ST345

UNIVERSITY OF WARWICK

THIRD YEAR EXAMINATION: SUMMER 2018

LIFE CONTINGENCIES

Time allowed: **2 hours**

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

Full marks may be obtained by correctly answering ALL questions.

The numbers in the margin indicate approximately how many marks are available for each part of a question. The total mark for all questions is 70.

Approved calculators may be used in this examination.

Actuarial tables are required.

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1. Calculate the standardized mortality ratio for the population of San Serriffe given the following data:

	Standard Population		**San Serriffe**	
Age	**Population**	**Deaths**	**Population**	**Deaths**
60	2,500,000	20,750	15,500	104
61	2,450,000	22,600	14,500	117
62	2,350,000	23,800	14,000	107

[**3**]

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2. Calculate the following:

(a) $\bar{A}_{\overline{40:30}}$,

(b) $\bar{a}_{40:30:\overline{25|}}$,

with the basis:

$\mu = 0.02$ throughout for the life aged 40 now,
$\mu = 0.01$ throughout for the life aged 30 now,
$\delta = 3\%$ per annum. [**6**]

Continued...

3. A life insurance company issues a non-profit assurance policy for a term of n years to a life aged x. For $t \in \{1, \ldots, n\}$:

- The level annual premium payable at the start of year t is P.
- The expense at the start of policy year t is E_t.
- The benefits payable at the end of the t^{th} year on death, surrender and survival are D_t, B_t and S_t respectively.
- The rate of interest earned on net cash flows during the t^{th} policy year is i_t
- The dependent rates of mortality and surrender at age $x + t$ are $(aq)^d_{x+t}$ and $(aq)^2_{x+t}$ respectively.

Assume that the insurance company **does not** set up a reserve for the policy.

(a) Write down an expression for $(\text{CF})_t$, the accumulation to the end of the t^{th} policy year of the expected net cash flow arising during the t^{th} policy year per policy in force at the start of the year.

(b) Derive an expression which could be used to calculate the level annual premium that the company should charge if the company requires the expected net present value of profit on the policy to be zero assuming a risk discount rate of $j\%$ per annum defining any notation used.

Assume that the insurance company **does** set up a reserve ${}_{t-1}V$ for the policy at the start of the t^{th} policy year.

c) Write down an expression for the expected profit at the end of the t^{th} policy year for each policy in force at the start of the year.

[**7**]

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4. A pension scheme provides a pension of $1/60^{\text{th}}$ of final pensionable salary on retirement, due to age or ill-health, for each year of service (part years included). Final pensionable salary is average salary over the three years before retirement. Normal retirement age is 65. Members contribute 5% of pensionable salary each year.

(a) Using the example pension scheme in the Formulae and Tables for Examinations, calculate the expected present value of the combined past and future benefits for a member aged 45 exact with 10 years of past service and salary in the previous year of £25,000.

(b) Calculate the present value of the member's future contributions. [**8**]

Continued...

5. (a) Explain the difference between select and ultimate mortality.

(b) Why does the ELT15 life table not include select mortality?

(c) In a particular mortality table we find that $q_{[18]} = 0.9q_{18}$ while $q_{[65]} = 0.25q_{65}$. Give one possible reason for the scaling difference.

[**8**]

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6. (a) An insurer uses Model 1 (see Figure 1) to calculate premiums and reserves for its disability policies. The transition intensities in Model 1 depend only on the individual's current age, x.

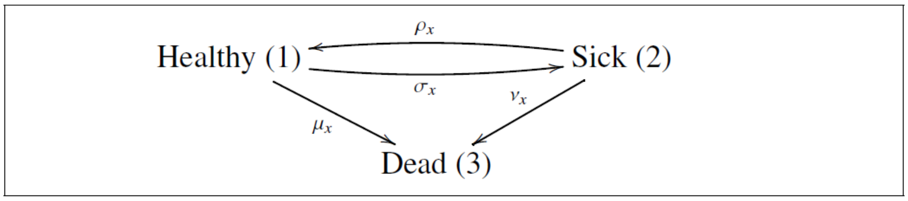

Figure 1: Model 1

Show that,

$$\frac{d}{dt} {}_tP_x^{12} = {}_tP_x^{11}\sigma_{x+t} - {}_tP_x^{12}(\rho_{x+t} + \nu_{x+t}).$$

(b) Now suppose the insurer uses Model 2 (see Figure 2), in which the intensities of recovery and mortality depend on the individual's current age, x, and on the duration of the current sickness, z.

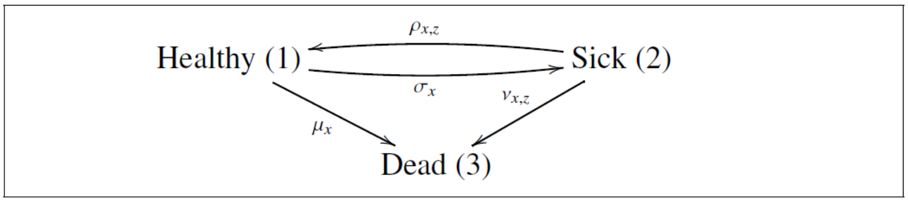

Figure 2: Model 2

The probability that an individual, who is currently aged x and has been sick for duration z, will be sick at age $x+t$ with duration of sickness less than or equal to $d\,(d < t)$ is denoted ${}_{d,t}p_{x,z}^{22}$. Write down an integral formula for ${}_{d,t}p_{x,z}^{22}$ involving the transition intensities and probabilities of the form ${}_wp_{y,u}^{gh}$ and ${}_wp_{y,u}^{\bar{g}\bar{g}}$. Explain carefully why your formula is correct.

[**8**]

Continued...

7. A life insurance company issues a 25-year with profits endowment assurance policy to a male life aged 40 exact. The sum assured of £100,000 plus declared reversionary bonuses are payable on survival to the end of the term or immediately on death, if earlier. Calculate the monthly premium payable in advance throughout the term of the policy if the company assumes that future reversionary bonuses will be declared at a rate of 1.92308% of the sum assured, compounded and vesting at the end of each policy year.

Basis:

- Interest 6% per annum
- Mortality AM92 Select
- Initial commission 87.5% of the total annual premium
- Initial expenses £175 paid at policy commencement date
- Renewal commission 2.5% of each monthly premium from the start of the second policy year
- Renewal expenses £65 at the start of the second and subsequent policy years
- Claim expenses 2.5% of the claim amount.

[**10**]

Continued...

8. A life insurance company issues a 4-year unit-linked endowment policy to a life aged 50 exact under which level premiums of £750 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year, 25% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 1% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of £3000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender, the bid value of the units is payable at the end of the policy year of exit. On maturity, 110% of the bid value of the units is payable. The company uses the following assumptions in carrying out profit tests of this contract:

- Rate of growth on assets in the unit fund: 6.5% p.a
- Rate of interest on non-unit fund cash flows: 5.5% p.a
- Mortality: AM92 Select
- Initial expenses: £150
- Renewal expenses: £65 p.a. on the second and subsequent premium dates
- Initial commission: 10% of first premium
- Renewal commission: 2.5% of premium after year 1.
- Risk discount rate: 8.5% p.a.

In addition assume that at the end of each of the first 3 years, 10% of all policies still in force then surrender.

(a) Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows.

(b) Suppose the company sets up reserves in order to zeroise future expected negative cash flows.

(i) Calculate the expected reserve that must be set up at the end of each policy year, per policy in force at the start of each policy year.

(ii) Calculate the profit margin allowing for the cost of setting up these reserves.

[20]

END.

ST345 Life Contingencies 2018: Solutions

1. Actual number of deaths in San Serriffe $= 104 + 117 + 107 = 328$.

Mortality rates in standard population are:

Age	**Rate**
60	$20,750/2,500,000 = 0.00830$
61	$22,600/2,450,000 = 0.00922$
62	$23,800/2,350,000 = 0.01013$

Therefore expected number of deaths
$= 0.00830\times 15,500 + 0.00922\times 14500 + 0.01013\times 14000 = 404.$

Hence SMR $= 328/404 = 0.812$, i.e. 81.2%. [**Total 3**]

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2. (a)

$$
\begin{aligned}
\bar{A}_{\overline{40:30}} &= \int_0^\infty e^{-.03t}\big[e^{-0.2t}(1-e^{-.01t})\times .02 \;+\; (1-e^{-.02t})e^{-.01t}\times .01\big]\,dt \\
&= \int_0^\infty \big[.02(e^{-.05t}-e^{-.06t}) \;+\; .01(e^{-.04t}-e^{-.06t})\big]\,dt \\
&= \int_0^\infty \big[.01e^{-.04t} + .02e^{-.05t} - .03e^{-.06t}\big]\,dt \\
&= \left[\frac{.01}{-.04}e^{-.04t} + \frac{.02}{-.05}.02e^{-.05t} + \frac{-.03}{-.06}e^{-.06t}\right]_0^\infty \\
&= \left[\frac{1}{4} + \frac{2}{5} - \frac{1}{2}\right] = \frac{3}{20} = 0.15.
\end{aligned}
$$

[4]

(b)

$$
\begin{aligned}
\bar{a}_{40:30:\overline{25}|} &= \int_0^{25} e^{-.03t}e^{-.02t}e^{-.01t}dt \\
&= \int_0^{25} e^{-0.6t}dt \;=\; \left[-\frac{1}{.06}e^{-.06t}\right]_0^{25} \\
&= \frac{1}{.06} - \frac{1}{.06}e^{-1.5} \;=\; 12.948.
\end{aligned}
$$

[2]

[**Total 6**]

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3. (a) $(\text{CF})_t = (P - E_t)(1 + i_t) - (aq)^d_{x+t-1} D_t - (aq)^w_{x+t-1} B_t - [1 - (aq)^d_{x+t-1} - (aq)^w_{x+t-1}] S_t.$

[2]

(b) We need to set the EPV of the profit signature of the policy equal to zero using a risk discount of j% p.a. Hence, if

$$\begin{aligned} (ap)_{x+k} &\approx [1 - (aq)^d_{x+k} - (aq)^w_{x+k}] \\ \Rightarrow \quad {}_t(ap)_x &= \prod_{k=0}^{t-1} (ap)_{x+k} \end{aligned}$$

then the level annual premium, P, is derived from the following:

$$\sum_{j=1}^{n} (\text{CF})_t \times {}_{t-1}(ap)_x \times v^t_{j\%} = 0.$$

[3]

(c) Expected profit at the end of the t^{th} policy year for each policy in force at the start of that year

$$= {}_{t-1}V(1 + i_t) + (\text{CF})_t - (ap)_{x+t-1} \times {}_tV.$$

[2]

[**Total 7**]

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4. (a)

$$\begin{aligned} &\frac{25000}{60}\left[\frac{10({}^zM^{ia}_{45} + {}^zM^{ra}_{45}) + ({}^z\bar{R}^{ia}_{45} + {}^z\bar{R}^{ra}_{45})}{s_{44}D_{45}}\right] \\ = &\frac{25000}{60}\left[\frac{10(52554 + 128026) + (609826 + 2244130)}{8.375 \times 2329}\right] \\ = &\ 99540. \end{aligned}$$

[5]

(b)

$$0.05 \times 25000\left(\frac{{}^s\bar{N}_{45}}{s_{44}D_{45}}\right) = 0.05 \times 25000 \times \left(\frac{253080}{8.375 \times 2329}\right) = 16219.$$

[3]

[**Total 8**]

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5. (a) Select mortality applies to individuals who have recently taken out a policy. New policyholders are typically accepted at the discretion of the insurance company. For instance, the underwriting department will exclude individuals at poorer risk. As such, mortality of a select life differs from (is typically lower than) the general population.

Ultimate mortality refers to the mortality of individuals in the general population.

Over time the effect of selection will degrade—resulting in both populations having similar mortality.

[3]

(b) ELT15 is a general populace (England + Wales) life table, all entering at age 0.

[2]

(c) Scaling from ultimate to select mortality represents the degree to which individuals with known adverse characteristics can be excluded. At younger ages death is typically accidental (hard to exclude); at older ages death is typically health related (easier to exclude).

[3]

[**Total 8**]

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6. (a) For $dt > 0$, we have ${}_{t+dt}P_x^{12} = {}_tP_x^{11}\sigma_{x+t}\,dt + {}_tP_x^{12}(1-\rho_{x+t}-\nu_{x+t})\,dt + o(dt)$.

Hence

$$\frac{{}_{t+dt}P_x^{12} - {}_tP_x^{12}}{dt} = {}_tP_x^{11}\sigma_{x+t} - {}_tP_x^{12}(\rho_{x+t}+\nu_{x+t}) + o(dt)/dt.$$

Letting $dt \to 0$, we have

$$\frac{d}{dt}{}_tP_x^{12} = {}_tP_x^{11}\sigma_{x+t} - {}_tP_x^{12}(\rho_{x+t}+\nu_{x+t}).$$

[4]

(b) ${}_{d,t}p_{x,z}^{22} = \int_{u=t-d}^{t} {}_up_{x,z}^{21}\sigma_{x+u}\cdot {}_{t-u}p_{x+u}^{\overline{22}}\,du$ for $d < t$.

N.B. the individual must not be sick at age $x+t$ with duration $\leq d(< t)$. Hence the individual must not fall sick (for the last time) between age $x+t-d$ and $x+t$ and stay sick until age $x+t$.

${}_up_{x,z}^{22}$ – probability of being healthy at $x = u$, given initial sickness.

$\sigma_{x+u}du$ – "prob" of falling sick at $x+u$

${}_{x-u}p_{x+u}^{\overline{22}}$ – "prob" of remaining sick

– integral is simply the sum over all states.

[4]

[**Total 8**]

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7. Let P be the monthly premium. Then

EPV of premiums:

$$12P\ddot{a}_{[40]:\overline{25}|}^{(12)} = 155.124\,P$$

$$\ddot{a}_{[40]:\overline{25}|}^{(12)} = \ddot{a}_{[40]:\overline{25}|} - \frac{11}{24}(1 - {}_{25}P_{[40]}v^{25})$$

$$= 13.290 - \frac{11}{24}\left(1 - (1.06)^{-25} \times \frac{8821.2612}{9854.3036}\right) = 12.927.$$

[2]

EPV of benefits:

$$\frac{100,000}{(1+b)} \times (1.06)^{1/2} \times \left[q_{[40]}(1+b)v + {}_{1|}q_{[40]}(1+b)^2v^2 + \cdots + {}_{24|}q_{[40]}(1+b)^{25}v^{25}\right]$$

$$+\,100,000\,{}_{25}p_{[40]}(1+b)^{25}v^{25} \qquad (\text{where } b = 0.0192308)$$

$$= \frac{100,000}{(1+b)} \times (1.06)^{1/2}A^{\;1}_{[40]:\overline{25}|}@i' + \frac{D_{65}}{D_{[40]}}@i' \qquad (\text{where } i' = 1.06/(1+b) - 1 = 0.04)$$

$$= \frac{100,000}{1.0192308} \times (1.06)^{1/2} \times (.38896 - .33579) + 100,000 \times .33579 = 38949.9$$

[3]

EPV of expenses:

$$.875 \times 12P + 175 + 0.025 \times 12 \times P\left(\ddot{a}_{[40]:\overline{25}|}^{(12)} - \ddot{a}_{[40]:\overline{1}|}^{(12)}\right) + 65(\ddot{a}_{[40]:\overline{25}|} - 1)$$

$$= 14.086P + 973.85;$$

$$\ddot{a}_{[40]:\overline{1}|}^{(12)} = \ddot{a}_{[40]:\overline{1}|} - \frac{11}{24}(1 - p_{[40]}v) = 1 - \frac{11}{24}\left(1 - (1.06)^{-1} \times \frac{9846.5384}{9854.3036}\right) = 0.974.$$

[2]

EPV of claim expenses:

$$.025 \times 38949.9 = 973.748.$$

[1]

Equation of value gives $155.124P = 38949.9 + 14.086P + 973.85 + 973.75$
and $P = £289.98$. [2]

[Total 10]

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8. Summary

- Annual premium: £750.
- Risk discount rate: 8.5%
- Interest on investments: 6.5%
- Interest on sterling provisions: 5.5%
- Minimum death benefit: £3000
- Allocation % (1st year): 25%
- Allocation % (2nd year): 102.5%
- Management charge: 1%
- B/O spread: 5%

Initial expense: $£150 + 10\%P \Rightarrow £225$.
Renewal expense: $£65 + 2.5\%P \Rightarrow £83.75$.

(a)

x	q_x^d	q_x^s	$(aq)_x^d$	$(aq)_x^s$	(ap)	$_{t-1}(ap)$
50	0.001971	0.1	0.001971	0.09980	0.898226	1
51	0.002732	0.1	0.002732	0.09973	0.897541	0.898226
52	0.003152	0.1	0.003152	0.09968	0.897163	0.806195
53	0.003539	0	0.003539	0	0.996461	0.723288

Unit fund

	year 1	*year 2*	*year 3*	*year 4*
value of units (boy)	0	187.806	968.018	1790.635
alloc.	187.5	768.75	768.75	768.75
B/O	9.375	38.4375	38.4375	38.4375
interest	11.578	59.678	110.392	163.862
management charge	1.897	9.778	18.087	26.848
value of units (eoy)	187.806	968.018	1790.635	2657.961

Cash flows

	year 1	*year 2*	*year 3*	*year 4*
unallocated premium	562.5	−18.75	−18.75	−18.75
B/O spread	9.375	38.4375	38.4375	38.4375
Expenses	225.0	83.75	83.75	83.75
Interest	19.078	−3.523	−3.523	−3.523
Man. charge	1.897	9.778	18.087	26.848
Extra death benefit	5.543	5.551	3.812	1.210
Benefit	0	0	0	264.855
End of year CF	362.307	−63.359	−53.311	−306.804
Prob. in force	1	0.898226	0.806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
EPV of profit	91.809			
premium signature	750.00	620.854	513.620	424.701
EPV of premiums	2309.215			
Profit margin	3.98%			

[13]

(b) (i) ${}_3V = \frac{306.804}{1.055} = 290.809$.

${}_2V \times 1.055 - (ap)_{52} \times {}_3V = 53.311 \;\Rightarrow\; {}_2V = 297.833$.

${}_1V \times 1.055 - (ap)_{51} \times {}_2V = 63.359 \;\Rightarrow\; {}_2V = 313.437$.

These need to be adjusted to BoY from EoY:

Year 3: ${}_3V \times (ap)_{52} = 260.903$

Year 2: ${}_2V \times (ap)_{51} = 267.38$

Year 1: ${}_1V \times (ap)_{50} = 281.538$.

(ii) Based on (a), years 2, 3, 4 will be zoroised.

Year 1: $362.307 - 281.538 = 80.769$.

$\Rightarrow$ Revision of (a)

	year 1	*year 2*	*year 3*	*year 4*
	⋮	⋮	⋮	⋮
End of year CF	80.769	0	0	0
Prob. in force	1	0.898226	0806195	0.723288
discount factor	0.921659	0.849455	0.782908	0.721574
premium signature	750.00	620.854	513.62	424.70
EPV of profit	74.442			
Profit margin	3.22%			

[7]

[**Total 20**]

END.