The 2nd London-Oxford-Warwick Financial Mathematics Workshop

Dates: **Mon 4th April** 2022 and **Tue 5th April** 2022.

Venue: MS.01 (Zeeman building), Warwick campus.

Organising Committee

London: John Armstrong, Roxana Dumitrescu, Blanka Horvath
Oxford: Sam Cohen
Warwick: Martin Herdegen, Gechun Liang

Conference Schedule

Monday 4 April	MS01 Zeeman Building, Warwick Campus
9:45-10:15	conference registration
10:15-11:15	Keynote talk: **Luitgard Veraart (LSE)** Systemic Risk in Markets with Multiple Central Counterparties
11:15-12:00	Contribution talk: **Raymond Pang (LSE)** Assessing and mitigating fire sales risk under partial information
12:00-13:30	lunch break
13:30-14:15	Invited talk: **Alex Tse (UCL)** Portfolio Selection, Periodic Evaluations and Risk Taking
14:15-15:00	Contribution talk: **Justin Gwee (LSE)** Equilibrium under Proportional Transaction Costs
15:00-15:30	coffee break
15:30-16:15	Contribution talk: **Osian Shelley (Warwick)** On Tauberian Theorems for Generalised Signed Measures
16:15-17:00	Contribution talk: **Andrei Ionescu (KCL)** Pathwise and model-free gamma hedging of exotic options using Malliavin calculus and rough path theory

19:00-21:00	conference dinner at Scarman Conference centre
Tuesday 5 April	MS01 Zeeman Building, Warwick Campus
9:00-10:00	Keynote talk: **Albina Danilova (LSE)** Option Markets in the Age of Robinhood (and before).
10:00-10:30	coffee break
10:30-11:15	Contribution talk: **Yuwei Wang (Warwick)** Predictable Forward Performance Processes: Infrequent Evaluation and Robo-Advising Applications
11:15-12:00	Contribution talk: **Huining Yang (Oxford)** Learning in Linear Quadratic Framework: From Single-agent to Multi-agent
12:00-13:30	lunch break
13:30-14:15	Invited talk: **Pietro Siorpaes (Imperial)** How to replace nets with sequences when dealing with martingales
14:15-15:00	Contribution talk: **Ruiqi Liu (Warwick)** The Optimal Control of Inventory and Production for a Hybrid Energy Producer
15:00	conference ends

Abstract

1. **Albina Danilova (LSE)** danilova@lse.ac.uk

Option Markets in the Age of Robinhood (and before).

We consider an economy in which some agents do not continuously hedge their position in derivative assets using the underlying assets market – i.e. we study the effects of an imbalanced derivate market. We show that, even in the presence of complete markets, the imbalance significantly alters the equilibrium price process of the underlying assets: risk premia and volatility become stochastically time varying, hence option implied volatility is characterized by smile and smirk pat- terns, momentum-like price dynamics arise as well as price spillovers across under- lying assets. Moreover, the derivative imbalance generates self-fulfilling equilibria, e.g. if the imbalance takes the form of a bet on an increase in asset volatility, then the equilibrium volatility does increase. Finally, since our formulation is extremely general, our results also apply to segmented markets where some investments are achievable only via financial intermediation.

2. **Justin Gwee (LSE)** J.Gwee@lse.ac.uk

Equilibrium under Proportional Transaction Costs

We study an equilibrium in which two heterogeneous agents are subjected to proportional transaction costs. We extend the setting of Gonon, Muhle-Karbe and Shi in which endowment rates are scalar multiples of a Brownian motion to twice continuously differentiable functions of a diffusion X (in which case the frictionless optimiser is also a function of X) which capture the sensitivity of the endowment rates to the state of the economy represented by X. The HJB equation associated with this singular stochastic control problem characterises a free boundary problem whose solution satisfies an ODE with variable coefficients in the interior as well as gradient constraints and smooth-pasting conditions at the boundaries. We solve for the unique equilibrium return rate and establish conditions for the existence of these boundaries when the frictionless optimiser is strictly increasing, or strictly increasing and then strictly decreasing in X. Finally, we derive small transaction cost asymptotics for the boundaries.

3. **Andrei Ionescu (KCL)** andrei.ionescu@kcl.ac.uk

Pathwise and model-free gamma hedging of exotic options using Malliavin calculus and rough path theory

We use the rough path integral to model hedging strategies without a probability model. We define gamma hedging in a more general sense by using both rough path theory and Malliavin calculus. The Fundamental Theorem of Derivatives Trading (FTDT) provides an expression for the delta hedging error in a probabilistic setting when the trader misestimates the volatility of the underlying. The FTDT is important because it proves that delta hedging is robust in the Black-Scholes model. In a probability-free framework, we prove that gamma hedging is necessary (and in fact sufficient) to keep said robustness. Classically, the gamma of an option is the second partial derivative of the option price with respect to the underlying. This definition works for European options or price processes that are Markov with respect to the underlying, but it does not apply for most exotic options. For Malliavin differentiable payoffs, we define the “rough” gamma of an option as the Gubinelli derivative of its delta. The delta itself is well defined via the Clark-Ocone formula. We prove that the existence of a rough gamma is equivalent to being able to hedge a payoff path-wise. For a class of payoffs, we prove a relationship between the rough gamma and a second order Malliavin derivative. We give examples of exotic options are path-wise hedgeable: notably Asian options and single/double barrier options with “smoothed” barrier levels. We present a path-wise generalisation of the Fundamental Theorem of Derivatives Trading. Finally, we show that a large class of payoffs can be superhedged path-wise by a sequence of rough hedgeable payoffs in the sense that one can choose this sequence to have a risk-neutral price that is arbitrarily close to the risk-neutral price of the original option. We believe that our results demonstrate the importance of gamma hedging as a truly robust, path-wise hedge.

4. **Ruiqi Liu (Warwick)** Ruiqi.liu@warwick.ac.uk

The Optimal Control of Inventory and Production for a Hybrid Energy Producer

We study a continuous-time, infinite horizon optimal energy storage and production problem. The primary source of production is modelled as an uncontrolled one-dimensional diffusion process with general dynamics. By controlling the secondary source of production and total energy output, which are both bounded variation processes, we aim to optimize the storage level under a general running reward function and maximize the profit generated from the production. Through associating the control problem with Dykin game, the optimal control is closely related to two free boundaries, and we show that one can be directly computed and the other is characterized via an integral equation. After establishing the smooth-pasting principle on these boundaries, a viscosity approach is used to prove the smoothness of the value function, which leads to the verification of the proposed optimal control.

5. **Raymond Pang (LSE)** r.pang@lse.ac.uk

Assessing and mitigating fire sales risk under partial information

We consider the problem of assessing and mitigating fire sales risk for banks under partial information. Using data from the European Banking Authority's stress tests, we consider the matrix of asset holdings of different banks. We first analyse fire sales risk under both full and partial information using different matrix reconstruction methods. We then investigate how well some policy interventions aimed at mitigating fire sales risk perform if they are applied based on only partial information. We compare the performance of policy interventions under full and partial information. We find that even under partial information, using suitable network reconstruction methods to decide on policy interventions can significantly mitigate risk from fire sales. Furthermore, we show that some interventions based on reconstructed networks significantly outperform ad hoc methods that decide on interventions only based on the size of an institution and do not account for overlapping portfolios.

6. **Pietro Siorpaes (Imperial)** p.siorpaes@imperial.ac.uk

How to replace nets with sequences when dealing with martingales

Given a uniformly integrable martingale (f_i)_{i\in I} indexed by a net I, we identify a sequence (i_n)_n of elements of I such that the limit of (f_{i_n})_n closes the whole net (f_i)_i. We then present applications of this result. We show how known results about converging martingales indexed by a net can be proved by using only sequences, thus making them more elementary. In particular, this allows to replace Kolmogorov's definition of conditional expectation E[f|G]

7. **Osian Shelley (Warwick)** o.d.shelley@warwick.ac.uk

On Tauberian Theorems for Generalised Signed Measures

In this talk, we describe an extension of Karamata's Tauberian to work for generalised signed measures and show its application in the context of stochastic control. On route to this goal, we highlight useful properties of vague convergence and its relationship to the convergence of distribution functions.

8. **Alex Tse (UCL)** Alex.tse@ucl.ac.uk

Portfolio Selection, Periodic Evaluations and Risk Taking

We present a continuous-time portfolio selection problem faced by an agent with S-shaped preference who maximizes the utilities derived from the portfolio's periodic performance over an infinite horizon. The periodic reward structure creates subtle incentive distortion. In some cases, local risk aversion is induced which discourages the agent from risk taking in the extreme bad states of the world. In some other cases, eventual ruin of the portfolio is inevitable and the agent underinvests in the good states of the world to manipulate the basis of subsequent performance evaluations. We outline several important elements of incentive design to contain the long-term portfolio risk. This is a joint work with Harry Zheng.

9. **Luitgard Veraart (LSE)** L.Veraart@lse.ac.uk

Systemic Risk in Markets with Multiple Central Counterparties

We provide a framework for modelling risk and quantifying payment shortfalls in cleared markets with multiple central counterparties (CCPs). Building on the stylised fact that clearing membership is shared among CCPs, we show that stress in this shared membership can transmit across markets through multiple CCPs. We provide stylised examples to lay out how such stress transmission can take place, as well as empirical evidence to illustrate that the mechanisms we study could be relevant in practice. Finally, we discuss how the framework can be used to enhance CCP stress-testing. This is joint work with Inaki Aldasoro (Bank for International Settlements).

10. **Yuwei Wang (Warwick)** yuwei.wang.2@warwick.ac.uk

Predictable Forward Performance Processes: Infrequent Evaluation and Robo-Advising Applications

We study discrete-time predictable forward processes when trading times do not coincide with performance evaluation times in the binomial tree model for the financial market. The key step in the construction of these processes is to solve a linear functional equation of higher order associated with the inverse problem driving the evolution of the predictable forward process. We provide sufficient conditions for the existence and uniqueness and an explicit construction of the predictable forward process under these conditions. Furthermore, we show that these processes are time-monotone in the evaluation period. Finally, we argue that predictable forward preferences are a viable framework to model preferences for robo-advising applications and determine an optimal interaction schedule between client and robo-advisor that balances a tradeoff between increasing uncertainty about the client's beliefs on the financial market and an interaction cost.

11. **Huining Yang (Oxford)** huining.yang@maths.ox.ac.uk

Learning in Linear Quadratic Framework: From Single-agent to Multi-agent

Linear quadratic framework is widely studied in the literature of stochastic control and game theory due to its simple structure, tractable solution, and various real-world applications. In this talk we discuss the theoretical convergence of policy gradient methods, one of the most popular reinforcement learning algorithms, in several linear quadratic problems. In the single agent setting, we show the global convergence of such methods to the optimal solution in the setting of known and unknown parameters. We also illustrate the performance of the algorithms in the optimal liquidation problem. In the multi-agent linear quadratic games,

we show that the policy gradient method enjoys global convergence to the Nash equilibrium provided that there is a certain amount of noise in the system. The noise can either come from the underlying dynamics, or carefully designed explorations from the agents. This talk is based on joint work with Prof. Ben Hambly (University of Oxford) and Prof. Renyuan Xu (University of Southern California).

Funded by