

Refresher - Background Material

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This refresher course is designed to help you ease the transition to University. We will revisit topics and concepts that you have possibly seen in your second-level studies, but everyone's mathematical journey is different. If you have not seen something before, or have forgotten it, don't worry; everything you need will be taught once you arrive.

1 Notation and Assumed Knowledge

First, we review some notation and knowledge from your A levels (or equivalent) that you may have seen before. When you begin at Warwick, this notation might be updated or even be used in different ways by different professors.

$\in$	is an element of
$\notin$	is not an element of
$\subseteq$	is a subset of
$\subset$ or $\subsetneq$	is a proper subset of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x \in A \mid \dots\}$ or $\{x \in A : \dots\}$	the set of all $x \in A$ such that $\dots$
$A \cup B$	the union of sets A and B
$A \cap B$	the intersection of sets A and B
A^c	the complement of A
$A \setminus B$	the elements in A that are not in B
$\emptyset$	the empty or null set
$\mathbb{N}$	the natural numbers $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	the integers $\{\dots - 2, -1, 0, 1, 2, \dots\}$
$\mathbb{Z}^+$	the non-negative integers $\{0, 1, 2, 3, \dots\}$
$\mathbb{R}$	the real numbers
$\mathbb{Q}$	the rational numbers $\{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{N}\}$
$\mathbb{C}$	the complex numbers $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$
$ A $	the number of elements in set A
(x, y)	the ordered pair x, y
$[a, b]$	the closed interval $\{x \in \mathbb{R} \mid a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} \mid a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} \mid a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} \mid a < x < b\}$
$P \Rightarrow Q$	P implies Q (if P then Q)
$P \iff Q$	P and Q are equivalent (one implies the other)
$\sum_{i=1}^n x_i$	The sum of $x_1, x_2, \dots, x_n$.
$\prod_{i=1}^n x_i$	The product of $x_1, x_2, \dots, x_n$.
$\forall x$	for all values of x
$\exists x$	there exists a value of x
$\binom{n}{k}$	n choose k

There will be other pieces of notation you have seen before but will be introduced again without much explanation (such as the conditional probability $P(A|B)$).

2 Experiments, Outcomes and Events

Let's start with some motivating examples.

Click [here](#) for video.

Example 2.1 Suppose we roll a 6-sided die and note the result. However, rather than the specific number, perhaps we're actually interested in a prime number being rolled?



Example 2.2 We're watching a football match; specifically the Manchester Derby between United and City. We are interested in the final result for United; win, lose or draw. We might also be interested in the score, and how that affects the result, or the total number of goals scored.

Sun 15 Dec 2024 · Premier League

Manchester City



1

|

2



Manchester United

FT

J. Gvardiol (36')

HT 1-0

Bruno Fernandes (88' pen) A. Diallo (90')

Assists

Lisandro Martínez (90')

These two examples lead to the following formal definitions.

Definition 2.1 A **random experiment** is a procedure that involves making observations of a process that is random and cannot be predicted precisely in advance.

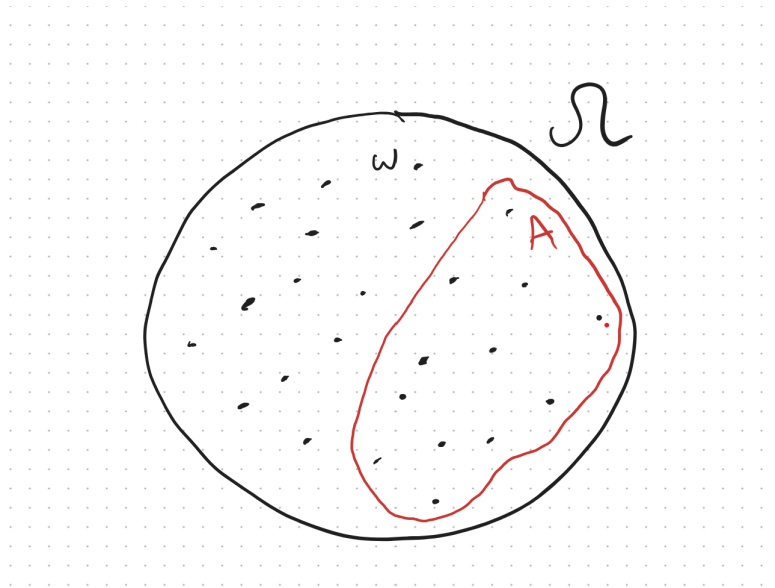
An experiment is anything that we don't know the outcome of beforehand. The classic example is flipping a coin, but rolling a dice or the playing of a football match are others.

Definition 2.2 An **outcome** ω is any possible result of an experiment. The set of all possible outcomes Ω is the **sample space**.

For an experiment, exactly one outcome must occur. When rolling a 6-sided die, the number we see is the outcome and the sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$. For the football match, if we are only interested in the result, then $\Omega = \{\text{Win, Lose, Draw}\}$. If the outcome of interest is the final score, then Ω will be the set of all possible scores (and a little harder to write down).

Definition 2.3 An **event** E is a set of outcomes and thus a subset of the sample space; $E \subseteq \Omega$. Let Σ be the **event space** (not to be confused with the summation notation), the set of events we wish to consider.

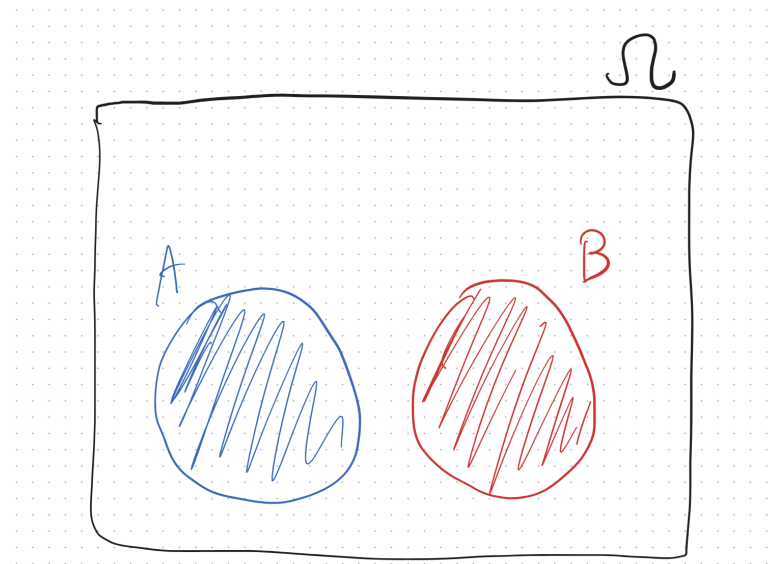
An event E is of the form $\{x_1, x_2, \dots, x_n\}$ where $x_i \in \Omega, \forall i$. Don't worry if you don't fully understand this notation.



In the above graph, think of each point as an outcome w in the sample space Ω . The event A is a group of these outcomes where we list each one.

Any outcome ω corresponds to an event, as a singleton set is still a set; $\{\omega\}$. For example, rolling a 6 is both an outcome and the event $\{6\}$. However, rolling a prime number is an event corresponding to the set $\{2, 3, 5\}$ or $\{2\} \cup \{3\} \cup \{5\}$; we can write events as unions of other events. The event that United don't lose the match is $\{\text{Win, Draw}\}$. We could also group scorelines into events such as more than 3 goals were scored.

Definition 2.4 Two events A and B are **mutually exclusive** or **disjoint** if they cannot happen at the same time. For two mutually exclusive events, $A \cap B = \emptyset$.



As only one outcome can happen each time, we can think of Ω as a union of mutually exclusive events. We cannot roll a 5 and a 6 at the same time. One team cannot both win and lose at the same time.

3 Probability

It's no coincidence that both experiments are games. Games lead to gambling, and the desire to win money off either your friends or the casino has, throughout history, motivated study in how to win the most. When deciding what outcome to pick, it's good to know how often these outcomes occur; their **probability**.

Example 3.1 In roulette, there are 37 numbered spaces. The numbers 1 to 36 are either red or black, with 18 of each colour. The number 0 is coloured green. Each number 0-36 represents an outcome, while each colour is an example of an event. Before deciding what event to bet on, a player should know the probability of that event first.



Probability is defined in terms of events. Each outcome ω has a probability of occurrence $P(\{\omega\}) = p$ such that the sum of these probabilities is 1, or:

$$\sum_{\omega \in \Omega} P(\{\omega\}) = 1 \quad (1)$$

If all outcomes are assumed **equally likely**, as for a **fair** die or the roulette wheel, then for $A \subseteq \Omega$:

$$P(A) = \frac{|A|}{|\Omega|}$$

If the outcomes are not equally likely, such as the probability of United winning or losing, we need to put a bit more thought into the calculation, by collecting data and building statistical models.

Example 3.2 We could represent the outcomes and probabilities for a fair 6-sided die as a table:

Outcome (ω)	1	2	3	4	5	6
Probability (p)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

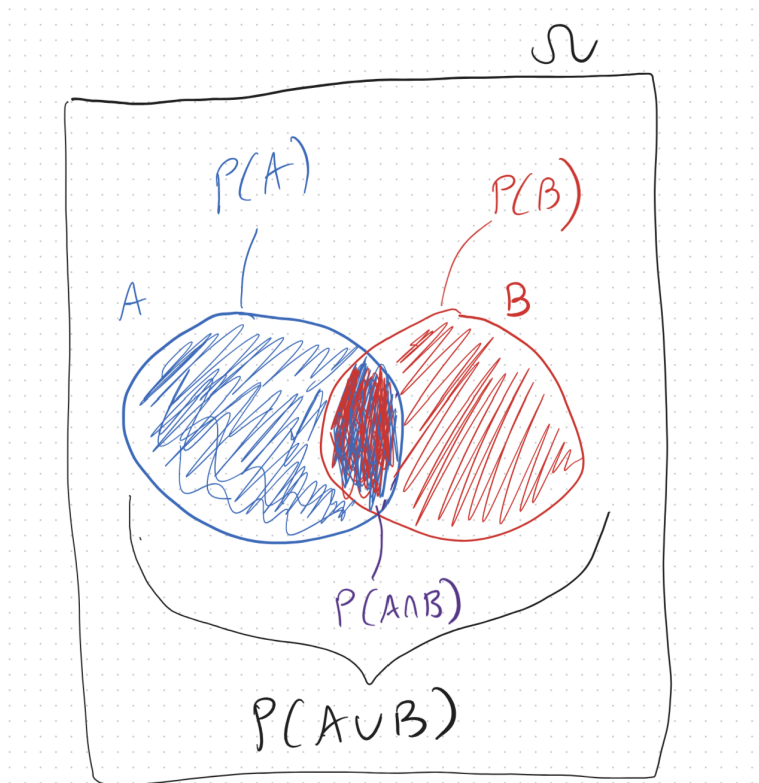
If the die is not fair, the individual probabilities must still be non-negative and sum to 1. We could roll the die 10,000 times and calculate the proportion each number comes up to estimate its probability. A hypothesis test could compare these proportions to $\frac{1}{6}$ to check if the die is actually or fair or not.

So how do we find probabilities for events when we only know the probabilities of the outcomes? We use the following law.

Definition 3.1 The **Law of Union** states, for two events A and B :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (2)$$

We subtract $P(A \cap B)$ to avoid double counting the intersection, which appears in both A and B .



If A and B are mutually exclusive, then $P(A \cap B) = 0$ and so $P(A \cup B) = P(A) + P(B)$.

So, to obtain the probability an event E occurs, we can break it up into a union of mutually exclusive events and sum their probabilities. The easiest such union is the outcomes within the event. That is, the probability of rolling an even number, which we might write $P(\text{Even})$, is:

$$P(\text{Even}) = P(\{2\}) + P(\{4\}) + P(\{6\}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

The probability United don't lose, $P(\text{Don't Lose})$, is:

$$P(\text{Don't Lose}) = P(\text{Win}) + P(\text{Draw})$$

Think of each event as a series of **OR** statements; rolling an even number is rolling a 2 OR rolling a 4 OR rolling a 6. We cannot roll a 2 and a 4 at the same time so there is no intersection. When we see OR or $\cup$, we **sum**, as in (2). We call the outcomes that make up an event **constituent**.

Key Takeaways:

- An event is a subset of possible outcomes of an experiment
- Mutually exclusive events cannot happen at the same time
- The easiest way to get the probability of event is to sum the probabilities of the constituent outcomes (OR / $\cup$)