Message-Passing Monte Carlo

Sam Power

Cambridge Centre for Analysis
Cantab Capital Institute for the Mathematics of Information

sp825@cam.ac.uk

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Sampling

- Task: Sample from smooth target distribution

$$\mathbf{P}(x) \propto \exp(-U(x)) \tag{1}$$

Factor Graphs

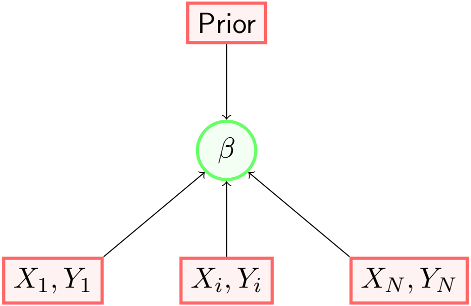
- Graphical representation of a probability measure with *local interactions*

$$\mathbf{P}(x) = \prod_{a \in F} \psi_a(x_{\partial a}) \tag{2}$$

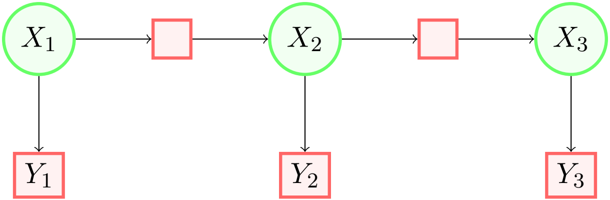
$$U(x) = \sum_{a \in F} U_a(x_{\partial a}) \tag{3}$$

- Includes many practical models

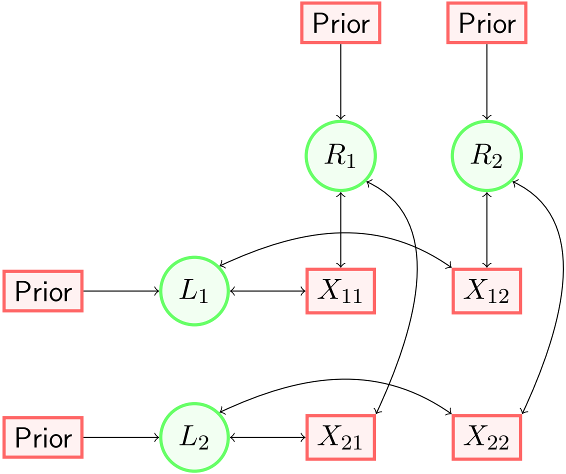
Bayesian GLM



Hidden Markov Model



Matrix Factorisation



Geometric MCMC

- High-dimensional models have delicate *geometry*
- Want clever proposals; use *geometrically-informed* dynamics
- **Hamiltonian Monte Carlo**
  - $\mathcal{H}(x,p) = U(x) + \frac{1}{2}p^T M^{-1} p$
  - Move *long distances*, *still* be accepted
- **Manifold HMC**
  - Use *position-dependent* mass matrix $M(x)$ to encode geometry
  - Navigate *complex* targets; **costly**

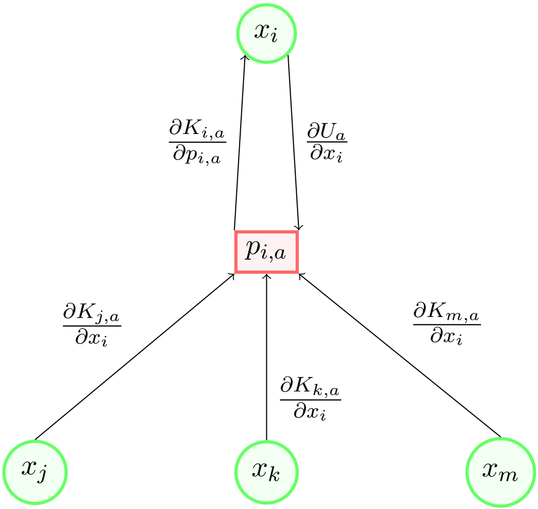
Semi-Separable HMC (Zhang, Sutton, 2014)

- Goal: Scale RMHMC to *hierarchical models*
- **Two** simplifying assumptions
  1. $M(x)$ block-diagonal
  2. p_i conditionally independent of x_i
- Dynamics: *Unconstrained* updating of subsystems
  - **Not** Gibbs sampling.

Message-Passing Monte Carlo

- *Extend* SS-HMC using *Factor Graph* structure
- Two adaptations
  1. **Split** momentum $p_i \mapsto \{p_{i,a}\}_{a \in \partial i}$ with
  $$p_{i,a} \sim \mathcal{N}(0, M_{i,a}) \tag{4}$$
  2. Assume $M_{i,a}$ depends only on $\{x_j\}_{j \in \partial a \setminus i}$
- Dynamics: Unconstrained '*message-passing dynamics*'

Message-Passing Dynamics



Summary

- *Geometric* structure
- *Local* operations
- *Tractable* computations
- $\cdots \rightarrow$ *Scalable* MCMC on Factor Graphs!

Thank you!