A matter of life and death: mortality estimation and prediction

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Joint work with Jakub Bijak, Erengul Dodd, Jason Hilton and Peter Smith
Why mortality matters

Mortality estimates forecasts are vitally important

- Tax and Expenditure
- Healthcare
- Pensions and Insurance

- **Goal**: Forecast future mortality with a realistic quantification of uncertainty.
The life table

A static summary of the distribution of age at death for a population:

**English Life Tables No 17**

**Period expectation of life**
Based on data for England and Wales for the years 2010-2012

<table>
<thead>
<tr>
<th>Age</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$m_x$</td>
<td>$q_x$</td>
</tr>
<tr>
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<td>111</td>
<td>0.725677</td>
<td>0.516003</td>
</tr>
<tr>
<td>112</td>
<td>0.750015</td>
<td>0.528125</td>
</tr>
</tbody>
</table>
Crude mortality rates 2010-2012
A basic smoothing model

We estimate the *Central Mortality Rate*

\[
m_x \equiv \frac{\text{Expected number of deaths aged } x \text{ in 1 year}}{\text{Population aged } x \text{ at risk}}
\]

In a generalised additive (smooth) statistical model, we estimate the \( m_x \) by

\[
\hat{m}_x = \exp s(x)
\]

where \( s \) is a function which ‘trades off’ smoothness as a function of age with how closely the \( \hat{m}_x \) are to the corresponding observed rates

\[
\frac{\text{Observed number of deaths aged } x \text{ in 1 year}}{\text{Population aged } x \text{ at risk}}
\]
Smooth mortality rates 2010-2012
Models for older ages and extrapolation

To obtain a more robust fit at older ages, and to extrapolate the mortality function $m_x$ beyond the range of the observed data, one might use the log-linear Gompertz model

$$\log m_x = \beta_0 + \beta_1 x, \quad x \geq x_0$$

where $x_0$ is a suitable threshold

or

$$m_x = \frac{\beta_2 \exp (\beta_0 + \beta_1 x)}{1 + \exp (\beta_0 + \beta_1 x)}, \quad x \geq x_0$$

where mortality rates flatten off, converging to the limit $\beta_2$ as $x \to \infty$. 
ELT17 modelling at high ages
Mortality rates are not static
Dynamic models and projection

Now mortality varies not just by age, but over time as well.

We denote by $m_{xt}$ the central mortality rate aged $x$, in year $t$, in population of interest, for $t = 1, \ldots, T =$ present.

Statistical mortality models provide a framework for projecting $m_{xt}$ etc for $t = T + 1, T + 2, \ldots$

Hence providing us with the information we need for planning.
A cohort is a subpopulation sharing a common birth-year. (1930 birth cohort identified above)
The model

Age-period-cohort (APC) GAM for mortality improvements

\[
\log \frac{m_{xt}}{m_{x \, t-1}} = s_\alpha(x) + \kappa_t + s_\gamma(t - x)
\]

For the highest ages \( x > x_0 \), use the structured model

\[
m_{xt} = \frac{\beta \exp(\mu_0 + \mu_1 x + (\alpha_0 + \alpha_1 x)t)}{1 + \exp(\mu_0 + \mu_1 x + (\alpha_0 + \alpha_1 x)t)} \exp(\kappa_t + s_\gamma(t - x))
\]
Forecast (with uncertainty)

Fit on data up to 2006, 10 year projection.

Log Rates, year = 2016

Female

Male

Age

Log rate

Interval

0.00

0.25

0.50

0.75
Forecast life expectancy (with uncertainty)

Fit on data up to 2006.