Dynamic Estimation of Static Neural Sources with Particle Filters

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Introduction

We use **particle filters** [1] to estimate dynamically the time varying parameters of brain activity, modelled with multiple current dipoles, from **MagnetoEncephaloGraphy** (MEG) data.
In a previous study [2] a dynamic model was used where current dipole locations moved according to a random walk. However, such dynamic model does not reflect the neurophysiological interpretation of a dipole as the activity of a (static) neural population.
Here we present a **new state-space model** of evolving dipoles, which appear and disappear, but whose locations are constant throughout their lifetime [4]. To conduct inference under this model we propose a strategy, based on the **Resample-Move** idea [3], that provides an effective solution for the stationary dipole model.

Statistical Model

As in [2], we use a state-space model in which the neural current is a time-varying set of current dipoles. Differently from [2], where the dipole dynamics was modeled as a random walk in the brain volume, we explicitly model dipole locations as stationary.

The neural current is a dipole set $j_t = \{(r_t^{(1)}, q_t^{(1)}), \dots, (r_t^{(N_t)}, q_t^{(N_t)})\}$

$r_t^{(k)}$ $q_t^{(k)}$ location and moment of the *k*-th dipole time *t*

N_t number of dipoles at time *t*

In our Bayesian setting, the prior for the neural current is given by

$$p(j_{0:T}) = p(j_0) \prod_{t=1 \dots T} p(j_t \mid j_{t-1})$$

In the transition kernel, new dipoles can appear, existing dipoles can die, all surviving dipoles evolve.

$$\begin{aligned} p(j_t \mid j_{t-1}) = {} & P_{birth}\, p^{new}(r_t^{(N_t)}, q_t^{(N_t)}) \prod_{k=1 \dots N_{t-1}} \delta(r_t^{(k)}, r_{t-1}^{(k)})\, p(q_t^{(k)} \mid q_{t-1}^{(k)}) + \\ & P_{death} \frac{1}{N_t} \sum_{k=1,\dots,N_t} \prod_{n=1..k..N_t} \delta(r_t^{(k)}, r_{t-1}^{(k)})\, p(q_t^{(k)} \mid q_{t-1}^{(k)}) + \\ & (1 - P_{birth} - P_{death}) \prod_{n=1 \dots N_t} \delta(r_t^{(k)}, r_{t-1}^{(k)})\, p(q_t^{(k)} \mid q_{t-1}^{(k)}) \end{aligned}$$

The likelihood function embodies the forward model and assumption on the noise distribution, here zero-mean Gaussian. $p(b_t \mid j_t)$

We are ultimately interested in the posterior distribution

$$p(j_{0:T} \mid b_{1:T}) = \frac{p(j_0) \prod_{t=1 \dots T} p(j_t \mid j_{t-1})\, p(b_t \mid j_t)}{p(b_{1:T})}$$

Algorithm

Particle filters [1] provide a general and powerful framework for inference in state-space models. The "standard" particle filters use importance sampling and resampling (steps 1 and 2 below) to produce samples distributed according to the posterior density; however, these basic particle filters are not suited for inference on static parameters.

We combined a S.I.R. with a Resample-Move particle filter to avoid degeneracy of the sample set; the algorithm consists of the sequential application, at each time *t*, of three steps:

1. Importance Sampling

Sample N dipole sets (*particles*) from a proposal $q(j_t \mid j_{t-1}, b_t)$ and assign them weights

$$w_t^i = \frac{p(b_t \mid j_t^i)\, p(j_t^i \mid j_{t-1}^i)}{q(j_t^i \mid j_{t-1}^i, b_t)}$$

Importance sampling aims at obtaining samples distributed according to the posterior; as this is not possible, importance sampling uses a proposal distribution and then weights the samples to correct for this. Thanks to the sequential structure of the problem, sampling and weighting can actually be performed on the "marginal" state at time t.

2. Resampling

Resample the weighted particles to obtain a uniformly weighted particle set.

Resampling removes particles with low weights, and concentrates particles in the high-probability region of the state space.

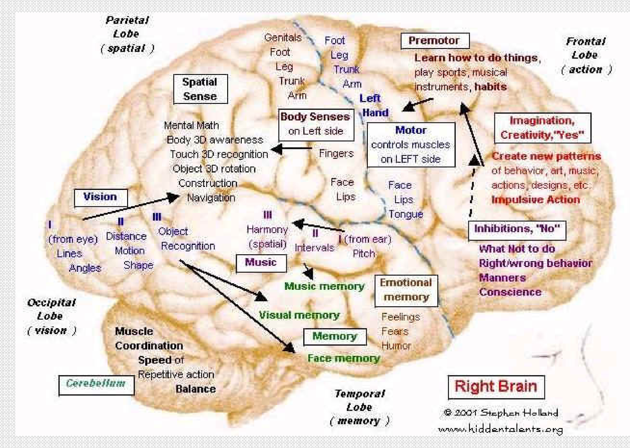
3. MCMC Move

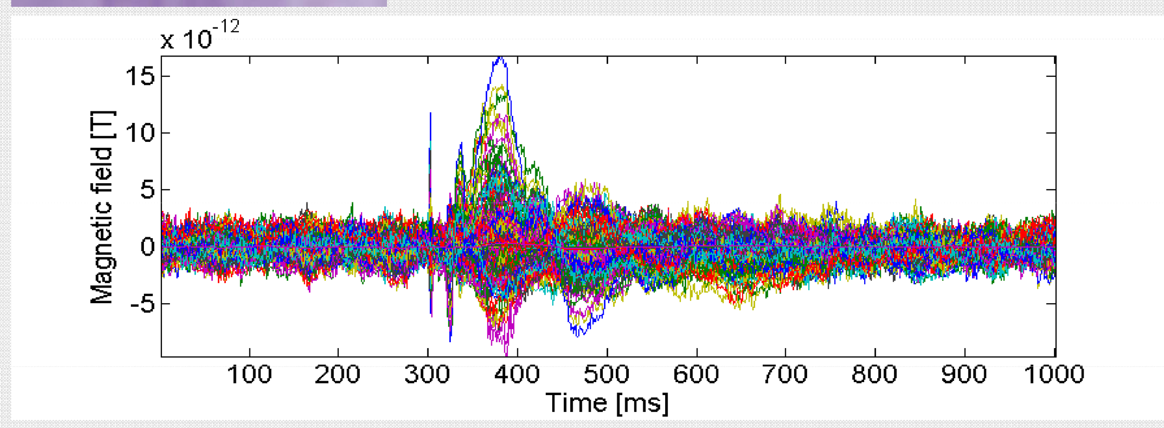
Perturb dipole locations by sampling from an MCMC kernel formally targeting $p(j_{0:t} \mid b_{1:t})$

This MCMC move makes it possible to track the static dipole locations. It works by proposing an updated location adjacent to the existing dipole, accepting with a probability assessed using the full time series. This allows exploration of the state space while working with (near-)static parameters.

MagnetoEncephaloGraphy and dipole model

Magnetoencephalography (MEG) is a functional imaging technique that measures non-invasively the magnetic field produced by the neural currents.
Modern MEG devices contain in between 100 and 500 sensors, each one sampling the magnetic field every millisecond.





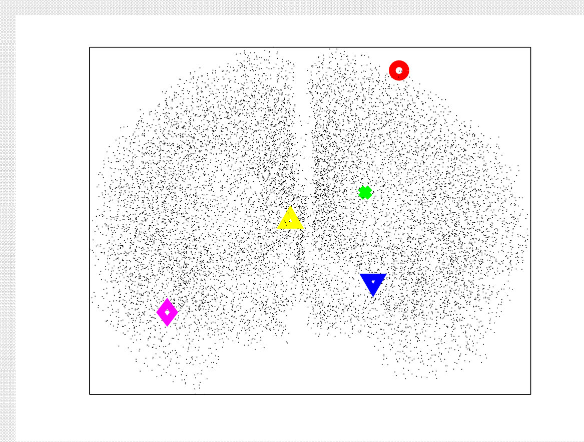
In typical MEG experiments, several brain areas activate and de-activate, with time-varying intensity.
In the dipolar approximation, the electrical current in a small brain area is modeled by a point source, named **current dipole**.
A current dipole is an applied vector; its neurophysiological interpretation regards the location as the location of a neural population, and the strength as the amount of synchronization of the neurons.

Simulations

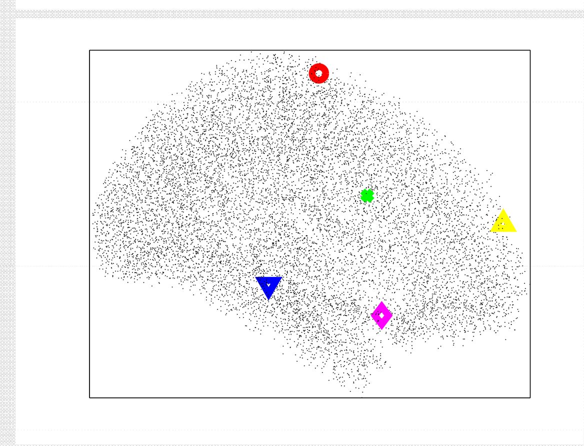
We generated 100 synthetic data sets containing activity produced by 1 to 5 sources. Source locations were randomized, but kept 3 cm apart from each other.

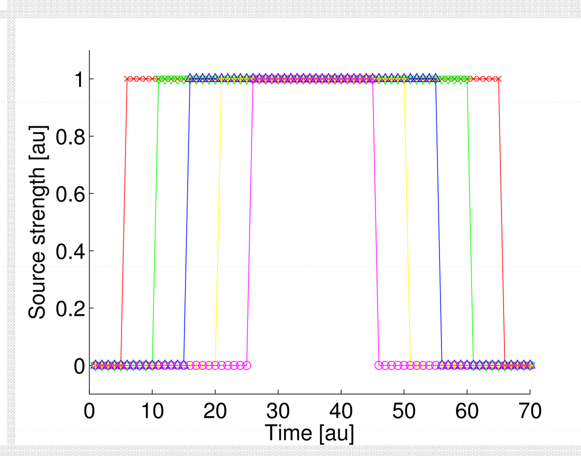
We applied the bootstrap and the resample-move particle filter. Point estimates were computed from the approximation to the posterior distribution provided by the particle filter(s). Mis-localisation metrics were computed to assess the difference between the estimated and the true source configurations.

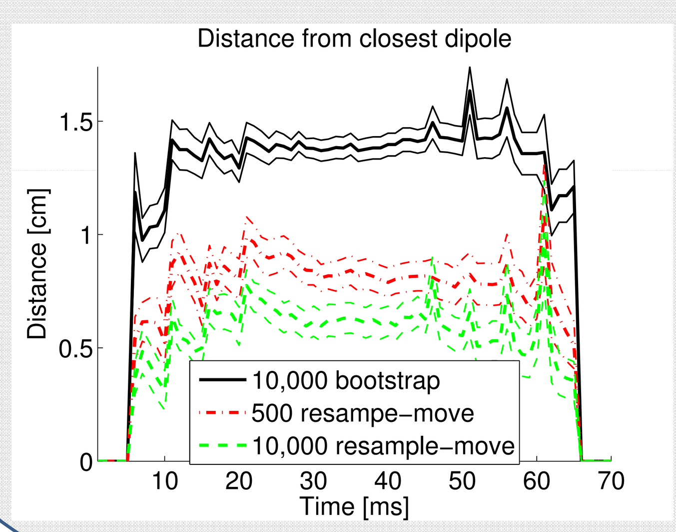
The bootstrap filter was run with 10,000 particles; the resample-move filter was run with 10,000 particles, as well as with 500 particles, to provide a comparison at the same computational cost.
Mis-localisation metrics indicate that the resample-move algorithm provides a substantial improvement over the bootstrap filter when applied to make inference for the static dipole model.
The higher conditional likelihood confirms that resample-move algorithm finds substantially more probability mass than the bootstrap.

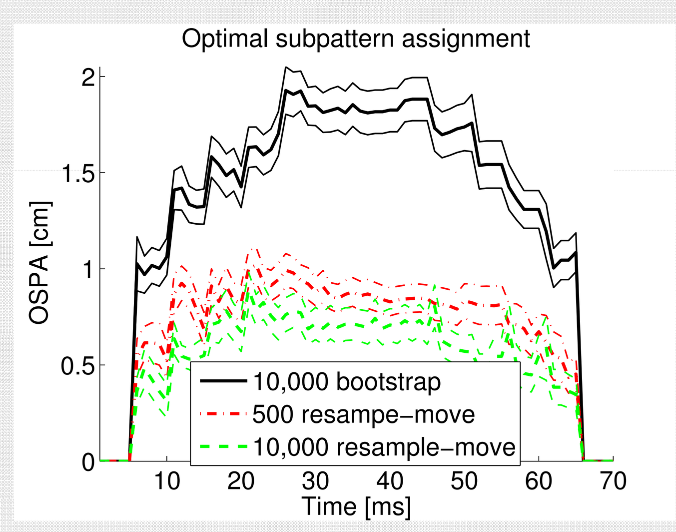


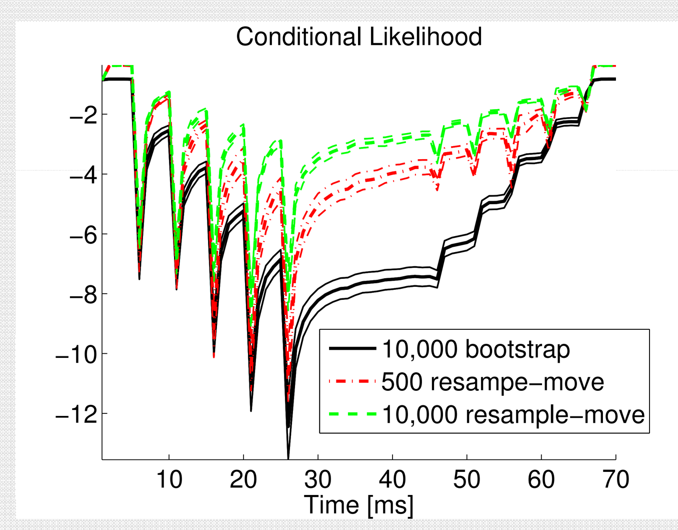
Sample source configuration and dipole time courses







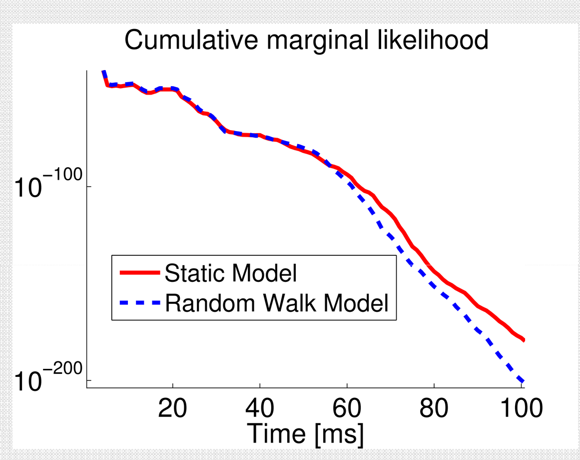




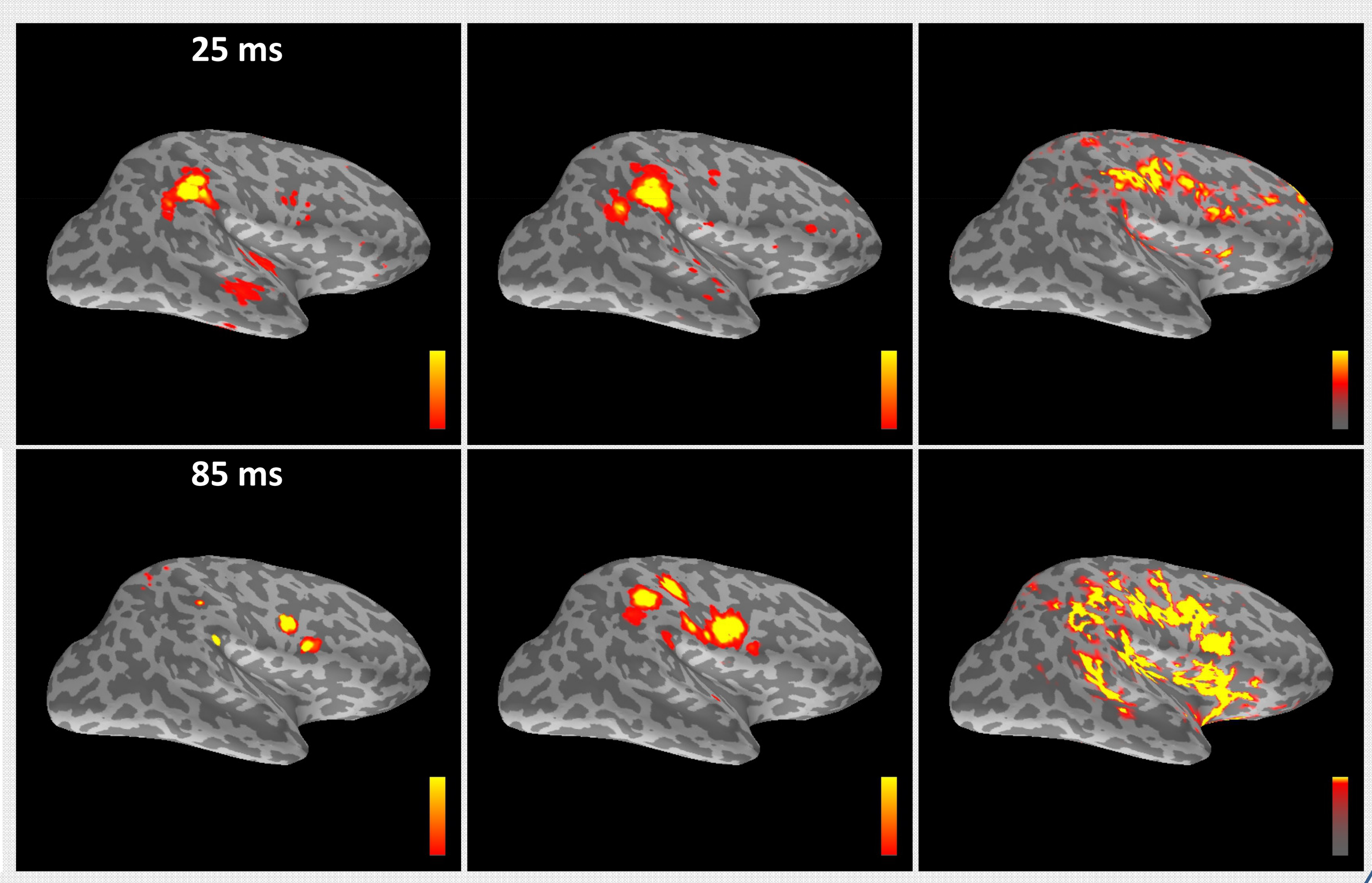
Somatosensory Evoked Fields

We applied the resample-move particle filter – based on the static dipole model – and (an improved version of) the bootstrap filter – based on the random walk model – to experimental data recorded during stimulation of the median nerve.
Visualisation of the posterior probability maps indicate that the static model gathers more information on the source location with time.

The cumulative marginal likelihood indicates that the static dipole model is a better representation of the underlying neurophysiological processes than the random walk model.



Resample-Move Static Model	Improved bootstrap Random Walk Model	dSPM Linear inverse



Conclusions

We have presented a new state space model for dynamic inference on current dipole parameters from MEG data. The model reflects the common neurophysiological interpretation of a current dipole as the activity of a small patch of cortex: dipoles appear and disappear, and dipole locations are fixed throughout their lifetimes. To compute the model, we have developed a resample-move type algorithm that overcomes the limitations of bootstrap filtering with our near-static parameters, and maintains diversity in the sample set for reasonable time spans.
Application to real data confirm that the static dipole model is comparatively more supported by the data than the previous random walk model.

References

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