

Favourite Song

Question:

You have a playlist with exactly 100 tracks (i.e., songs), numbered 1 to 100. To go to another track, there are two buttons you can press:

- “Next” will take you to the next track in the list or back to song 1 if you are currently on track 100.
- “Random” will take you to a track chosen uniformly from among the 100 tracks. Pressing “Random” can restart the track you’re already listening to — this will happen 1 percent of the time you press the “Random” button.

For example, if you started on track 73, and you pressed the buttons in the sequence “Random, Next, Random, Random, Next, Next, Random, Next,” you might get the following sequence of track numbers: 73, 30, 31, 67, 12, 13, 14, 89, 90. You always know the number of the track you are currently listening to.

Your goal is to get to your favourite song (on track 42, of course) with as few button presses as possible.

What should your general strategy be? Assuming you start on a random track, what is the average number of button presses you would need to make to reach your favourite song?

Hints:

Think of a simple strategy first. How long does that take on average? Thinking of the cases that slow this down, can you come up with a more complex strategy?