Counterexamples for optimal scaling of Metropolis-Hastings chains with rough target densities

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30-th April 2021, 1pm UK time

Abstract
For sufficiently smooth targets of product form it is known that the variance of a single coordinate of the proposal in RWM (random walk Metropolis) and MALA (Metropolis adjusted Langevin algorithm) should optimally scale as n^{-1} and as $n^{-1/3}$ with dimension n, and that the acceptance rates should be tuned to 0.234 and 0.574. We establish counterexamples to demonstrate that smoothness assumptions of the order of $\mathcal{C}^1(\mathbb{R})$ for RWM and $\mathcal{C}^3(\mathbb{R})$ for MALA are indeed required if these scaling rates are to hold. The counterexamples identify classes of marginal targets for which these guidelines are violated, obtained by perturbing a standard normal density (at the level of the potential for RWM and the second derivative of the potential for MALA) using roughness generated by a path of fractional Brownian motion with Hurst exponent H. For such targets the RWM and MALA proposal variances should optimally be scaled as $n^{1/H}$ and as $n^{-1/(2+H)}$ and will then obey anomalous acceptance rate guidelines. We will discuss the framework developed to deliver these counterexamples and its possible applications.

This is joint work with Wilfrid Kendall

References

[1] Vogrinc, Jure, and Wilfrid S. Kendall. "Counterexamples for optimal scaling of Metropolis–Hastings chains with rough target densities." The Annals of Applied Probability 31, no. 2 (2021): 972-1019.