CRiSM Workshop, University of Warwick
Statistics for Differential Equations Driven by Rough Paths

PROGRAMME

Wednesday, 7th September 2016

12:30-1:30	LUNCH
1:30-2:00	Horatio Boedihardjo
2:00-2:30	Theo Papamarkou
2:30-3:00	Danyu Yang
3:00-3:30	TEA & COFFEE
3:30-4:15	Sotiris Sabanis
4:15-5:00	Christian Bayer

Thursday, 8th September 2016

9:30-10:15	Tom Cass
10:15-11:00	Harald Oberhauser
11:00-11:30	TEA & COFFEE
11:30-12:00	Ilya Chevyrev
12:00-12:30	Thodoris Manikas
12:30-1:30	LUNCH
1:30-2:15	Andreas Neuenkirch
2:15-3:00	Bruno Saussereau
3:00-3:30	TEA & COFFEE
3:30-4:00	Andrey Kormilitzin
4:00-4:30	Jeremy Reizenstein

Friday, 9th September 2016

9:30-10:15	Tessy Papavasiliou
10:15-11:00	Joscha Diehl
11:00-11:30	TEA & COFFEE
11:30-12:00	Hao Ni
12:00-12:30	Terry Lyons (TBC)

Abstracts

Christian Bayer

Title: Pricing under rough volatility

From an analysis of the time series of realized variance (RV) using recent high frequency data, Gatheral, Jaisson and Rosenbaum (2014) previously showed that log-RV behaves essentially as a fractional Brownian motion with Hurst exponent H of order 0.1, at any reasonable time scale. The resulting Rough Fractional Stochastic Volatility (RFSV) model is remarkably consistent with financial time series data. We now show how the RFSV model can be used to price claims on both the underlying and integrated variance. We analyze in detail a simple case of this model, the rBergomi model. In particular, we find that the rBergomi model fits the SPX volatility markedly better than conventional Markovian stochastic volatility models, and with fewer parameters. Finally, we show that actual SPX variance swap curves seem to be consistent with model forecasts, with particular dramatic examples from the weekend of the collapse of Lehman Brothers and the Flash Crash.

Horatio Boedihardjo

Title: Expected signature of fractional Brownian motion

Ladroue-Papavasiliou's "expected signature matching" method of parameter estimation requires a way of computing the expected signatures of stochastic processes. Since Fawcett's result on the expected signature of Brownian motion, there has been results on the expected signature of Levy processes (Friz-Shekhar), diffusions (Ni), SLE curves (Werness) and fractional Brownian motion with $H > 1/2$ (Coutin-Baudoin). In this talk we will explore the questions around the expected signatures of Gaussian processes.

Tom Cass

Title: A Stratonovich-to-Skorohod conversion formula for Gaussian rough paths

Lyons' theory of rough paths allows us to solve stochastic differential equations driven by a Gaussian processes X under certain conditions on the covariance function. The rough integral of these solutions against X again exist, and a natural question is to find a closed-form correction formula between this rough integral and the Skorohod integral of the solution. This is particularly useful in applications, in which typically one wants to compute (or estimate) the expectation of the rough integral. In the case of Brownian motion our formula reduced to the classical Stratonovich-Ito conversion formula. Previous works in the literature assumes the integrand to be the gradient of a smooth function of X_t; our formula again recovers these results as special cases.

Joint work with Nengli Lim (Imperial College London and National University of Singapore).

Ilya Chevyrev

Title: Densities of RDEs with Markovian noise under Hörmander's condition

We consider a class of diffusions which arise from symmetric sub-elliptic Dirichlet forms and which admit canonical lifts as rough paths. We present a result on densities of solutions to RDEs driven by such Markovian rough paths under a Hörmander-type condition. In contrast to similar results known for RDEs with Gaussian noise where Malliavin calculus plays a central role, our methods are mostly based on sub-Riemannian geometry and analysis of (non-symmetric) Dirichlet forms.

Joscha Diehl

Title: Filtering, MLE and the need to measure a rough path

We show that in the problem of stochastic filtering and the maximum likelihood estimation of diffusion coefficients it is in general necessary to treat the signal as a rough path. This leaves the practitioner with the formidable task of measuring a rough path. We show that this indeed possible, if the signal is made to act on a physical system which reveals its higher order terms.

Based on joint works with I. Bailleul, D. Crisan, P. Friz, H. Mai, H. Oberhauser.

Andreas Neuenkirch

Title: Simulating Rough Volatility Models

In this talk, we will study the simulation of the rough volatility model introduced by Gatheral, Jaisson and Rosenbaum in 2014. We will present two findings:

- A complexity result for the strong approximation of the log-asset price given an equidistant discretisation of the log-volatility process and the driving Brownian motions.
- A covariance structure for the log-volatility and Brownian motions that allows an efficient joint simulation.

This is joint work with Taras Shalaiko (Universitt Mannheim).

Hao Ni

Title: An adaptive estimation algorithm for an integral along a path

We consider the problem of regression when the input covariate X is a continuous path from a non-parametric function class and the output response Y is the integral of the unknown integrand function along the input X. It covers a large family of applications where X represents a continuous data stream and Y is the corresponding effect of X on a complex system. In this article, we propose a novel adaptive and non-parametric algorithm to learn the functional relationship between X and Y from the data, which is based on a theory for an effective approximation to the integral along a path. The novelty of our approach is to introduce a structured feature set of path, which allows us to reduce a non-linear problem into a multi-scale linear one and ensures the performance of the global approximation. The algorithm is demonstrated on a financial application where the task is to predict the P&L of the unknown trading strategy.

Harald Oberhauser

Title: Rough paths in high dimension

Similar to polynomials, the number of coefficients in the signature of a path grows exponentially with dimension in the truncation degree. This poses a challenge to use it on standard benchmark results in the machine learning literature such as multivariate time series (in dimensions $\gg 10$). I will show that by using kernelization and concentration inequalities allows to efficiently build signature features for high dimensional sequential data. Joint works with Franz Kiraly and Terry Lyons.

Theo Papamarkou

Title: An algorithm for the approximate construction of the driving path of rough differential equations given discrete observations of the output

Solving a differential equation typically means acquiring its driver given its discretely observed output signal. This implies solving an inverse problem, which takes the form of an integral equation. Various methods based on numerical integration exist that allow solving for the driving path. This talk will put forward an iterative numerical algorithm for approximating the driving path given discrete observations of the output. The proposed constructive algorithm alternates iteratively between approximate solutions of the involved forward and inverse problems. It operates under the postulate that successive approximate solutions of the inverse problem uncover the driving path, while respective interpolations of the discretely observed output approximate the solution of the forward problem. The novelty of the algorithm is to correct the interpolated solution of the forward problem by contracting it and by connecting the resulting contraction to the observed output using rough path properties. Empirical validation demonstrates that the algorithm converges, thus approximating the driving path. Work in progress focuses on comparing the proposed algorithm to existing numerical methods in terms of numerical accuracy and computational complexity. Ultimately, the goal is to employ the approximate driving path for parameter inference in the context of inverse problems.

Jeremy Reizenstein

Title: Computation of signatures

We are using the iterated integral signature or log signature in machine learning, with the goal of making problems like handwriting recognition closer to easy classification problems. In these experiments, we need to calculate the signatures of paths very frequently. I will discuss some of the issues involved in getting this done efficiently, which we tackle in the iisignature library. Derivatives are easier to obtain than one might think, letting us place signature calculations inside neural networks, and I will suggest this is useful.

Sotiris Sabanis

Title: A new generation of explicit numerical schemes for SDEs with super-linear coefficients.

Convergence results for a new class of explicit, high-order schemes, which approximate stochastic differential equations (SDEs) with super-linear coefficients, will be presented. Theoretical results will be illustrated through the implementation of these algorithms to known non-linear models. Some references:
- S. Sabanis. Euler approximations with varying coefficients: the case of superlinearly growing diffusion coefficients. To appear, *Annals of Applied Probability.*
- K. Dareiotis, C. Kumar and S. Sabanis. On Tamed Euler Approximations of SDEs Driven by Lévy Noise with Applications to Delay Equations. *SIAM Journal of Numerical Analysis*, 2016, 54 (3) pp. 1840-1872.
- I. Gyongy, S. Sabanis and D. Siska. Convergence of tamed Euler schemes for a class of stochastic evolution equations. *Stochastic Partial Differential Equations: Analysis and Computations*, 2016, 4 (2) pp. 225-245.

Bruno Saussereau

Title: Nonparametric inference for fractional diffusion

A non-parametric diffusion model with an additive fractional Brownian motion noise is considered in this work. The drift is a non-parametric function that will be estimated by two methods. On one hand, we propose a locally linear estimator based on the local approximation of the drift by a linear function. On the other hand, a Nadaraya-Watson kernel type estimator is studied. In both cases, some non-asymptotic results are proposed by means of deviation probability bound. The consistency property of the estimators are obtained under a one sided dissipative Lipschitz condition on the drift that insures the ergodic property for the stochastic differential equation. Our estimators are first constructed under continuous observations. The drift function is then estimated with discrete time observations that is of the most importance for practical applications.

Danyu Yang

Title: Integration of geometric rough paths

We interpret the integration of geometric rough paths as the integration of slowly-varying exact one-forms along a continuous path on a group. Consider a differentiable function between two groups G1 and G2, and an exact one-form that is the differential of this function. The integration of the exact one-form along a continuous path on G1 can be defined as the difference of the values of the function at the end points of the path. Generally, the exact one-form can vary slowly along the path, and the integration is still well-defined.