

THE UNIVERSITY OF WARWICK

THIRD YEAR EXAMINATION: CLASS TEST 1, 5TH FEBRUARY 2019

ST345 LIFE CONTINGENCIES

Time Allowed: **45 hours**

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

Full marks will be obtained by correctly answering ALL questions.

The numbers in the margin indicate approximately how many marks are available for each part of a question. The total mark for all questions is 70.

Actuarial tables are required.

1. Calculate using AM92 Select with interest at 6% per annum,

a) ${}_{42}P_{[18]+1}$

b) ${}_{35}q_{86}$

c) $a_{17:\overline{9}}^{(3)}$

d) $A_{51:\overline{23}}^1$

[6]

2. a) Give an intuitive explanation for why the following recursion may hold:

$$(IA)_{x:\overline{n}}^1 = v \cdot q_x + v \cdot p_x \cdot \left[(IA)_{x+1:\overline{n-1}}^1 + A_{x+1:\overline{n-1}}^1 \right]$$

b) Derive the recursion from first principles.

[8]

3. a) State what is meant by both overhead and direct expenses incurred by a life insurer with respect to a policy.

b) Describe three categories of direct expenses, giving an example of each.

c) How might an insurer account for overhead expenses when setting premiums for a policy?

[6]

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Model Solution No: 1

a) ${}_{42}P_{[18]+1} = \ell_{51}/\ell_{[18]+1} = 9212.7143/9987.6338 = 0.922412.$ **[1 mark]**

b) ${}_{35}q_{86} = 0$, noting that the maximal attainable age in AM92 is 120. **[1 mark]**

c) $a_{\overline{17}|}^{(3)} = a_{17}^{(3)} - v^9 \cdot (\ell_{26}/\ell_{17}) \cdot a_{26}^{(3)} \approx (\ddot{a}_{17} - 2/3) - 1.06^{-9} \cdot (9947.9807/10000) \cdot (\ddot{a}_{26} - 2/3) =$
 $(16.974 - 2/3) - 0.5888194 \cdot (16.609 - 2/3) = 6.920178.$ **[2 marks]**

d) $A_{\overline{51}|}^1 = A_{51} - v^{23} \cdot (\ell_{74}/\ell_{51}) \cdot A_{74} = 0.21542 - 0.2617973 \cdot (7150.2401/9687.7149) \cdot$
 $0.54896 = 0.109347.$ **[2 marks]**

[Total 6 Marks]

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Model Solution No: 2

- a) Consider the decomposition of the EPV of the n -year increasing term assurance based on the survival / death of an individual aged (x) in year one. The EPV of the death benefit in year 1 is simply $v \cdot q_x$, and the EPV of the benefits conditional on survival are the EPV of the remaining cashflows discounted to the end of year one, multiplied by $v \cdot p_x$ to account for mortality and interest arising in year one.

The remaining benefits from the end of year one can now be considered as an $(n-1)$ -year increasing term assurance to a life aged $(x+1)$, but incrementing from an initial level of 2 (as opposed to the typical initial level of 1). This increasing term assurance can itself be decomposed into a term assurance, plus an increasing term assurance incrementing from an initial level of 1. Following the above rationale, the recursion will hold. **[3 marks]**

- b) We have,

$$\begin{aligned}
 (IA)_{x:\overline{n}|}^1 &:= \sum_{k=0}^{n-1} (k+1) \cdot v^{k+1} \cdot {}_k|q_x \\
 &= v \cdot q_x + v \cdot p_x \cdot \sum_{k=1}^{n-1} (k+1) \cdot v^k \cdot {}_{k-1}|q_{x+1} \\
 &= v \cdot q_x + v \cdot p_x \cdot \sum_{k=1}^{n-1} \left[k \cdot v^k \cdot {}_{k-1}|q_{x+1} + v^k \cdot {}_{k-1}|q_{x+1} \right] \\
 &= v \cdot q_x + v \cdot p_x \cdot \left[\left(\sum_{k=0}^{n-2} (k+1) \cdot v^{k+1} \cdot {}_k|q_{x+1} \right) + \left(\sum_{k=0}^{n-2} v^{k+1} \cdot {}_k|q_{x+1} \right) \right] \\
 &=: v \cdot q_x + v \cdot p_x \cdot \left[(IA)_{x+1:\overline{n-1}|}^1 + A_{x+1:\overline{n-1}|}^1 \right].
 \end{aligned}$$

[5 marks]

[Total 8 Marks]

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Model Solution No: 3

- a) Direct expenses vary with the amount of business written, overhead expenses do not. In the long term, large changes in the amount of business written will result in all expenses being direct. **[2 marks]**

- b) The three categories are: i) Initial expenses, arising when a policy is issued (for instance, underwriting costs); ii) Renewal expenses, arising during the policy term (for instance, bank charges); iii) Termination expenses, arising on policy termination due to the occurrence of a contingent event (for instance, medical examination costs). **[3 marks]**

- c) Overhead expenses are usually charged as a renewal expense on a per policy basis. The valuation of expenses uses the same methodology as that used for the valuation of benefits. **[1 mark]**

[Total 6 Marks]