Identifying Direct and Indirect effects

S.Geneletti

Imperial College Department of Epidemiology and Public Health

15/04/2009

Outline

- Basics
- DAGs and intervention
- Causal?
- Examples
- Regimes
  - Heuristic
  - Formal
- Definition of effects
- Identification of effects
- Extensions
- Conclusions

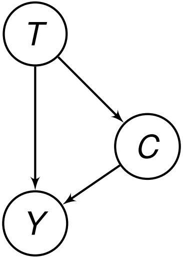
Basics

Conditional Independence

$$p(A, B|C) = p(A|C) \times p(B|C) \Rightarrow A \perp\!\!\!\perp B|C$$

DAGs

- Directed Acyclic Graphs
- Encode conditional independence
- More than one can encode same CI - like intuitve DAG for direct and indirect effects.



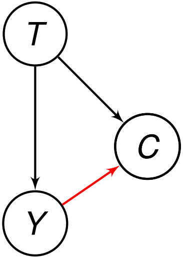
Basics

Conditional Independence

$$p(A, B|C) = p(A|C) \times p(B|C) \Rightarrow A \perp\!\!\!\perp B|C$$

DAGs

- Directed Acyclic Graphs
- Encode conditional independence
- More than one can encode same CI - like intuitve DAG for direct and indirect effects.



Basics continued

Notation

- T treatment: 0,1
- F_T treatment assignment: 0,1,$\emptyset$
- C intermediate variable: 0,1
- Y outcome/response variable: y

Basics continued

Notation

- T treatment: 0,1
- F_T treatment assignment: 0,1,$\emptyset$
- C intermediate variable: 0,1
- Y outcome/response variable: y

F_T: the regime indicator

- $p(T = t|X, F_T = t) = 1$ and
- $p(T = t|X, F_T = \emptyset) = p(T = t|X)$
  i.e. T is drawn from its "natural" or "observational" distribution
- F_T is an intervention variable, treatment assignment indicator, regime indicator.
- DAGs with intervention variables = Augmented DAGs

Causality and ADs

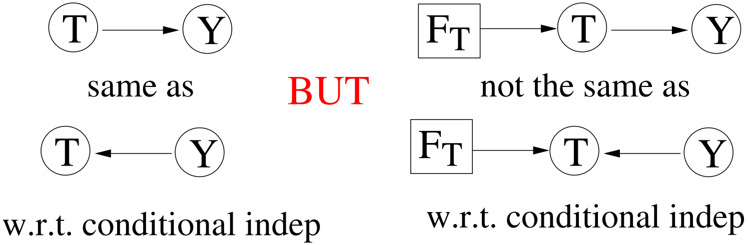
Causes as interventions

- In the framework proposed here, causes are equated to intervention e.g. randomization
- i.e when $F_T \neq \emptyset$ then we are talking cause!

Causality and ADs

Causes as interventions

- In the framework proposed here, causes are equated to intervention e.g. randomization
- i.e when $F_T \neq \emptyset$ then we are talking cause!



- LHS - T and Y are dependent

Causality and ADs

Causes as interventions

- In the framework proposed here, causes are equated to intervention e.g. randomization
- i.e when $F_T \neq \emptyset$ then we are talking cause!

$$T \longrightarrow Y \qquad\qquad F_T \longrightarrow T \longrightarrow Y$$

same as **BUT** not the same as

$$T \longleftarrow Y \qquad\qquad F_T \longrightarrow T \longleftarrow Y$$

w.r.t. conditional indep w.r.t. conditional indep

- LHS - T and Y are dependent
- RHS - top says $Y \perp\!\!\!\perp F_T | T$ so T causes Y

Causality and ADs

Causes as interventions

- In the framework proposed here, causes are equated to intervention e.g. randomization
- i.e when $F_T \neq \emptyset$ then we are talking cause!

$T \longrightarrow Y$

same as

$T \longleftarrow Y$

w.r.t. conditional indep

BUT

$F_T \longrightarrow T \longrightarrow Y$

not the same as

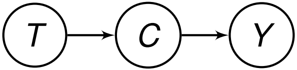
$F_T \longrightarrow T \longleftarrow Y$

w.r.t. conditional indep

- LHS - T and Y are dependent
- RHS - top says $Y \perp\!\!\!\perp F_T | T$ so T causes Y
- RHS - bottom says $Y \perp\!\!\!\perp F_T$ so T does not cause Y

DAGs for direct and indirect effects

(i)

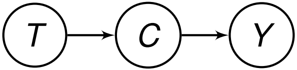


The DAGs encode conditional independence

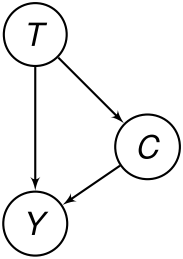
(i) says $T \perp\!\!\!\perp Y | C$ i.e. if we know C, T cannot tell us anything more about Y

DAGs for direct and indirect effects

(i)



(ii)



The DAGs encode conditional independence

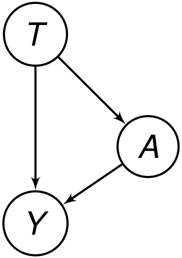
(i) says $T \perp\!\!\!\perp Y | C$ i.e. if we know C, T cannot tell us anything more about Y

(ii) says nothing ... but that silence speaks volumes! We can extract more out of it if we add more variables - see how later

Example 1: side-effects

Story

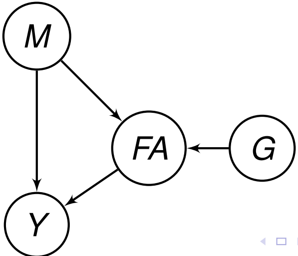
- A drug has strong head-aches as a side-effect
- Patients take aspirin to alleviate this
- It is thought that the aspirin might affect the outcome
- Drug company wants to know what are direct/indirect effects?



Example 2: meat and colon cancer

Story

- Meat consumption thought to be associated to colon cancer
- Also associated to colon cancer are genes that are responsible for breaking down fatty acids (CD36) via complex chemistry involving proteins etc
- Can we change/emulate gene behaviour
- for those with genes that predispose them to cancer?



Causal?

Remember Causes as interventions

- i.e. sex, age, race are not causes - we cannot change them!
- treatments and other things we can in principle intervene on (education, weight) are potential causes
- genes can be causes if we can conceive of intervening on them or their function

Causal?

Remember Causes as interventions

- i.e. sex, age, race are not causes - we cannot change them!
- treatments and other things we can in principle intervene on (education, weight) are potential causes
- genes can be causes if we can conceive of intervening on them or their function

C can be intervened upon

- For there to be a direct/indirect effect, it must be possible to intervene on C
- Ex 1: aspirin intake of patients can in principle be controlled
- Ex 2: gene function changed or emulated

Identifiable?

Problem

- OK C can be intervened upon in principle....
- but in practice often not possible:
- cannot deny a patient with head-ache pain relief
- cannot intervene on a human (yet)

Identifiable?

Problem

- OK C can be intervened upon in principle....
- but in practice often not possible:
- cannot deny a patient with head-ache pain relief
- cannot intervene on a human (yet)

Solution

- It all depends on the conditional independences!
- Using these and some cunning, we can identify the direct and indirect effects

Identifiable?

Problem

- OK C can be intervened upon in principle....
- but in practice often not possible:
- cannot deny a patient with head-ache pain relief
- cannot intervene on a human (yet)

Solution

- It all depends on the conditional independences!
- Using these and some cunning, we can identify the direct and indirect effects
- and often estimate them using data that are purely observational!

Direct & indirect effects: a heuristic overview

The type of direct and indirect effect you get depends on what you do to C

Ways of manipulating C

- Fix C

Direct & indirect effects: a heuristic overview

The type of direct and indirect effect you get depends on what you do to C

Ways of manipulating C

- Fix C
- Draw C from $p(C|T)$

Direct & indirect effects: a heuristic overview

The type of direct and indirect effect you get depends on what you do to C

Ways of manipulating C

- Fix C
- Draw C from $p(C|T)$
- Draw C from an appropriate distribution

Each one of these manipulations can be described by a regime.

Fixing regime: heuristic

Ex 1: imaginary experiment I

- Imagine you can fix aspirin intake e.g. everyone has to take no aspirins

Fixing regime: heuristic

Ex 1: imaginary experiment I

- Imagine you can fix aspirin intake e.g. everyone has to take no aspirins
- then can estimate the direct effect of treatment for no aspirins.

Fixing regime: heuristic

Ex 1: imaginary experiment I

- Imagine you can fix aspirin intake e.g. everyone has to take no aspirins
- then can estimate the direct effect of treatment for no aspirins.
- Imagine you can randomise aspirins as well as treatment

Fixing regime: heuristic

Ex 1: imaginary experiment I

- Imagine you can fix aspirin intake e.g. everyone has to take no aspirins
- then can estimate the direct effect of treatment for no aspirins.
- Imagine you can randomise aspirins as well as treatment
- then can estimate the direct effect of treatment for 2 levels of aspirin and some kind of indirect effect by comparing the two direct effects.

Fixing regime: heuristic

Ex 1: imaginary experiment I

- Imagine you can fix aspirin intake e.g. everyone has to take no aspirins
- then can estimate the direct effect of treatment for no aspirins.
- Imagine you can randomise aspirins as well as treatment
- then can estimate the direct effect of treatment for 2 levels of aspirin and some kind of indirect effect by comparing the two direct effects.

Problems

- Not ethical!
- Not realistic – people "out there" in the population do not behave like this.

Drawing C from $p(C|T)$ regime: heuristic

Ex 1: imaginary experiment II

- One random group of patients are given the control $F_T = 0$ and we record their "natural" aspirin intake $p(C|F_T = 0)$

Drawing C from $p(C|T)$ regime: heuristic

Ex 1: imaginary experiment **II**

- One random group of patients are given the control $F_T = 0$ and we record their "natural" aspirin intake $p(C|F_T = 0)$
- next we take the rest of the patients and
  - If a patient gets $F_T = 1$, they are administered aspirin according to $p(C|F_T = 0)$

Drawing C from $p(C|T)$ regime: heuristic

Ex 1: imaginary experiment II

- One random group of patients are given the control $F_T = 0$ and we record their "natural" aspirin intake $p(C|F_T = 0)$
- next we take the rest of the patients and
  - If a patient gets $F_T = 1$, they are administered aspirin according to $p(C|F_T = 0)$
  - If a patient gets $F_T = 0$, we leave them alone, and assume that they take aspirin according to $p(C|F_T = 0)$

Drawing C from $p(C|T)$ regime: heuristic

Ex 1: imaginary experiment II

- One random group of patients are given the control $F_T = 0$ and we record their "natural" aspirin intake $p(C|F_T = 0)$
- next we take the rest of the patients and
  - If a patient gets $F_T = 1$, they are administered aspirin according to $p(C|F_T = 0)$
  - If a patient gets $F_T = 0$, we leave them alone, and assume that they take aspirin according to $p(C|F_T = 0)$
- This way we can compare direct effects fixing not the aspirin intake, but its (control) distribution

Problem

- Not ethical!

Drawing C from $p(D)$ regime: heuristic

Ex 1: imaginary experiment **III**

- We cannot estimate $p(C|T)$ - lack of funds!
- So we pick a suitable distribution $p(D)$ such that domain of C is same as D.
  - If a patient gets $F_T = 1$, they are administered aspirin according to $p(D)$
  - If a patient gets $F_T = 0$, they are also administered aspirin according to $p(D)$
- In this case, the comparison is not as easily interpretable, but we can still get a handle on the direct and indirect effects.

Drawing C from $p(D)$ regime: heuristic

Ex 1: imaginary experiment III

- We cannot estimate $p(C|T)$ - lack of funds!
- So we pick a suitable distribution $p(D)$ such that domain of C is same as D.
  - If a patient gets $F_T = 1$, they are administered aspirin according to $p(D)$
  - If a patient gets $F_T = 0$, they are also administered aspirin according to $p(D)$
- In this case, the comparison is not as easily interpretable, but we can still get a handle on the direct and indirect effects.

Standardisation

- The 2^{nd} and 3^{rd} regimes are forms of direct standardisation.

Formal definition of Regimes

Manipulating C

- Define M_C the variable of manipulations of C
- M_C has 4 regimes:
   1. $\mathbf{M_C} = \emptyset$: C is left alone and comes from $p(C|T = t, F_T = \emptyset, M_C = \emptyset) = P_t$

Formal definition of Regimes

Manipulating C

- Define M_C the variable of manipulations of C
- M_C has 4 regimes:
  1. $\mathbf{M_C} = \emptyset$: C is left alone and comes from $p(C|T = t, F_T = \emptyset, M_C = \emptyset) = P_t$
  2. $\mathbf{M_C} = \mathbf{c}$: C is set to c with no uncertainty, distribution of C is δ_c

Formal definition of Regimes

Manipulating C

- Define M_C the variable of manipulations of C
- M_C has 4 regimes:
  1. $\mathbf{M_C} = \emptyset$: C is left alone and comes from $p(C|T = t, F_T = \emptyset, M_C = \emptyset) = P_t$
  2. $\mathbf{M_C} = \mathbf{c}$: C is set to c with no uncertainty, distribution of C is δ_c
  3. $\mathbf{M_C} = \mathbf{t}^*$: C is drawn from $p(C|F_T = \emptyset, M_C = t^*) = P_{t^*}$

Formal definition of Regimes

Manipulating C

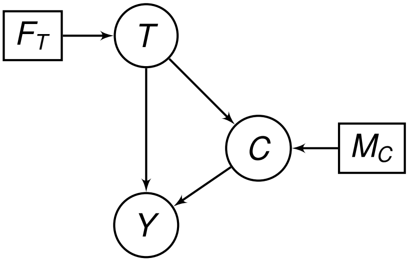
- Define M_C the variable of manipulations of C
- M_C has 4 regimes:
  1. $\mathbf{M_C} = \emptyset$: C is left alone and comes from $p(C|T = t, F_T = \emptyset, M_C = \emptyset) = P_t$
  2. $\mathbf{M_C} = \mathbf{c}$: C is set to c with no uncertainty, distribution of C is δ_c
  3. $\mathbf{M_C} = \mathbf{t}^*$: C is drawn from $p(C|F_T = \emptyset, M_C = t^*) = P_{t^*}$
  4. $\mathbf{M_C} = \mathbf{D}$: C is drawn from $p(D) = P_D$
- Note that Regime 2 is a special case of regime 3 which in turn is a special case of regime 4. Regime 1 is also a special case of 4.

Formal definition of Regimes continued

Conditional independences

$$
\begin{aligned}
(Y, C) &\perp\!\!\!\perp F_T \mid T; &(1)\\
Y &\perp\!\!\!\perp M_C \mid (C, T); &(2)\\
C &\perp\!\!\!\perp (F_T, T) \mid M_C \neq \emptyset. &(3)
\end{aligned}
$$

(3) only for regimes 2-4



Some points

- The framework of these regimes is determined by M_C as defined

Some points

- The framework of these regimes is determined by M_C as defined
- Adding other variables might require a redefinition of M_C

Some points

- The framework of these regimes is determined by M_C as defined
- Adding other variables might require a redefinition of M_C
- As there are only 3 variables we consider here, we call this the 3DI framework

Some points

- The framework of these regimes is determined by M_C as defined
- Adding other variables might require a redefinition of M_C
- As there are only 3 variables we consider here, we call this the 3DI framework
- Extensions are quite straightforward, might show some later

Some simplyfing notation

$$E(Y|F_T = a, M_C = b) = \qquad E(a, b)$$

Some simplyfing notation

$$E(Y|F_T = a, M_C = b) = \qquad E(a, b)$$

Some simplyfing notation

$$
\begin{aligned}
E(Y|F_T = a, M_C = b) &= & E(a,b) \\
E(Y|F_T = a, M_C = b, C = c) &= & E(a,b,c) \\
Pr(C = c|F_T = a, M_C = b) &= & p(c|a,b)
\end{aligned}
$$

Definition of direct & indirect effects

DE for C set at c - Set effect (SE_c)

The direct effect of $F_T = t$ with respect to baseline $F_T = t^*$ on response Y for C set at c is given by

$$\begin{aligned} E(Y|F_T = t, M_C = c) &- E(Y|F_T = t^*, M_C = c) \quad (4) \\ = E(t, c) &- E(t^*, c). \end{aligned}$$

Definition of direct & indirect effects

DE for C set at c - Set effect (SE_c)

The direct effect of $F_T = t$ with respect to baseline $F_T = t^*$ on response Y for C set at c is given by

$$\begin{aligned} E(Y|F_T = t, M_C = c) &- E(Y|F_T = t^*, M_C = c) \quad (4) \\ = E(t, c) &- E(t^*, c). \end{aligned}$$

DE for C drawn from P_{t^*}: Generated effect (GDE_{t^*})

The direct effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response for C generated from P_{t^*} the distribution of C conditional on $F_T = t^*$ the baseline treatment is given by

$$E(t, t^*) - E(t^*, t^*)$$

Definition of direct & indirect effects

DE for C set at c - Set effect (SE_c)

The direct effect of $F_T = t$ with respect to baseline $F_T = t^*$ on response Y for C set at c is given by

$$\begin{aligned} E(Y|F_T = t, M_C = c) &- E(Y|F_T = t^*, M_C = c) \\ = E(t, c) &- E(t^*, c). \end{aligned} \tag{4}$$

DE for C drawn from P_{t^*}: Generated effect (GDE_{t^*})

The direct effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response for C generated from P_{t^*} the distribution of C conditional on $F_T = t^*$ the baseline treatment is given by

$$E(t, t^*) - E(t^*, t^*) \equiv E(t, t^*) - E(t^*, \emptyset). \tag{5}$$

Definition of direct & indirect effects cont

DE for C drawn from P_D: Generated effect (GDE_D)

The direct effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response for C drawn from a specified distribution over D is given by:

$$E(t, D) - E(t^*, D) \tag{6}$$

Definition of direct & indirect effects cont

DE for C drawn from P_D: Generated effect (GDE_D)

The direct effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response for C drawn from a specified distribution over D is given by:

$$E(t, D) - E(t^*, D) \tag{6}$$

Total effect: TE

The total effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response is given by

$$E(Y|F_T = t) - E(Y|F_T = t^*). \tag{7}$$

Definition of direct & indirect effects cont

DE for C drawn from P_D: Generated effect (GDE_D)

The direct effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response for C drawn from a specified distribution over D is given by:

$$E(t, D) - E(t^*, D) \tag{6}$$

Total effect: TE

The total effect of $F_T = t$ with respect to the baseline $F_T = t^*$ on response is given by

$$E(Y|F_T = t) - E(Y|F_T = t^*). \tag{7}$$

IDE for C drawn from P ($P \in \{\delta_c, P_{t^*}, P_D\}$)

$$IDE = TE - DE$$

Identifying effects in 3DI

- The TE is always identifiable, even when $F_T = \emptyset$ if there are no confounders
- If we can identify the DE then it follows we can identify the IDE, so focus is on DE
- We look first at scenario where there are no confounders and $F_T \neq \emptyset$
- Then consider what happens if there are confounders i.e. when

  $$Y \perp\!\!\!\perp M_C | (C, T)$$

  does not hold
- AND $F_T = \emptyset$.

Identifying GDE_{t^*}

$$GDE_{t^*} = \underbrace{E(t, t^*)}_{(a)} - \overbrace{E(t^*, \emptyset)}^{(b)}$$

Identifying GDE_{t^*}

$$GDE_{t^*} = \underbrace{E(t, t^*)}_{(a)} - \overbrace{E(t^*, \emptyset)}^{(b)}$$

(b) is easy $= E(Y|T = t^*, F_T = \emptyset, M_C = \emptyset)$ by (1)

$(a) =$

$$= \sum_c E(t, t^*, c) \times p(c|t, t^*) \tag{8}$$

Identifying GDE_{t^*}

$$GDE_{t^*} = \underbrace{E(t, t^*)}_{(a)} - \overbrace{E(t^*, \emptyset)}^{(b)}$$

(b) is easy $= E(Y|T = t^*, F_T = \emptyset, M_C = \emptyset)$ by (1)

$(a) =$

$$= \sum_c E(t, t^*, c) \times p(c|t, t^*) \tag{8}$$

$$= \sum_c E(t, \emptyset, c) \times p(c|\emptyset, t^*) \tag{9}$$

- from (8)-(9) by (2) & (3)

Identifying GDE_{t^*}

$$GDE_{t^*} = \underbrace{E(t, t^*)}_{(a)} - \overbrace{E(t^*, \emptyset)}^{(b)}$$

(b) is easy $= E(Y|T = t^*, F_T = \emptyset, M_C = \emptyset)$ by (1)

$(a) =$

$$= \sum_c E(t, t^*, c) \times p(c|t, t^*) \tag{8}$$

$$= \sum_c E(t, \emptyset, c) \times p(c|\emptyset, t^*) \tag{9}$$

$$= \sum_c E(Y|T = t, F_T = \emptyset, M_C = \emptyset, C = c)$$
$$\times\, p(C = c|T = t^* F_T = \emptyset, M_C = \emptyset) \tag{10}$$

- from (8)-(9) by (2) & (3)
- from (9)-(10) by (1)

Identifying continued

GDE_D is identified in same way as GDE_{t*} with P_D replacing P_{t*}

Pros

- So it is possible to identify and hence estimate the 3DI effects from data that are purely observational for both T and C as F_T and M_C are both $\emptyset$.

Identifying continued

GDE_D is identified in same way as GDE_{t*} with P_D replacing P_{t*}

Pros

- So it is possible to identify and hence estimate the 3DI effects from data that are purely observational for both T and C as F_T and M_C are both $\emptyset$.
- Simple formulae, easy to understand

Identifying continued

GDE_D is identified in same way as GDE_{t*} with P_D replacing P_{t*}

Pros

- So it is possible to identify and hence estimate the 3DI effects from data that are purely observational for both T and C as F_T and M_C are both $\emptyset$.
- Simple formulae, easy to understand - once you get the hang of it

Identifying continued

GDE_D is identified in same way as GDE_{t*} with P_D replacing P_{t*}

Pros

- So it is possible to identify and hence estimate the 3DI effects from data that are purely observational for both T and C as F_T and M_C are both $\emptyset$.
- Simple formulae, easy to understand - once you get the hang of it

Cons

- No counfounder assumption is unlikely to hold in practice

Identifying with confounders

Confounder between T and C or T and Y

- This is only really an issue if $F_T = \emptyset$

Identifying with confounders

Confounder between T and C or T and Y

- This is only really an issue if $F_T = \emptyset$
- In this situation it is necessary to observe the confounders - if the are unobserved, we're in trouble

Identifying with confounders

Confounder between T and C or T and Y

- This is only really an issue if $F_T = \emptyset$
- In this situation it is necessary to observe the confounders - if the are unobserved, we're in trouble
- If observed, we treat them as usual - condition and sum over

Identifying with confounders

Confounder between T and C or T and Y

- This is only really an issue if $F_T = \emptyset$
- In this situation it is necessary to observe the confounders - if the are unobserved, we're in trouble
- If observed, we treat them as usual - condition and sum over

Confounder between C and Y

- This requires a redefiniton of M_C as the distribution of C now depends on this confounder (call it W) as well as T.

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.
- One knows that the side-effect is headache, tells patients not to take aspirin

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.
- One knows that the side-effect is headache, tells patients not to take aspirin
- The other knows nothing so his patients are ignorant

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.
- One knows that the side-effect is headache, tells patients not to take aspirin
- The other knows nothing so his patients are ignorant
- In this situation we want to get rid of W

W is relevant

- Ex 1: W is sex: women suffer more side-effect than men.

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.
- One knows that the side-effect is headache, tells patients not to take aspirin
- The other knows nothing so his patients are ignorant
- In this situation we want to get rid of W

W is relevant

- Ex 1: W is sex: women suffer more side-effect than men.
- We might want to adjust for W (or even intervene – not in this example!)

W confounder between C and Y

W is a niusance

- Ex 1: Two doctors administer the treatment.
- One knows that the side-effect is headache, tells patients not to take aspirin
- The other knows nothing so his patients are ignorant
- In this situation we want to get rid of W

W is relevant

- Ex 1: W is sex: women suffer more side-effect than men.
- We might want to adjust for W (or even intervene – not in this example!)

The two roles of W need to be treated in slightly different ways.

W as sex

Just consider this - the other case in the paper

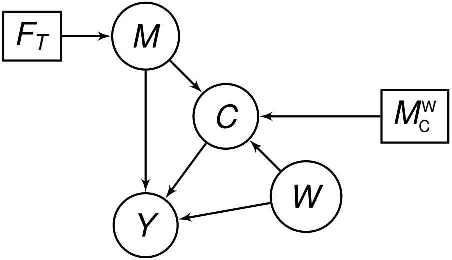
New variable and conditional independence - 4DI

Define M_C^W as a manipulation variable on C s.t.

$$(Y, C, W) \perp\!\!\!\perp F_T | T \quad (11)$$

$$W \perp\!\!\!\perp (F_T, M_C^W) \quad (12)$$

$$Y \perp\!\!\!\perp M_C^W | (T, C, W). \quad (13)$$



W as sex continued

Regimes of M_C^W

1. $M_C^W = \emptyset$: C is left alone
2. $M_C^W = t^*$: C is drawn from $p(C|T = t^*, W = w, F_T = \emptyset)$ where w corresponds to the realised value of W.

W as sex continued

Regimes of M_C^W

1. $M_C^W = \emptyset$: C is left alone
2. $M_C^W = t^*$: C is drawn from $p(C|T = t^*, W = w, F_T = \emptyset)$ where w corresponds to the realised value of W.

- No need for other regimes as they sever relationship between C and W and are equivalent to 3DI
- that means that there is only 1 extra DE
- Regime 2 induces

$$C \perp\!\!\!\perp (F_T, T) | M_C^W = t^* \tag{14}$$

Identifying DE's

We want the woman ($W = 1$) specific GDE_{t*}

$$E(t, t^{*}, W = 1) \quad - \quad E(t^{*}, t^{*}, W = 1)$$

Identifying DE's

We want the woman ($W = 1$) specific GDE_{t*}

$$E(t, t^*, W = 1) \quad - \quad E(t^*, t^*, W = 1)$$
$$= \sum_c [E(t, \emptyset, c, W = 1)$$

Identifying DE's

We want the woman ($W = 1$) specific GDE_{t^*}

$$E(t, t^*, W = 1) \quad - \quad E(t^*, t^*, W = 1)$$
$$= \sum_c [E(t, \emptyset, c, W = 1) \times p(c|t^*, \emptyset, W = 1)]$$

Identifying DE's

We want the woman ($W = 1$) specific GDE_{t^*}

$$
\begin{aligned}
& E(t, t^*, W = 1) \quad - \quad E(t^*, t^*, W = 1) \\
& \quad = \sum_c [E(t, \emptyset, c, W = 1) \times p(c|t^*, \emptyset, W = 1)] - E(t^*, \emptyset, W = 1).
\end{aligned}
$$

Identifying DE's

We want the woman ($W = 1$) specific GDE_{t^*}

$$
\begin{aligned}
E(t, t^*, W = 1) \quad &- \quad E(t^*, t^*, W = 1) \\
&= \sum_c [E(t, \emptyset, c, W = 1) \times p(c|t^*, \emptyset, W = 1)] - E(t^*, \emptyset, W = 1).
\end{aligned}
$$

This can be further expressed with $F_T = \emptyset$ and can thus also be estimated from purely observational data.

Conclusions

- DAGs and conditional independences are useful for expressing D&I effects
- We need to be clear about what is going on though - what CI's hold?
- Provided we do this, we have a wealth of DEs we can estimate
- Also remember that the concepts don't make much sense if we cannot intervene on the intermediate at all.
- The 3DI and 4DI are tools that can be used to clarify what we mean.
- The framework can be extended further.

References

- Dawid, A.P. (2000). Causal Inference without Counterfactuals (with comments and rejoinder). *JASA* 95(450), 407-448.
- Dawid A.P. (2002). Influence diagrams for causal modelling and inference. Intern. Statist. Rev. 70:161-189.
- Geneletti S. (2007). Identifying direct and indirect effects in a non-counterfactual framework. *J ROY STAT SOC B.* 69:199-215
- Kuriki K. et al. (2005). Increased risk of colorectal cancer due to interactions between meat consumption and the CD36 gene A52C polymorphism among Japanese. *Nutr Cancer*;51(2):170-7
- Pearl, J. (2005). Direct and indirect effects, *Proceedings of the American Statistical Association Joint Statistical Meetings*