information). They have their own native agendas and values, and don't exist primarily to complement the needs of decision-making under uncertainty. Nevertheless, from our perspective, such cultures are important in several respects. First, they can exert an influence on graphic literacy. That is, they form spaces in society where certain visual signifiers can be developed and popularised, and where people can learn and reinforce certain ways of thinking visually. In other parts of their lives, the users of decision support systems may have had significant exposure to such conventions. Second, by looking to visual innovation within domains such art, design, cinema, the digital humanities, gaming and game studies, and so on, we can gain direct inspiration for novel formats, signifiers, techniques, and so on to test out in an empirically robust way. Finally, it is plausible that participation in these wider visual cultures enriches our visual imaginations more generally, putting us in contact with a diversity of visual forms that indirectly supports the both the tasks of empathy-building and the creative leaps that are often necessary to develop an effective visualisation format.

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11 | RESOURCES

ADDITIONAL IMAGES/ ILLUSTRATIONS

from www.flaticon.com, made by

authors/Freepic 3

authors/Freepic 12

authors/Eucalyp 13

authors/Eucalyp 24

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authors/Eucalyp 33

authors/monkik 35

authors/Freepic 35

authors/wanicon 36

authors/Eucalyp 37

authors/Eucalyp 38

GENERAL BACKGROUND

How to create interactive data visualisations.

- **Nesta Sparks lecture by Cath Sleeman**
  https://www.youtube.com/ watch?v=ctSl8tYEEDY

- **Visualising the uncertainty in data by Nathan Yau (UCLA)**
  https://flowingdata. com/2018/01/08/visualizing- the-uncertainty-in-data/

- **Visualising conflict data**
  https://www.acleddata.com/

DESIGN

- **Free images**
  https://pixabay.com/
  https://unsplash.com/
  https://thenounproject.com/

- **Catalogue of examples**
  http://www.rethinkingvis. com/#all
  https://datavizcatalogue.com/

- **Tutorials**
  https://flowingdata.com/ category/tutorials/

PLATFORMS FOR DATA VISUALISATIONS

- **Microsoft**
  https://powerbi.microsoft. com/en-us/

- **R shiny**
  https://shiny.rstudio.com/gallery/

- **Tableau**
  https://www.tableau.com/

- **D3**
  https://github.com/d3/ d3/wiki/Gallery

SOCIAL MEDIA ON VISUALISING DATA

- Financial Times @ftdata

- NYT Graphics @nytgraphics

- https://twitter.com/ GuardianVisuals

- The Pudding @puddingviz or https://pudding.cool/

- Andy Kirk @visualisingdata or http://www.visualisingdata.com/

A

APPENDIX
UNCERTAINTY REPRESENTATIONS
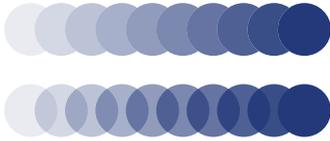
SIMULATIONS

Some uncertainty formats attempt to present a range of possible future realities simultaneously. Another option is to present them sequentially. Many decision-makers and analysts will construct a base case, an upside, and a downside scenario. By examining these in turn, it is possible to develop a sense of where risks and opportunities lie.

Uncertainty can also be represented in a simulation where randomness is built into the model at appropriate points. Running the model again and again, and comparing the different outputs, can provide intuition for the fuzziness of predictions.

Pros

Showing simulations provides a sense of build-up and a link with individual outcomes.

Cons

Too much weight might be placed on individual outcomes and obscure the overall picture.

Showing all data at once can be challenging for interpretation and lead to data overload.

OBSCURITY

Blurriness is a powerful visual metaphor for displaying uncertainty (fog).

Pros

The metaphor makes sense: results that are more uncertain are displayed with a blurry (not sharp/clear) edge, which makes it less visually prominent.

Cons

How is fuzziness or obscurity perceived?

Are various levels actually interpreted, and to what degree?

ERROR BARS

Graphical representations of the variability of data; used to indicate the error or uncertainty in a reported measurement.

Pros

Lines or bars represent a range of values, so you can see that a mean or median represents only part of an estimate. Especially useful when comparing multiple estimates.

Widely used, therefore easily understood.

Con

Details in the data can get lost.

"Within-the-bar bias": viewers judge points that fall within the coloured bar as being more likely than points equidistant from the mean, but above the bar.
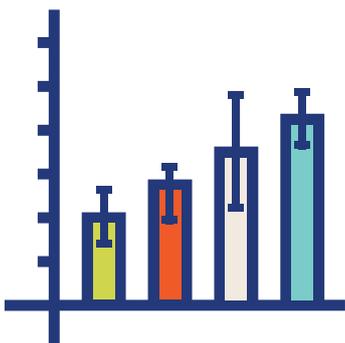
DISTRIBUTIONS

Show the spread of possible values with a histogram or a variant of it. You might see something a median would never show.

Pros

By showing the variation, a user can make a more educated judgement about the accuracy and trustworthiness a sample. It is oddly skewed? Are there multiple peaks? Or is it an expected bell curve?

Cons

Many people don't understand distributions, so a careful explanation needs to be given in the annotations.

Sometimes variation is just noise, or the details might obscure the overall view, impression, or key point.
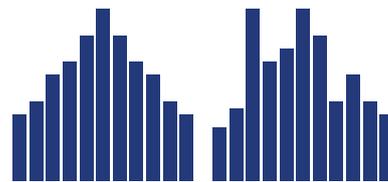
MULTIPLE OUTCOMES

For projections and forecasts, it can be helpful to see various outcomes of what might happen.

Pros

Uncertainty is displayed more explicitly; it is shown that there is not one set path, but multiple possible paths which may diverge and/or converge.

Cons

The chart can become confusing if there is too much noise or too many possibilities. Like many visualisation formats, it is easily manipulated to promote a desired analysis or strategy.

DECISION TREES

In this context, a decision tree refers to a visualisation format made up of nodes and branches. It is often laid out to be read left-to-right, or top-down; with a single root node as the starting point, branching out into possible futures.

One common convention is to use squares to represent decisions and circles to represent chance events. A decision tree can be useful for understanding how a plethora of interconnected decisions, whose different outcomes can be assigned estimated probabilities, will play out under different future scenarios.
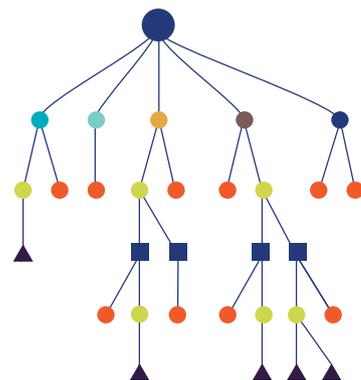
WORDS

Not everything has to be visualized. Sometimes words better describe uncertainty. Some rules of thumb: avoid absolutes when describing numbers; treat estimates as such when you use them; account for the uncertainty in the numbers.

B APPENDIX
CHARTS, GRAPHS, SYMBOLS

VENN DIAGRAM	PIE CHART	CONCENTRIC DIAGRAM	FAN CHART	DONUT CHART
SUNBURST DIAGRAM	CHORD DIAGRAM	RADAR CHART	CIRCULAR CHART	WINDROSE CHART
DENSITY PLOY	AREA CHART	BAR CHART	LINE CHART	SCATTER PLOT
BOX & WHISKER PLOT	BUBBLE PLOT	SPAN CHART	GANTT CHART	CANDLESTICK CHART
VIOLIN PLOT	ARC DIAGRAM	FLOW CHART	SPIRAL GRAPH	TIMELINE
HEAT MAP	MARIMEKKO CHART	ALLUVIAL DIAGRAM	SANKEY CHART	STREAM GRAPH
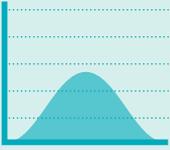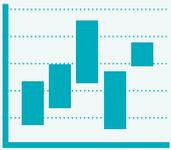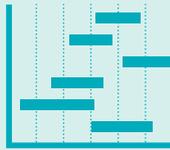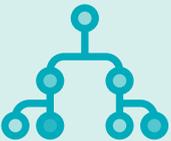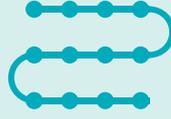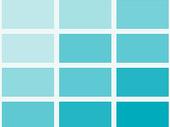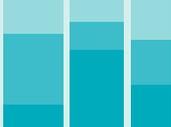
SYMBOLS, METAPHORS, VISUAL ANALOGIES

CHAIN/LINK	AGREEMENT	IDEA	BRAINSTORM	NETWORK
FUTURE/TAKEOFF	STITCHES	PATH/DIRECTION	CLOUD	UMBRELLA
TIP OF THE ICEBERG	LAYERS OF THE ONION	DOMINO EFFECT	TIPPING THE SCALES	SANDWICH LAYERS
ROOTS/ORIGIN	SOLAR SYSTEM	LABYRINTH/PUZZLE	TREE/TREE OF LIFE	GEARS/WORKING THINKING
ROLLER COASTER	CLOCK/TIME	EXPERIMENT	TOOLS	PUZZLE
MOUNTAINTOP	TARGET/GOAL	PLAYING FIELD	FOOD CHAIN	FACTORY
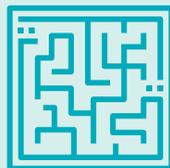
AU4DM
Network

Imperial College London

cep

Warwick
Statistics

love every drop
anglianwater

Psychology | Lancaster University

ssee   UNIVERSITY OF OXFORD

E3G

Government Office for Science

LSE Department of **Management**

FORESIGHT TRANSITIONS

The Alan Turing Institute

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VISUALISING UNCERTAINTY A SHORT INTRODUCTION

A publication of the
Analysis Under Uncertainty *for* Decision Makers Network.

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DECISION SUPPORT TOOLS *for* COMPLEX DECISIONS UNDER UNCERTAINTY

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Analysis Under Uncertainty *for* Decision Makers Network.

CBA
Evidential Reasoning
Hurdle rate Analysis/RARoC
NPV
AHP
Value-Focused Thinking
LCA
TOPSIS
SMART
MCDAid
Fuzzy decision analysis
UTA
MAV/UT
Conflict analysis
Agent Based Modelling
Scenario analysis
Promethee
Cognitive maps
Game Theory
Adversarial Risk Analysis
Risk Register
Impact Uncertainty Mapping
Argument maps
Influence diagrams & decision trees
ERM
Decision Tables
Delphi
SE3
Real Options
Cynefin
Belief Nets
Horizon Scanning
Robust Decision making
Dempster-Shafer Theory

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Network