Performing Bayesian inference in a 'Big Data' setting is a challenging problem. A modern researcher may have a posterior distribution of the form \( \pi(x) \propto \prod_{i=1}^{N} f_i(x) \) with \( N \) incredibly large: in such a setting merely evaluating the posterior density is an inhibitive \( O(N) \) calculation. The recent Scalable Langevin Exact (ScaLE) Algorithm [2] proposes a new paradigm for Big-Data Bayesian inference: by constructing a killed diffusion with the appropriate quasistationary distribution and utilising subsampling techniques, this novel algorithm attains an \( O(1) \) runtime in the size of the data and retains exactness of the target distribution. In this poster we give theoretical results underpinning quasistationary Monte Carlo methods and describe an alternative stochastic approximation method, known as Regenerating ScaLE (ReScaLE).

**Simulation via Stochastic Approximation**

Suppose \( X \) has quasistationary distribution \( \pi \). How might we try to draw from \( \pi \)? Rejection sampling. ScaLE: continuous-time sequential Monte Carlo [22]. Regenerating ScaLE (ReScaLE): stochastic approximation, cf. [23] for compact state space, discrete-time case. See Figure 2.

Initialise \( X_0 \sim \mu_0 \) and fix \( x > 0 \). Our diffusion \( X \) is killed at rate \( t \mapsto \kappa(X_t) \). At each death event \( T_n \), the process is instantaneously reborn according to \( X_{T_n} \sim \mu_{T_n} \), independently, where

\[
\mu_{T_n} = \frac{\mu_0 + \int_0^{T_n} \kappa(X_s) \, ds}{T_n + 1} \quad \text{for each } T_n \geq 0 .
\]

Conjecture: \( \mu_T \rightarrow \pi \) (weakly) almost surely under fairly general settings. We have established this in the restricted case of compact state space with bounded killing. We follow the approach of [1].

Make a time-change: \( h(t) = \exp(-t) - t \). \( \zeta = \Phi(\mu) \). Then it turns out that \( \zeta \) satisfies a (weak) ODE:

\[
\dot{\zeta} = - \zeta + \Pi(\zeta) + \left( X_{T_{n+1}} - \Sigma_{T_n} \right) .
\]

[That is, for any continuous, bounded test function \( f \), \( t \mapsto \langle f, \zeta \rangle \) satisfies the appropriate ODE on \( \mathbb{R} \).]

Here \( \Pi(\mu) \) denotes the invariant distribution of the fixed rebirth process \( X^0 \) with rebirth distribution \( \mu \); that is, the diffusion killed at rate \( t \mapsto \kappa(X_t) \) and instantaneously reborn according to \( \mu \).

We argue that the second bracket is "small", and analyse the resulting deterministic ODE.

**Convergence to Quasistationarity**

**Proposition.** We can define a continuous flow \( R_t \times \mathcal{P}([0, \infty)) \rightarrow \mathcal{P}([0, \infty)) \) such that \( t \mapsto \Phi(\mu_t) \) solves the weak, measure-valued ODE \( \dot{\mu}_t = -\kappa(X_t) + \Pi(\mu_t) \). Furthermore we have that \( \Phi(\mu_t) \rightarrow \pi \) as \( t \rightarrow \infty \) for any compactly support \( \mu \).

We say that \( t \mapsto \zeta \) is an asymptotic pseudo-trajectory for \( \Phi \) if \( \zeta \) is tight and for all \( T > 0 \),

\[
\lim_{t \rightarrow \infty} \sup_{\mu \in \mathcal{P}([0,T])} d(\mu_t, \Phi(\mu)) = 0 .
\]

\([d \] is a metric measuring weak convergence of probability measures.\]

This holds if \( \zeta \) is tight and the signed measures \( \nu_t(x) = \int_0^t \mathbf{1}_{\{X_s = x\}} \, ds - \Pi(\zeta) \, ds \) satisfies for each continuous bounded \( f \) and \( T > 0 \),

\[
\lim_{t \rightarrow \infty} \sup_{f \in C_b} |\nu_t(f)| = 0 .
\]

Establishing tightness involves showing \( \int_0^T |\kappa(X_s) \, ds = O(1) \) almost surely; have urn-like dynamics. Controlling \( \nu_t(x) \) involves showing \( \int_0^T \mathbb{E}_0(\kappa(X_s) \, ds = O(T) ) \). Can utilise renewal-like structure.

**References**


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