

## 1. FAMILY SIZES

Alex Average started work as a statistician at the ONSO (the Office for National Statistics for Outopia) on 1st January 2020. Alex's first job was to find the distribution of family sizes in Outopia. After discussion, Alex decided to define the family of an individual,  $x$ , as meaning the number of people (including  $x$ ) who are related to  $x$  and live in the same dwelling as  $x$ .

Alex decided to initially survey a random sample of 20 individuals and ask their family size, recording the answers as  $F_1, \dots, F_{20}$ .

The survey results were 6, 1, 6, 6, 5, 3, 4, 3, 2, 4, 5, 6, 6, 5, 6, 5, 5, 6, 3, 6. This gives a mean family size of 4.35. Alex is very surprised by this, since the last full census of Outopia (in December 2019) found an average family size of 3.75.

Explain what's gone wrong.

**Hint** Try taking a random sample of size 3 from a population with just two families of sizes 1 and 5 (you can use a single die to do this).

### Extensions

- (1) Obtain an estimate of Outopia's average family size from Alex's data.

**Hint** If the *proportion* of families in Outopia of size  $j$  is  $p_j$ , what is the proportion,  $s_j$ , of people that are in a family of size  $j$ ?

- (2) What does this tell you about the time you spend waiting for a bus at the bus-stop? Can the average waiting time be very bad?

**Hint** You might think of seconds between buses (interarrival times) as family size.