Message-Passing Monte Carlo

Sam Power

Cambridge Centre for Analysis
Cantab Capital Institute for the Mathematics of Information

sp825@cam.ac.uk

July 9, 2018
Task: Sample from smooth target distribution

\[ P(x) \propto \exp(-U(x)) \] (1)
Graphical representation of a probability measure with local interactions

\[ \mathbf{P}(x) = \prod_{a \in F} \psi_a(x_\partial a) \]  

\[ U(x) = \sum_{a \in F} U_a(x_\partial a) \]  

Includes many practical models
Hidden Markov Model

\[ X_1 \rightarrow Y_1 \rightarrow X_2 \rightarrow Y_2 \rightarrow X_3 \rightarrow Y_3 \]
Matrix Factorisation

Prior

\[ L_1 \]

\[ X_{11} \]

\[ X_{12} \]

Prior

\[ L_2 \]

\[ X_{21} \]

\[ X_{22} \]

Prior

\[ R_1 \]

\[ R_2 \]
High-dimensional models have delicate geometry

Want clever proposals; use geometrically-informed dynamics

**Hamiltonian Monte Carlo**

\[ \mathcal{H}(x, p) = U(x) + \frac{1}{2} p^T M^{-1} p \]

Move *long distances, still* be accepted

**Manifold HMC**

- Use *position-dependent* mass matrix \( M(x) \) to encode geometry
- Navigate *complex* targets; **costly**
Goal: Scale RMHMC to *hierarchical models*

Two simplifying assumptions:

1. $M(x)$ block-diagonal
2. $p_i$ conditionally independent of $x_i$

Dynamics: *Unconstrained* updating of subsystems

- Not Gibbs sampling.
Extend SS-HMC using Factor Graph structure

Two adaptations

1. **Split** momentum $p_i \mapsto \{p_{i,a}\}_{a \in \partial i}$ with

   $$p_{i,a} \sim \mathcal{N}(0, M_{i,a})$$

2. Assume $M_{i,a}$ depends only on $\{x_j\}_{j \in \partial a \setminus i}$

Dynamics: Unconstrained ‘message-passing dynamics’
Message-Passing Dynamics

\[ \frac{\partial K_{i,a}}{\partial p_{i,a}} \quad \frac{\partial U_a}{\partial x_i} \quad \frac{\partial K_{j,a}}{\partial x_i} \quad \frac{\partial K_{k,a}}{\partial x_i} \quad \frac{\partial K_{m,a}}{\partial x_i} \]

\[ x_i \quad p_{i,a} \quad x_j \quad x_k \quad x_m \]
Summary

- Geometric structure
- Local operations
- Tractable computations
- \(\cdots \rightarrow\) Scalable MCMC on Factor Graphs!
Thank you!