Blue puzzle: Black and white circle

Question: A circle of radius 1/2 is painted black and white, with the total length of the black region being 1. Show that there must be three white points that form the vertices of an equilateral triangle.

Extensions:

- (1) Is it necessarily possible to find four white points that form a square?
- (2) Is it necessarily possible to find four white points that form a rectangle?