Week 3 Personal Tutor Activity

These activities are to be completed in groups of two or three.

Part 1

Consider the following problem.

Ten passengers board an aeroplane with ten numbered seats. Passenger 1 is assigned seat 1, passenger 2 is assigned seat 2, and so on.

Passenger 1 chooses one of the ten seats uniformly at random. Each subsequent passenger sits in their assigned seat if it is free. If it is occupied, they choose uniformly at random from the remaining seats.

What is the probability that passenger 10 gets seat 10?

Activity. In your groups, work together to attempt this problem.

While attempting the problem, make a note of:

- what you did when you became stuck and what helped you progress;
- any gaps or claims that still need justification;
- anything you found difficult to explain clearly.

At the end of this part, write a brief account of your group's progress. Distinguish clearly between:

- what you have established;
- what you currently conjecture;
- what remains uncertain or unjustified.

You are not expected to have a complete solution.

Part 2

The following is an AI-generated solution to the above problem.

> The probability is $\frac{1}{2}$.
>
> If passenger 1 chooses seat 1, every later passenger can sit in their assigned seat, so passenger 10 gets seat 10. If passenger 1 chooses seat 10, passenger 10 cannot get their assigned seat.
>
> Suppose instead that passenger 1 chooses an intermediate seat, say seat j. Passengers $2, \ldots, j-1$ sit correctly, but passenger j is displaced and must choose another available seat. If they choose the seat belonging to a later passenger, that passenger will eventually be displaced in turn. This creates a chain of displaced passengers.
>
> The chain continues until somebody selects either seat 1 or seat 10. If seat 1 is selected first, the remaining passengers can sit correctly and passenger 10 gets seat 10. If seat 10 is selected first, passenger 10 does not.
>
> Throughout the process, seats 1 and 10 remain available until one of them is chosen. Whenever a displaced passenger chooses randomly, the two seats are treated in exactly the same way. They are therefore equally likely to be selected first.
>
> Hence
>
> $$\mathbb{P}(\text{passenger 10 gets seat 10}) = \frac{1}{2}.$$

Read through the above solution. Highlight:

- steps that are adequately explained;
- claims whose justification is incomplete or unclear;
- anything you would add, remove or reorganise before submitting it.

Part 3

Using your annotations and your work from Part 1, write a complete solution to the original problem in your own words. Your solution should make clear:

- how a chain of displaced passengers develops;
- why the chain must end;
- why its two possible endings correspond to success and failure;
- why those endings have equal probability;

- how the conclusion follows.

Part 4

After comparing your work with the model solution, discuss the following:

1. What valuable parts of the learning process would you have missed if you had received the response immediately?
2. What can you try when you are stuck before looking for a complete solution?
3. How can you tell the difference between recognising a convincing solution and understanding it well enough to explain it yourself?
4. What does a written solution need to show a lecturer about your understanding?
5. What will you do differently the next time you encounter a difficult mathematical problem?