Blue puzzle: Black and white circle

Question: A circle of radius 1/2 is painted black and white, with the total length of the black region being 1. Show that there must be three white points that form the vertices of an equilateral triangle.

Extensions:

(1) Is it necessarily possible to find four white points that form a square?
(2) Is it necessarily possible to find four white points that form a rectangle?

Hints:

- Rephrase the task: You want to show that there is an equilateral triangle inscribed in the circle such that all its vertices coincide with white points (on the circle).
- Rather than actually constructing such a triangle, show that the probability that there is such a triangle is larger than 0. (This way you actually show that there are infinitely many triangles of the kind you are after. In particular that implies there is one of them.)
- Start by choosing at random an equilateral triangle inscribed in the circle.
- Consider that all such triangles are equally likely to be chosen.
- Don't forget the obvious, the opposite of a white point is a black point (in the context of this puzzle).
- What is the probability that one of the triangle's vertices is black? What about all three?

Hints to extensions:

(1) Find a way to paint the circle black and white so that no such square exists. Of course, there are ways to paint the circle black and white so that the square exists. But the question was whether it exist no matter how exactly the regions for black and white are chosen.
(2) A rectangle is defined by its two diagonals. Proceed similarly to the solution to the original question.