Computational Information Geometry:

Geometry of Model Choice

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Big Picture

- Introduction to Computational Information Geometry

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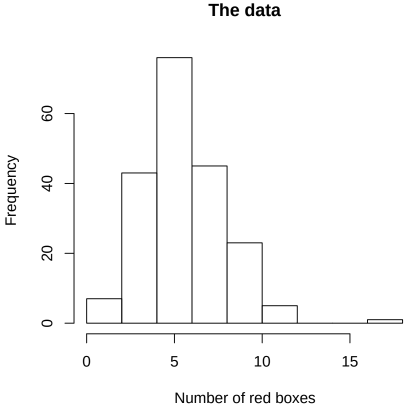
- Thanks to EPSRC Grant Number EP/E017878/1

Problem of Interest



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- How do modelling assumptions effect inference about mean?
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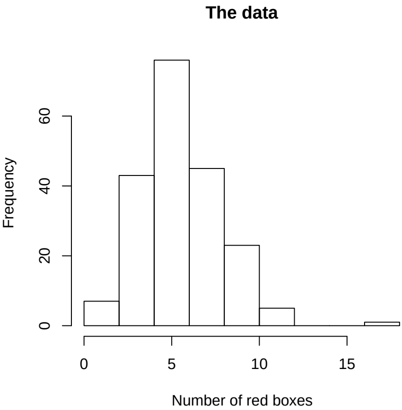

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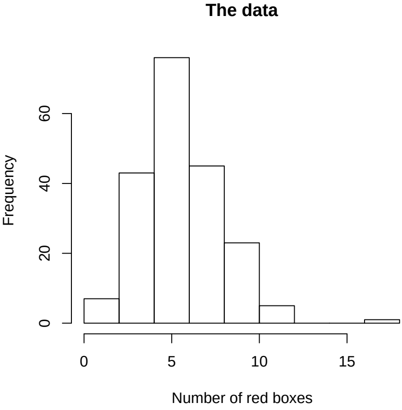

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- SEM proxy for universal space of all distributions
- Can be finite or infinite dimensional

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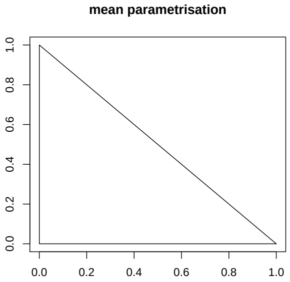

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- Different support sets
- Union of exponential families each with corresponding natural $(+1)$ parameters

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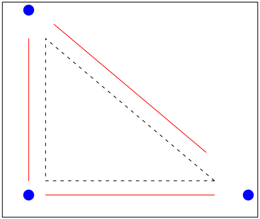

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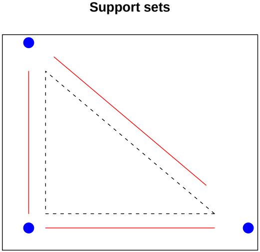

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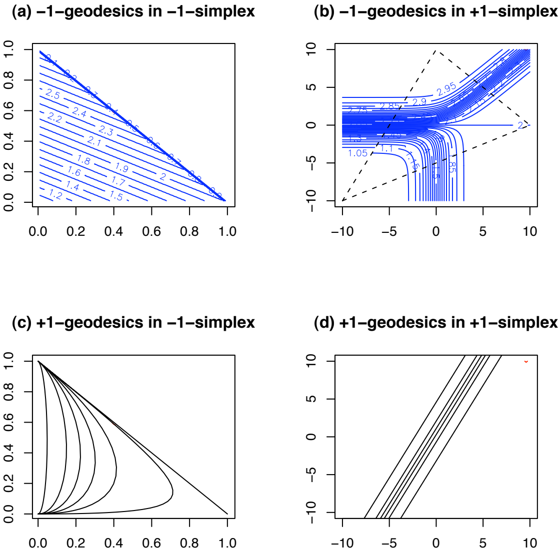
- Use the dual structure of information geometry

Dual Parameterisations



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- This is our computational framework

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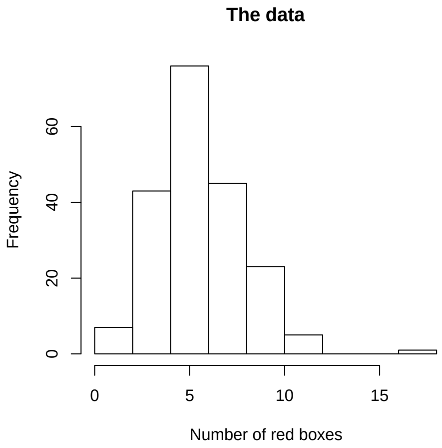

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- Statistician B: Looks at ‘raw’ data and talks to scientists

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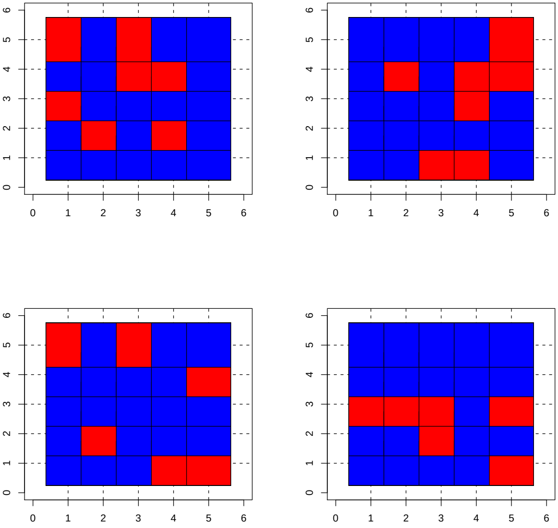

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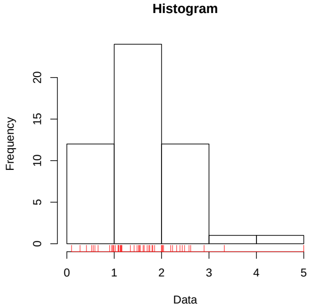
- Statistician D: uses robust methods
- Main Question: can the universality and geometry the SEM give a framework in which A, B, C and D can communicate their differing views about the mean number of red boxes?

Shape of likelihood in SEM



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- Working in high dimensional multinomial with many zero counts
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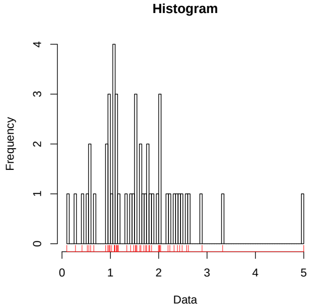

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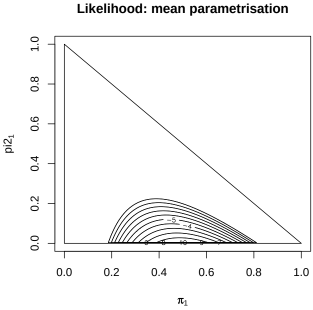

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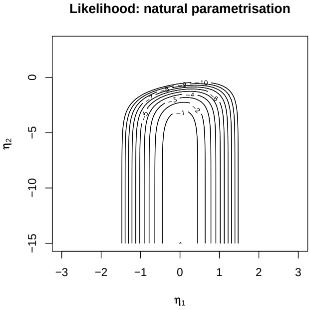

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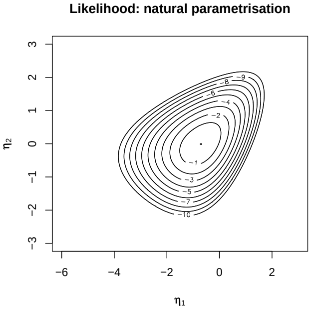

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- Such tests are necessary but not sufficient $\cdots$

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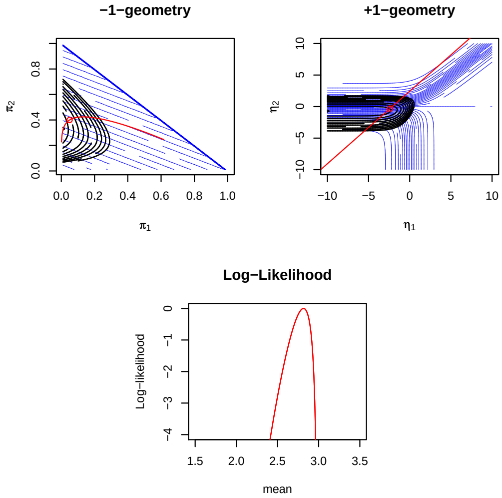

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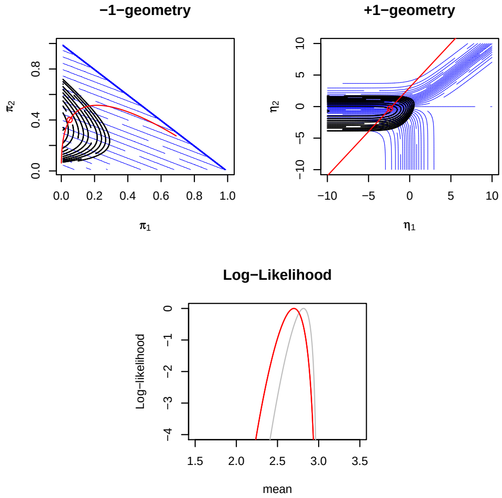

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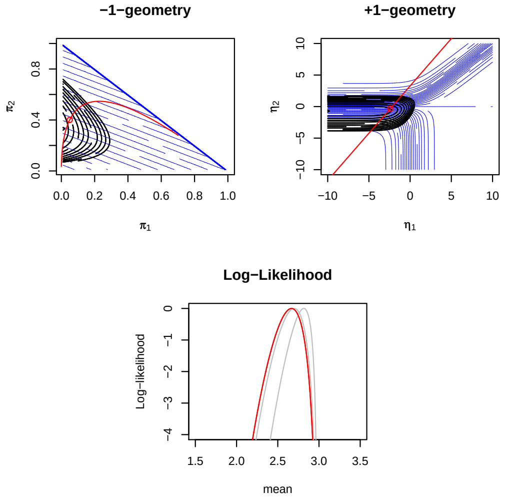

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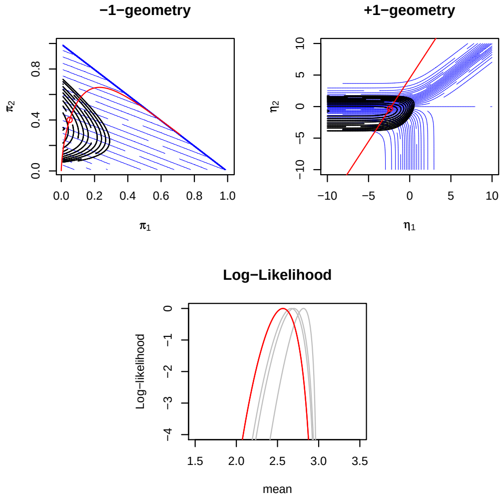

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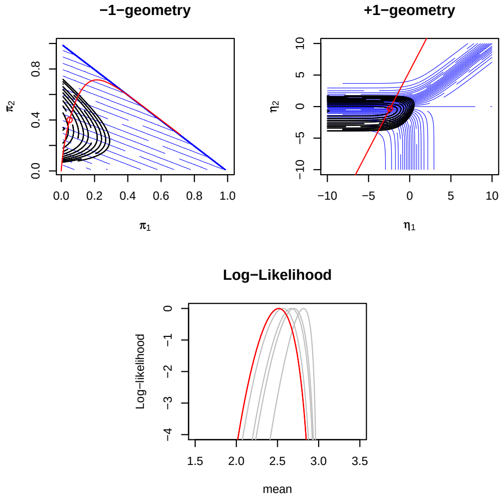

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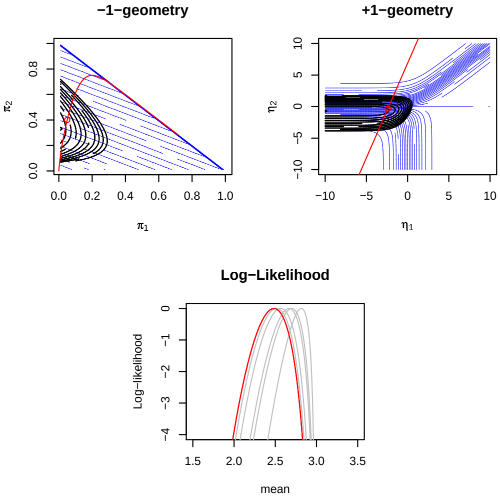

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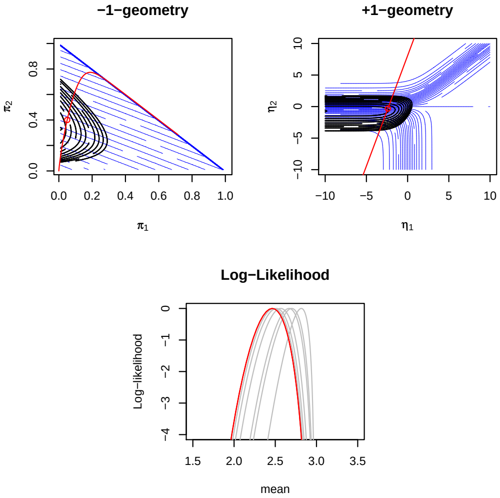

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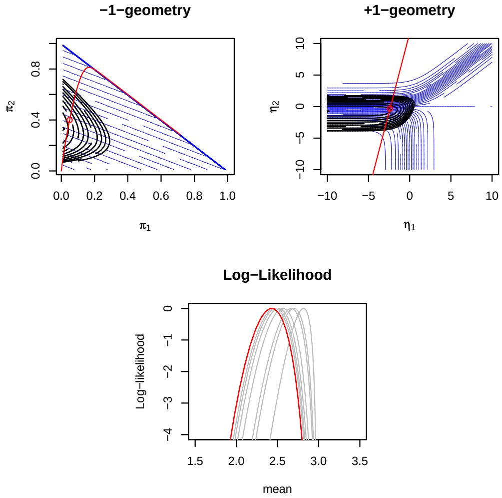

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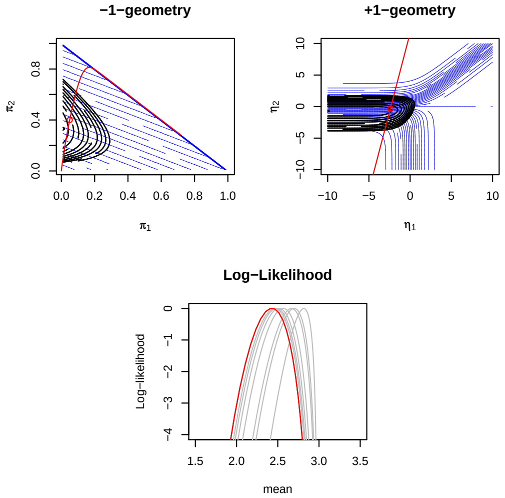

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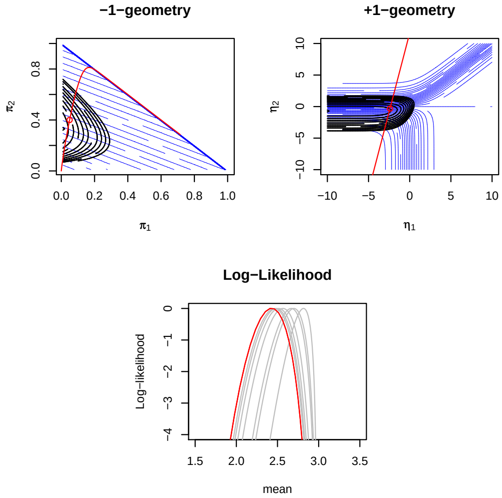

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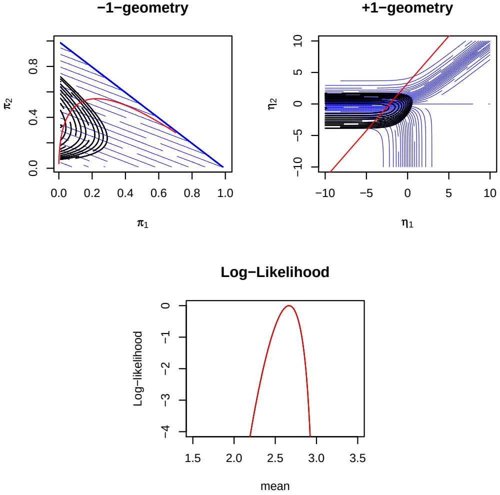
Effect of translation



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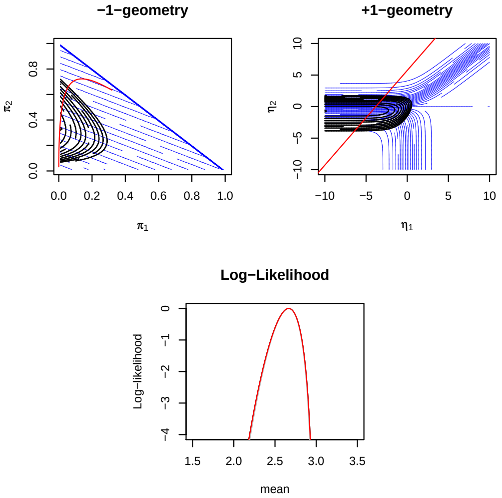

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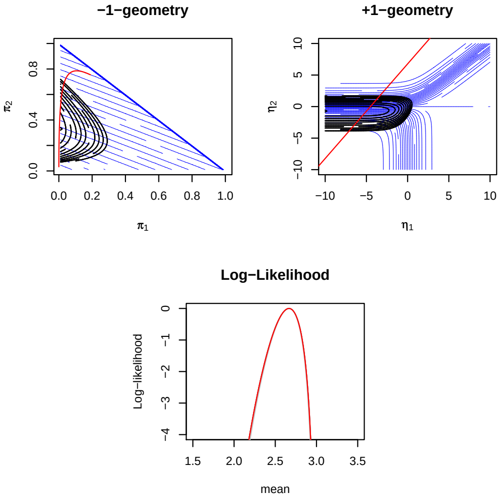

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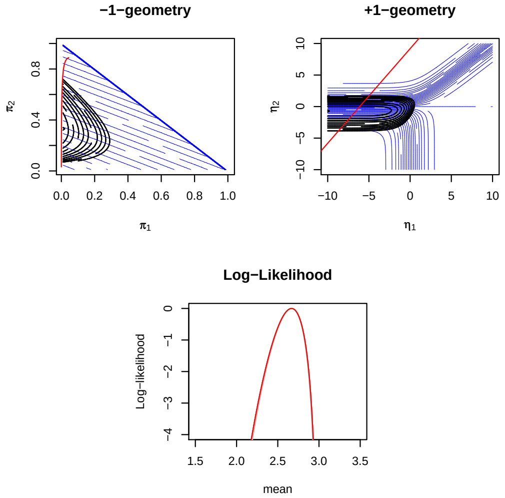

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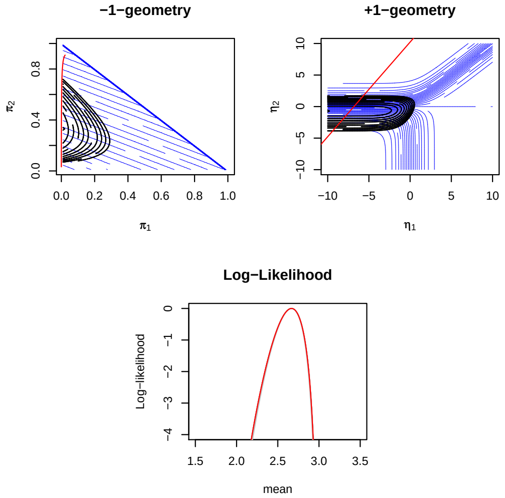

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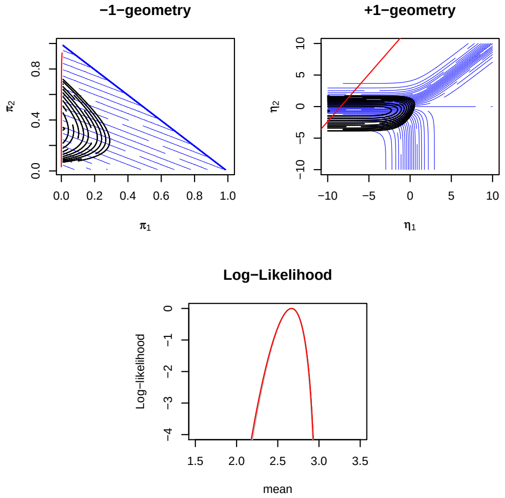

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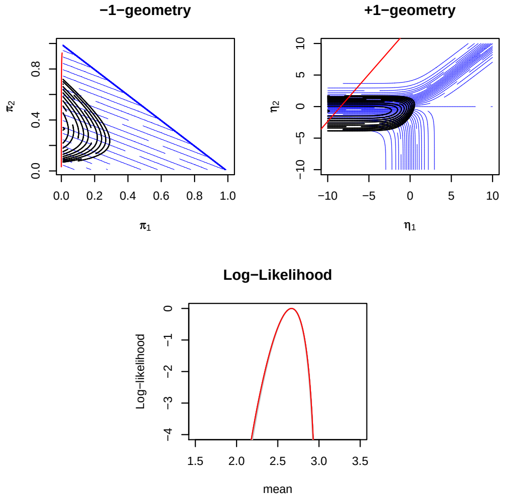

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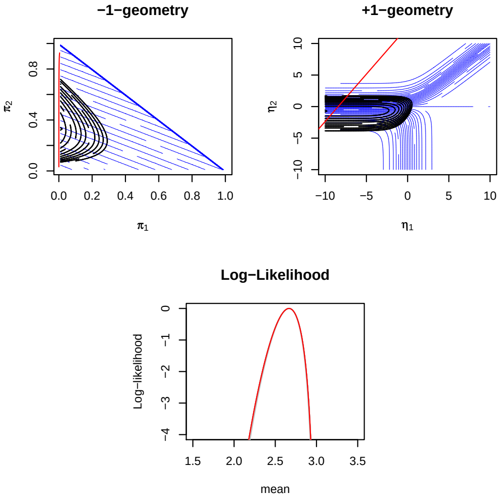

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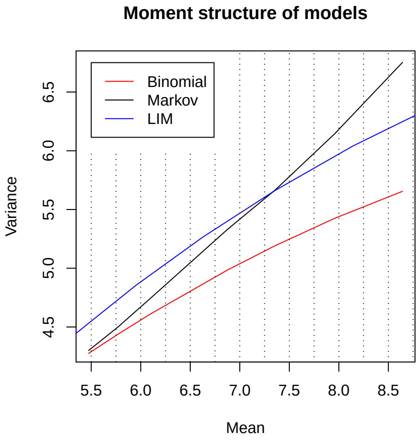

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- The set of sensitive directions defines a framework in which the four Statisticians can communicate, see Karim's talk

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- Affine geometry allows local and global analysis

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