REPORT: New Scaling Limits and other recent developments in Probability

University of Warwick, Monday 31st March – Friday 4th April 2008

Organized by: David Croydon, David Elworthy, Martin Hairer, Saul Jacka, Wilfrid Kendall, Vassili Kolokoltsov, Jon Warren

The purpose of this workshop was to provide opportunities for UK researchers to learn from and to collaborate with experts in four areas of modern probability, namely coagulation & fragmentation, Schramm-Loewner evolution, random matrices, and stochastic partial differential equations. These areas have been developing rapidly and in interlinked ways over the last few years: their international recognition may be best illustrated by the recent award of the Fields medal to Werner for his work on SLE. The growth of these ideas is expected to be one of the dominant features of early 21st century probability theory, and they should have a major impact on wide ranges of applied probability as we discover how to exploit the new capacity for modelling.

To maximize the value of the workshop for a wide variety of UK participants, a major part of the week was reserved for four mini-courses, each composed of 3 hour-long lectures, presented by Bertoin, Sheffield, Johannsson, and Flandoli. There were also 22 forty-minute research talks on associated subjects, presented by 9 overseas and 13 UK speakers, and a poster session at which local and PhD-level researchers presented their recent work.

The 69 participants were composed of 16 overseas participants (1 US, 15 Europe), 29 from UK non-Warwick, 14 from Warwick, and 10 PhD students (4 from Warwick and 6 elsewhere). Of the 29 UK non-Warwick tenured participants, 12 were senior academics (professorial or reader): while not all senior UK probabilists were able to attend, a major fraction were present. The UK EPSRC funded attendance by 13 of the overseas participants and 23 of the UK participants; PhD attendance was facilitated by underwriting from Warwick Statistics department and a grant from the London Mathematical Society. The low attendance from North America is attributable to clashes with teaching commitments according to the North American timetable.

Scientific content

Coagulation and fragmentation

Fragmentation is a natural phenomenon that can be observed at a great variety of scales. Conversely, coagulation processes describe the merging of `clusters' of various sizes. While there is a clear intuitive link between both types of processes (coagulation should just be fragmentation `running backwards') its mathematical meaning remained elusive until Pitman showed in 1999 that there was indeed a duality between a large class of coagulation and fragmentation processes. It is natural to consider self-similar instances of these processes, where the behaviour at smaller scales can be linked by a simple scaling transformation to the behaviour at larger scales. Bertoin's mini-course began by explaining the fundamental notion of coalescents, based on significant genetic applications to the Wright-Fisher model, and developed an exposition of recent joint work with Le Gall which connects generalized coalescents to certain stochastic flows of bridges. This enables a powerful and intuitive perspective on the behaviour of coalescence and fragmentation. Related research talks included presentations by Fournier concerning the emergence of giant particles in coagulation, Norris on the derivation of Smoluchkowski's coagulation equation as a scaling limit, and Goldschmidt on asymptotics of allele frequency using coalescence theory.

Schramm-Loewner evolution

While fragmentation and coagulation processes describe the decay / growth of structures on various scales, they are not designed to take into account their spatial structure. On the other hand, it is possible to construct spatial models that exhibit not only scale invariance, but invariance (or covariance) under any conformal transformation. In 1999 Schramm constructed a parametrised family of random curves in the plane that are now known as SLE_κ (Schramm-Loewner Evolution with parameter κ), and argued that these gave the only possible scaling limits for interfaces in two dimensional models with conformal invariance. Since then there has been great success in using SLE to study such models. Sheffield's mini-course centered on the notion of a random combinatorial surface with conformal structure, motivated by considerations of quantum gravity, and surveyed striking links with SLE in the scaling limit. Smirnov discussed connections between conformal invariance and planar Ising models, using the notion of a discrete holomorphic map. Beffara explored relationships between scaling limits for percolation and circle packings.

Random matrices

Random matrix theory (RMT) is often considered to have its origins in the 1950s in the work of the physicists Dyson and Wigner which was motivated by studying spectroscopic data in nuclear physics. Today, RMT has grown into a remarkable field with diverse applications across Physics and Mathematics. The probability distributions that describe the eigenvalues of random matrices belong to a class known as determinantal processes and provide fundamental mathematical models for complex systems in which there is repulsion between particles. As the size of the matrix grows, the limiting distributions and processes associated with the eigenvalues that arise possess surprising universality properties. There are striking relationships with random tilings and queueing system models. Johannsson's mini-course set out the foundations of random matrix theory based on random complex hermitean matrices, and described how these surprising relationships arise. Konig, Biane, and O'Connell discussed various developments centered around related Brownian motion results; Rouault expounded links with the classical random moment problem, while Martin and Ferrari presented work on related growth models.

Stochastic partial differential equations

Stochastic partial differential equations (SPDEs) arise naturally in a wide spectrum of areas. One area of particular current importance is the study of turbulence, now widely recognised as of the great theoretical challenges of the 21st century. A clear picture is still far beyond the grasp of rigorous mathematics, but over the last decade or so, the importance of stochastic models of turbulence has gained considerable recognition, one of the main reasons being that stochastic models often provide clean mathematical formulations for problems and / or phenomena that can only be formulated in a very vague way in a deterministic setting. Flandoli's mini-course developed the stochastic theory which provides an approach to the Navier-Stokes equations, with clear discussion of the substantial theoretical difficulties remaining. Cerrai discussed asymptotics for SPDEs arising in the study of reaction-diffusion; Brzezniak presented new work on the stochastic Landau-Lifshitz equation.

Other areas

It is of the nature of probability that major themes generate a wide range of possible links, and therefore appropriate for a workshop such as this to allow a broad representation of current work; indeed the workshop invitation specifically remarked that talks would not be narrowly limited to the prescribed themes. Other topics covered at the workshop were: Hambly on the local limit behaviour of random walks on graphs; Goldschied on random walks in random environments, Turner on scaling limits for diffusion-limited aggregation, Kyprianou on Levy processes, Morters on the parabolic Anderson model, Zambotti on random polymers, Winkel on scaling limits for random

trees, Harris on branching Brownian motion and Friz on rough path analysis.

Feedback and conclusions

Many participants commented favourably on the mini-courses in returned feedback forms, with particular praise being directed at the course on coagulation and fragmentation, for example "Derived much benefit from coagulation mini-course, this being a subject which I am learning right now." Several participants reported significant research discussions or progress on collaborations. Others identified possible topics for future mini-courses, including free probability, applications in computer science, random graphs, Levy processes, and interface with statistics especially in respect of statistics for diffusion processes. There was a strong sense at the meeting that the vitality and activity in probability merited further meetings following this general formula, and plans are being laid for meetings of a similar format at Bath and then at Oxford over the next four years to address other major developments in the subject area; that people are prepared to put in the effort required in organizing such meetings is a very concrete endorsement of the success of the workshop. Indeed it is apparent from general remarks by attenders that the workshop has been very successful, and has fulfilled its aims of introducing UK researchers to the four topic areas and promoting collaborations within the UK and with overseas experts. The mini-course formula worked well in developing strong focus on specific linked subject areas, without dominating the workshop in terms of time. There is always a trade-off in organization of such a meeting between time for research talks and time for interaction; despite a substantial programme workshop attenders found time to interact and collaborate, and it was of particular importance that UK researchers had opportunity to make contact with leading researchers in the very strong French probability community. The support of the EPSRC and the LMS, and of Warwick Statistics Department (who underwrote the workshop at an early stage) is much appreciated.

New Scaling Limits and other recent developments in Probability: List of Participants

Larbi Alili	University of Warwick	Wolfgang Konig	Leipzig
David Applebaum	University of Sheffield	Andreas Kyprianou	University of Bath
Ismael Bailleul	Cambridge University	Huiling Le	University of Nottingham
Vincent Beffara	Lyons	Xue Mei Li	University of Warwick
Jean Bertoin	Paris	James Martin	Oxford University
Philippe Biane	Paris	Anthony Metcalfe	University College Cork
Karim BounebacheSaid	Paris	John Moriarty	University of Manchester
Zdzislaw Brzezniak	York University	Peter Morters	University of Bath
Sandra Cerrai	Florence	James Norris	Cambridge University
Stephen Connor	University of Warwick	Neil O'Connell	University of Warwick
David Croydon	University of Warwick	Tessy Papasiviliou	University of Warwick
Denis Denisov	Heriot Watt	Juan Carlos Pardo Millan	University of Bath
Ron Doney	University of Manchester	Goran Peskir	University of Manchester
David Elworthy	University of Warwick	Jacques Printems	Paris
Patrik Ferrari	Berlin	Andrew Richards	Heriot-Watt
Sam Finch	University of Warwick	Markus Riedle	University of Manchester
Franco Flandoli	Pisa	Victor Rivero	Bath
Nicolas Fournier	Paris	Matthew Roberts	University of Bath
Lisa Fox	Oxford University	Alain Rouault	Versailles
Peter Friz	Cambridge University	Mladen Savov	Manchester
llya Goldshied	QMUL London	Tom Schmitz	Max-Planck Institute
Christina Goldshmidt	Oxford University	Rafael Serrano	York University
Nastasiya Grinberg	University of Warwick	Scott Sheffield	New York
Martin Hairer	University of Warwick	Nadia Sidorova	UCL
Ben Hambly	Oxford University	Stas Smirnov	Geneva
John Harris	University of Bristol	Perla Sousi	University of Cambridge
Simon Harris	University of Bath	Dario Spano	University of Warwick
Erika Hausenblad	Salzburg	Amanda Turner	Lancaster University
David Hobson	University of Warwick	Andrew Wade	University of Bristol
Robin Hudson	Loughborough University	Jon Warren	University of Warwick
Saul Jacka	University of Warwick	Peter Windridge	University of Warwick
Kurt Johansson	Sweden	Matthias Winkel	Oxford University
Jonathan Jordan	University of Sheffield	Yuxin Yang	University of Warwick
Wilfrid Kendall	University of Warwick	Lorenzo Zambotti	Paris
Vassili Kolokoltsov	University of Warwick		

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	Monday 31.03.08	Tuesday 01.04.08	Wednesday 02.04.08	Thursday 03.04.08	Friday 04.04.08
0900-1000		Bertoin	Johansson	Johansson	Johansson
1000-1100		Flandoli	Flandoli	Bertoin	Bertoin
1100-1130	Registration	Coffee			
1130-1210	Smirnov	Biane	Zambotti	Beffara	Brzezniak
1210-1250	Hambly	Moerters	O'Connell	Norris	Friz
1250-1415	Lunch				
1415-1515	Flandoli	Sheffield	Sheffield	Sheffield	
1515-1555	König	Cerrai	Fournier	Rouault	
1555-1615	Tea				
1615-1655	Turner	Ferrari	Winkel	Goldschmidt	
1655-1735	Kyprianou	Goldscheid	Harris	Martin	
		Poster session			
	Dinner				