REPORT: New Scaling Limits and other recent developments in Probability

University of Warwick, Monday 31st March – Friday 4th April 2008

Organized by: David Croydon, David Elworthy, Martin Hairer, Saul Jacka, Wilfrid Kendall, Vassili Kolokoltsov, Jon Warren

The purpose of this workshop was to provide opportunities for UK researchers to learn from and to collaborate with experts in four areas of modern probability, namely coagulation & fragmentation, Schramm-Loewner evolution, random matrices, and stochastic partial differential equations. These areas have been developing rapidly and in interlinked ways over the last few years: their international recognition may be best illustrated by the recent award of the Fields medal to Werner for his work on SLE. The growth of these ideas is expected to be one of the dominant features of early 21st century probability theory, and they should have a major impact on wide ranges of applied probability as we discover how to exploit the new capacity for modelling.

To maximize the value of the workshop for a wide variety of UK participants, a major part of the week was reserved for four mini-courses, each composed of 3 hour-long lectures, presented by Bertoin, Sheffield, Johannsson, and Flandoli. There were also 22 forty-minute research talks on associated subjects, presented by 9 overseas and 13 UK speakers, and a poster session at which local and PhD-level researchers presented their recent work.

The 69 participants were composed of 16 overseas participants (1 US, 15 Europe), 29 from UK non-Warwick, 14 from Warwick, and 10 PhD students (4 from Warwick and 6 elsewhere). Of the 29 UK non-Warwick tenured participants, 12 were senior academics (professorial or reader): while not all senior UK probabilists were able to attend, a major fraction were present. The UK EPSRC funded attendance by 13 of the overseas participants and 23 of the UK participants; PhD attendance was facilitated by underwriting from Warwick Statistics department and a grant from the London Mathematical Society. The low attendance from North America is attributable to clashes with teaching commitments according to the North American timetable.

Scientific content

Coagulation and fragmentation

Fragmentation is a natural phenomenon that can be observed at a great variety of scales. Conversely, coagulation processes describe the merging of 'clusters' of various sizes. While there is a clear intuitive link between both types of processes (coagulation should just be fragmentation 'running backwards') its mathematical meaning remained elusive until Pitman showed in 1999 that there was indeed a duality between a large class of coagulation and fragmentation processes. It is natural to consider self-similar instances of these processes, where the behaviour at smaller scales can be linked by a simple scaling transformation to the behaviour at larger scales. Bertoin's mini-course began by explaining the fundamental notion of coalescents, based on significant genetic applications to the Wright-Fisher model, and developed an exposition of recent joint work with Le Gall which connects generalized coalescents to certain stochastic flows of bridges. This enables a powerful and intuitive perspective on the behaviour of coalescence and fragmentation. Related research talks included presentations by Fourrier concerning the emergence of giant particles in coagulation, Norris on the derivation of Smoluchkowskii's coagulation equation as a scaling limit, and Goldschmidt on asymptotics of allele frequency using coalescence theory.
Schramm-Loewner evolution

While fragmentation and coagulation processes describe the decay / growth of structures on various scales, they are not designed to take into account their spatial structure. On the other hand, it is possible to construct spatial models that exhibit not only scale invariance, but invariance (or covariance) under any conformal transformation. In 1999 Schramm constructed a parametrised family of random curves in the plane that are now known as SLE\(_{\kappa}\) (Schramm-Loewner Evolution with parameter \(\kappa\)), and argued that these gave the only possible scaling limits for interfaces in two dimensional models with conformal invariance. Since then there has been great success in using SLE to study such models. Sheffield's mini-course centered on the notion of a random combinatorial surface with conformal structure, motivated by considerations of quantum gravity, and surveyed striking links with SLE in the scaling limit. Smirnov discussed connections between conformal invariance and planar Ising models, using the notion of a discrete holomorphic map. Beffara explored relationships between scaling limits for percolation and circle packings.

Random matrices

Random matrix theory (RMT) is often considered to have its origins in the 1950s in the work of the physicists Dyson and Wigner which was motivated by studying spectroscopic data in nuclear physics. Today, RMT has grown into a remarkable field with diverse applications across Physics and Mathematics. The probability distributions that describe the eigenvalues of random matrices belong to a class known as determinantal processes and provide fundamental mathematical models for complex systems in which there is repulsion between particles. As the size of the matrix grows, the limiting distributions and processes associated with the eigenvalues that arise possess surprising universality properties. There are striking relationships with random tilings and queueing system models. Johannsson's mini-course set out the foundations of random matrix theory based on random complex hermitean matrices, and described how these surprising relationships arise. König, Biane, and O'Connell discussed various developments centered around related Brownian motion results; Rouault expounded links with the classical random moment problem, while Martin and Ferrari presented work on related growth models.

Stochastic partial differential equations

Stochastic partial differential equations (SPDEs) arise naturally in a wide spectrum of areas. One area of particular current importance is the study of turbulence, now widely recognised as of the great theoretical challenges of the 21st century. A clear picture is still far beyond the grasp of rigorous mathematics, but over the last decade or so, the importance of stochastic models of turbulence has gained considerable recognition, one of the main reasons being that stochastic models often provide clean mathematical formulations for problems and / or phenomena that can only be formulated in a very vague way in a deterministic setting. Flandoli's mini-course developed the stochastic theory which provides an approach to the Navier-Stokes equations, with clear discussion of the substantial theoretical difficulties remaining. Cerrai discussed asymptotics for SPDEs arising in the study of reaction-diffusion; Brzezniak presented new work on the stochastic Landau-Lifshitz equation.

Other areas

It is of the nature of probability that major themes generate a wide range of possible links, and therefore appropriate for a workshop such as this to allow a broad representation of current work; indeed the workshop invitation specifically remarked that talks would not be narrowly limited to the prescribed themes. Other topics covered at the workshop were: Hambly on the local limit behaviour of random walks on graphs; Goldschied on random walks in random environments, Turner on scaling limits for diffusion-limited aggregation, Kyprianou on Levy processes, Morters on the parabolic Anderson model, Zambotti on random polymers, Winkel on scaling limits for random
Feedback and conclusions

Many participants commented favourably on the mini-courses in returned feedback forms, with particular praise being directed at the course on coagulation and fragmentation, for example “Derived much benefit from coagulation mini-course, this being a subject which I am learning right now.” Several participants reported significant research discussions or progress on collaborations. Others identified possible topics for future mini-courses, including free probability, applications in computer science, random graphs, Levy processes, and interface with statistics especially in respect of statistics for diffusion processes. There was a strong sense at the meeting that the vitality and activity in probability merited further meetings following this general formula, and plans are being laid for meetings of a similar format at Bath and then at Oxford over the next four years to address other major developments in the subject area; that people are prepared to put in the effort required in organizing such meetings is a very concrete endorsement of the success of the workshop. Indeed it is apparent from general remarks by attenders that the workshop has been very successful, and has fulfilled its aims of introducing UK researchers to the four topic areas and promoting collaborations within the UK and with overseas experts. The mini-course formula worked well in developing strong focus on specific linked subject areas, without dominating the workshop in terms of time. There is always a trade-off in organization of such a meeting between time for research talks and time for interaction; despite a substantial programme workshop attenders found time to interact and collaborate, and it was of particular importance that UK researchers had opportunity to make contact with leading researchers in the very strong French probability community. The support of the EPSRC and the LMS, and of Warwick Statistics Department (who underwrote the workshop at an early stage) is much appreciated.
New Scaling Limits and other recent developments in Probability: List of Participants

Larbi Alili  University of Warwick  Wolfgang König  Leipzig
David Applebaum  University of Sheffield  Andreas Kyprianou  University of Bath
Ismael Bailleul  Cambridge University  Huiling Le  University of Nottingham
Vincent Beffara  Lyons  Xue Mei Li  University of Warwick
Jean Bertoin  Paris  James Martin  Oxford University
Philippe Biane  Paris  Anthony Metcalfe  University College Cork
Karim BounebacheSaid  Paris  John Moriarty  University of Manchester
Zdzislaw Brzezniak  York University  Peter Morters  University of Bath
Sandra Cerrai  Florence  James Norris  Cambridge University
Stephen Connor  University of Warwick  Neil O'Connell  University of Warwick
David Croydon  University of Warwick  Tessy Papasviliou  University of Warwick
Denis Denisov  Heriot Watt  Juan Carlos Pardo Millan  University of Bath
Ron Doney  University of Manchester  Goran Peskir  University of Manchester
David Elworthy  University of Warwick  Jacques Printems  Paris
Patrik Ferrari  Berlin  Andrew Richards  Heriot-Watt
Sam Finch  University of Warwick  Markus Riedle  University of Manchester
Franco Flandoli  Pisa  Victor Rivero  Bath
Nicolas Fournier  Paris  Matthew Roberts  University of Bath
Lisa Fox  Oxford University  Alain Rouault  Versailles
Peter Friz  Cambridge University  Mladen Savov  Manchester
Ilya Goldshied  QMUL London  Tom Schmitz  Max-Planck Institute
Christina Goldshmidt  Oxford University  Rafael Serrano  York University
Nastasiya Grinberg  University of Warwick  Scott Sheffield  New York
Martin Hairer  University of Warwick  Nadia Sidorova  UCL
Ben Hambly  Oxford University  Stas Smirnov  Geneva
John Harris  University of Bristol  Perla Sousi  University of Cambridge
Simon Harris  University of Bath  Dario Spano  University of Warwick
Erika Hausenblad  Salzburg  Amanda Turner  Lancaster University
David Hobson  University of Warwick  Andrew Wade  University of Bristol
Robin Hudson  Loughborough University  Jon Warren  University of Warwick
Saul Jacka  University of Warwick  Peter Windridge  University of Warwick
Kurt Johansson  Sweden  Matthias Winkel  Oxford University
Jonathan Jordan  University of Sheffield  Yuxin Yang  University of Warwick
Wilfrid Kendall  University of Warwick  Lorenzo Zambotti  Paris
Vassili Kolokoltsov  University of Warwick
### New scaling limits and other recent developments in probability

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