THE UNIVERSITY OF WARWICK

THIRD YEAR EXAMINATION: CLASS TEST 1, 6TH FEBRUARY 2018

ST345 LIFE CONTINGENCIES

Time Allowed: **45 minutes**

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

Full marks will be obtained by correctly answering ALL questions.

The numbers in the margin indicate approximately how many marks are available for each part of a question. The total mark for all questions is 20.

Actuarial tables are required.

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1. a) Give the definitions, and one example, for each of the following types of selection with regard to mortality,

- Class Selection
- Self Selection
- Temporary Initial Selection

b) State, in words, the definition of ${}_tp_{[x]+r}$ and give an example of conditions under which, for mortality, we can observe

$$q_{[x]} > q_{[x-1]+1} > q_{[x-2]+2}.$$

[7]

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2. Suppose for a stochastic model of mortality we have ${}_tp_0 = 1 + 0.01(1 - \exp\{0.0461512 \cdot t\})$. What is the maximum attainable age? **[2]**

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3. Calculate using AM92 Select with interest at 6% per annum,

a) ${}_{5|10}q_{[52]}$

b) $\ddot{a}^{(104)}_{[55]:\overline{45|}}$

[5]

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4. A term assurance contract for a life aged 60 exact for a term of 20 years provides a benefit of £ 100,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract using AM92 Select with interest at 4% per annum. **[6]**

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Model Solution No: 1

a)
- Class selection refers to permanent differences in sub-populations. For instance, mortality between males and females.
- Self selection refers to mortality differences due to the actions of individuals selecting themselves into a population. For instance, an individual actively seeks and takes out life insurance, as opposed to one who has life insurance provided by their employer, may experience higher mortality.
- Temporary initial selection refers to differences in duration since selection into the population. For instance, lower mortality experienced by those recently underwritten for insurance. **[5 marks]**

b) ${}_tp_{[x]+r}$ is the probability that a life, currently aged $x + r$, who was selected r years ago, will survive at least t years.

The ordering in mortality rates indicates that those selected more recently have higher mortality. This could be observed for instance in the mortality of those taking early retirement on ill-health grounds. **[2 marks]**

[**Total 7 Marks**]

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Model Solution No: 2

a) Require t such that ${}_tp_0 = 0 = 1 + b(1 - \exp\{a \cdot t\}$, where $a = 0.0461512$ and $b = 0.01$. Re-arranging we have, $t_{\max} = -\frac{1}{a} \log \frac{b}{1+b} = 100$

[Total 2 Marks]

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Model Solution No: 3

a) ${}_{5|10}q_{[52]} = (\ell_{57} - \ell_{67})/\ell_{[52]} = (9467.2906 - 8557.0118)/9652.6965 = 0.094303$. **[2 marks]**

b) $\ddot{a}^{(104)}_{[55]:\overline{45}|} = \ddot{a}^{(104)}_{[55]} - \dfrac{\ell_{100}}{\ell_{[55]}} \cdot v^{45} \cdot \ddot{a}^{(104)}_{100} \approx \left(\ddot{a}_{[55]} - \dfrac{103}{208}\right) - \dfrac{\ell_{100}}{\ell_{[55]}} \cdot v^{45} \cdot \left(\ddot{a}_{100} - \dfrac{103}{208}\right) = \left(13.072 - \dfrac{103}{208}\right) - \dfrac{95.8476}{9545.9929} \cdot 0.07265007 \cdot \left(2.380 - \dfrac{103}{208}\right) = 12.57543.$ **[3 marks]**

[**Total 5 Marks**]

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Model Solution No: 4

a) Expected Present Value: $100000A_{[60]:\overline{20}|}^{\ 1} = 100000 \cdot \left(A_{[60]} - v^{20} \cdot \frac{\ell_{80}}{\ell_{[60]}} \cdot A_{80}\right)$

$= 100000 \cdot \left(0.45510 - 0.4563869 \cdot \frac{5266.4606}{9263.1422} \cdot 0.73775\right) = 26367.31.$ **[3 marks]**

b) Variance: $100000^2 \cdot \left({}^2A_{[60]:\overline{20}|}^{\ 1} - (A_{[60]:\overline{20}|}^{\ 1})^2\right)$

$= 100000^2 \cdot \left(\left({}^2A_{[60]} - v^{40} \cdot \frac{\ell_{80}}{\ell_{[60]}} \cdot {}^2A_{80}\right) - (26417.03/100000)^2\right)$

$= 100000^2 \cdot \left(0.23547 - 0.208289 \cdot \frac{5266.4606}{9263.1422} \cdot 0.56432 - (26367.31/100000)^2\right)$

$= (31483.24)^2.$ **[3 marks]**

[**Total 6 Marks**]