ST2220

UNIVERSITY OF WARWICK

FIRST YEAR EXAMINATION: April Examination Paper 2017

GAMES, DECISIONS AND BEHAVIOUR

Time allowed: 2 hours

ANSWER 3 QUESTIONS.

Full marks will be obtained by correctly answering THREE complete questions. Candidates may attempt all questions. Marks will be awarded for the best three answers only.

Calculators are not permitted in this examination.

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1. (a) (11 points) You consider investing an amount $\pounds C > \pounds 100$ in stocks of the company *Xax* for a period of three months. There is a possibility for *Xax* to merge with *Yoy*, in which case you expect the investment to appreciate £100, otherwise you expect it to depreciate £200. Alternatively to investing you would leave your $\pounds C$ in a bank account at zero interest.

   i. Set up the decision space and the outcome space, and draw a decision tree showing the considerations explained above.

   ii. You consider asking a market expert about the possibility that the companies *Xax* and *Yoy* merge. Describe a method for eliciting the probability for this event from the market expert using bets.

   iii. Using the expected monetary value strategy, determine a threshold probability p_0 making it is worthwhile to put $\pounds C$ into *Xax* stocks if the probability for the two companies to merge is larger than p_0. Describe the relationship between p_0 and C.

   iv. Modify the previous calculation to obtain a modified minimal probability p_0^U using expected utility maximisation with a utility function $U(x) = \log x$. Describe the relationship between p_0 and C.

   (b) (6 points) Consider the following reward matrix in a zero-sum game:

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$$

   i. What does it mean for this game to be separable?

   ii. Determine under which conditions it is separable.

   (c) (3 points) In a survey conducted at the *Second International Congress on Forecasting in July of 1982* 115 professional analysts, employed by industry, universities, or research institutes were split into two groups and asked to rate the probability of a statement about the future.

   The first group was given the statement:

   > *A complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983.*

   The second group was given the statement:

   > *A Russian invasion of Poland, and a complete suspension of diplomatic relations between the USA and the Soviet Union, sometime in 1983.*

   Estimates of probability were low for both statements, but significantly lower for the first group than for the second.

   i. Name and explain the underlying fallacy that causes the experts' probability statements. Why is this wrong from the point of view of normative probability?

   ii. Suggest which probabilities the experts have confused in this example?

Continues on next page

2. (a) (7 points) Model a sequence of n fair coin tosses by a sequence of independent random variables X_i $(i = 1, \ldots, n)$ with values in $\{0, 1\}$ and with $P(X_i = 1) = P(X_i = 0) = 1/2$ for all $i = 1, \ldots, n$.

   i. Let Z_r be the number of runs of heads or of tails of length r in n tosses of the coin. Calculate the expectation of Z_r for $r, n \in \mathbb{N}$ with $1 \leq r \leq n$.

   ii. Explain the relevance of the length of the longest run in a binary sequence for judging whether or not the sequence could have been obtained by independent tosses of a fair coin.

(b) (7 points) Answer the following questions using the model of a binary sequence of random variables X_i $(i = 1, 2, \ldots)$ with values in a state space $S = \{s_1, s_2\}$.

   i. What is the 'gambler's fallacy'? Give a simple condition under which this belief is incorrect.

   ii. How could an incorrect interpretation of the law of large numbers lead to the gambler's fallacy? *Use no more than 20 words.*

   iii. Give a real world example (excluding casinos!) where the gambler's fallacy may interfere with objective judgement.

   iv. In a study by Gilovich et al. (1985) on basketball fans 91% agreed that a player has 'a better chance of making a shot after having just made his last two or three shots'. This is the opposite belief to what the gambler's fallacy suggest, but the latter is always widely spread. How can these two principle both exist in parallel? State two bullet crucial points that shed light on this apparent paradox. *Use no more than 40 words.*

(c) (6 points) Consider a zero sum game with the following payoff matrix for player 1.

	δ_1	δ_2	δ_3	δ_4	δ_5
d_1	0	1	5	6	8
d_2	10	6	5	2	3

   i. What is player 1's maximin mixed strategy? *It is helpful to use a graphical method.*

   ii. What is player 2's maximin mixed strategy?

   iii. What is the value of this game?

Continues on next page

3. (a) (10 points) The 1998 German thriller film *Lola rennt* (engl.: Run Lola Run) by Tom Tykwer is based on the philosophy of Krzysztof Kieślowski's 1981 film *Przypadek* (engl.: Blind Chance) which examines the role of randomness in an individual's life. The plot starts with a woman called Lola receiving a frantic phone call from her boyfriend. He needs her to raise 100,000 *Marks* in 20 minutes to save his life. (He is a small-time criminal who accidentally left this sum in an underground train on his way to deliver it to the boss of his gang.) The film presents three separate storylines. In her third run, Lola enters a casino where roulette is played.

   In a roulette game, each number has a chance of $1/37$ to win. Staking x on a single number results in a payout of $35x$ if the number wins, and the stake is returned. If the number loses there is no payout and the stake is lost.

   Lola has enough money to obtain a 100 *Marks* chip. She considers the following betting strategy: Stake the marks chip on the single number 20. If the numbers wins, stake everything on the single number 20 again.

   i. If 20 comes up in each round, how much money will she have raised?

   ii. Should Lola enter this game? Answer this question using the expected monetary value approach. (*To avoid unnecessary calculations, do not carry out all the numerical operations immediately. Answer the question by comparing the expressions instead.*)

   iii. Does that coincide with common sense?

   iv. What utility function would you advise Lola to use in this circumstance, assuming she wants to keep her boyfriend alive and this is her sole concern?

   v. Using your utility function from part (iv), what is the expected utility of the two possible decisions? And what is optimal decision following expected utility?

(b) (6 points) Consider a lottery in which, with probability p you win £100 and with probability $1-p$ you lose £1. You may decide to enter or not to enter.

   i. What is the maximax decision as a function of p?

   ii. What is the expected monetary value decision rule as a function of p?

   iii. What is the maximin decision as a function of p?

(c) (4 points) In a study with American UG students the following question was asked:

   Consider a regular six-sided die with four green faces and two red faces. The die will be rolled 20 times and the sequences of greens (G) and reds (R) will be recorded. You are asked to select one sequence, from a set of three, and you will win \$25 if the sequence you chose appears on successive rolls of the die. Please circle the sequence of greens and reds on which you prefer to bet.

   RGRRR   *GRGRRR*   *GRRRRR*

   i. Using normative theory of probability, which of the three choices gives you the highest chance of winning?

   ii. A large majority of students chose the second option. Give reasons why they would do so? Explain which concepts they confuse.

Continues on next page

4. (a) (8 points) Consider three choices between two lotteries each.

A: Which of the following options do you prefer?
(a) 25% chance to win $30 *(b) 20% chance to win $45*

B: Which of the following options do you prefer?
(a) sure win of $30 *(b) 80% chance to win $45*

C: In the first stage, there is a 75% chance to end the game without winning anything, and a 25% chance to move into the second stage. If you reach the second stage you have a choice:
(a) a sure win of $30 *(b) 80% chance to win $45*
Your choice must be made before the game starts.

i. For each of these three choices, would the expected monetary value approach recommend to select (a) or to select (b)? Give explicit calculations to support your answers.

ii. Studies asking people to make these choices have shown that in A a majority of people prefers (b) and in B and in C a majority of people prefers (a). For each of these three choices, discuss the potential reasons and principles underlying it. Comment also on the questions whether or not the majority's choices are consistent.

iii. Now assume that people apply a utility function to the payoffs and use expected utility theory to make their choices. How would that impact the choices in A, B and C, and how are these choices related?

(b) (7 points) The lexicographical order relation on $\mathcal{R}^2$ is defined as follows

$$(x_1, x_2) \succ (y_1, y_2) \quad \Longleftrightarrow \quad x_1 > y_1 \ \vee \ (x_1 = y_1 \ \wedge \ x_2 > y_2).$$

(This is using the notation $x = (x_1, x_2)$ for $x \in \mathcal{R}^2$.)

i. Show that the lexicographical order relation is complete

ii. Show that the lexicographical order relation is transitive.

iii. Does it have the Archimedean property? Proof it or demonstrate that it is not true.

(c) (5 points) As part of their health check during the hiring process, a company secretly conducts tests for certain recreational drugs used by about 1% of the relevant population. The test are able to detect drug use with certainty, but also have a probability of falsely detecting drug use in 10% of people who do not actually use drugs.

If someone tests positive the company will not hire the applicant. They argue that based on the test, they can be at least 90% sure the person used drugs based on their test.

i. Calculate the probability that an applicant who tested positive is actually a drug user.

ii. The company has stated this number is much higher. Which fallacy is behind this?

End

Solutions ST222, April 2017

1. (a) **[11 points: Typical type of question about EMV and EUT decision making, from a past paper.]**

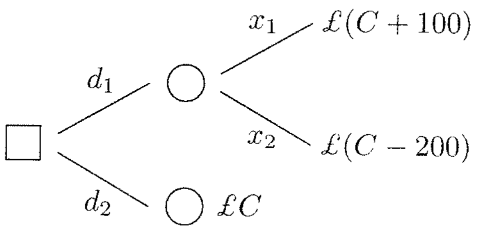
You consider investing an amount $£C > £100$ in stocks of the company *Xax* for a period of three months. There is a possibility for *Xax* to merge with *Yoy,* in which case you expect the investment to appreciate £100, otherwise you expect it to depreciate £100. Alternatively to investing you would leave your $£C$ in a bank account at zero interest.

i. **[3 pts]** Set up the decision space and the outcome space, and draw a decision tree showing the considerations explained above.

Decision space $D = \{d_1, d_2\}$ with d_1 ="invest £C in *Xax*" and d_2 ="leave £C in bank account".

Outcome space $\Omega = \{\omega_1, \omega_2\}$ with ω_1 ="stock appreciates £100" and ω_2 ="stock depreciates £100".

Decision tree:



ii. **[3 pts]** For elicitation by ball-in-bag bet define the event A = "*Xax* merges with *Yoy*" and pick $M > 0$.

Let b_A be the bet that pays $£M$ if A happens and nothing otherwise.

Let $b(n, r)$ be the bet that pays $£M$ if a ball drawn at random from a bag with r black balls and $n - r$ not black balls is black.

Start with $r =$ and increase to r^* until the market expert prefers $b(n, r^*)$ to b_A. Then the market expert's probability p_E for A fullfills

$$(r^* - 1)/n \leq p_E \leq r^*/n$$

Choose n large enough to obtain he precision you desire.

Alternative answers are accepted (e.g. using spinners).

iii. **[3 pts]** Define the gain function $G(d_i, x_j)$ ="value of investment after three months" for $i = 1, 2$ and $j = 1, 2$. For simplicity, drop £ from the calculations.

$$G(d_1, x_1) = C + 100,\ G(d_1, x_2) = c - 200,\ G(d_2, x_1) = G(d_2, x_2) = C$$

Let p be the probability for A. Then

$$E[G(d_1, \cdot)] = p(C + 100) + (1 - p)(C - 200) = 200p + C - 100$$
$$E[G(d_2, \cdot)] = C$$

EMV means to maximise expected gain. So, choosing d_1 is equivalent with

$$E[G(d_1, \cdot)] > E[G(d_2, \cdot)]$$

which is equivalent with

$$p(C+100) + (1-p)(C-200) > C$$

which resolves to $p > 2/3$. Hence the threshold is $p_0 = 2/3$.

p_0 does not depend on C.

iv. **[2 pts]** Similar to above, but now the condition becomes

$$E[U(G(d_1, \cdot))] > E[U(G(d_2, \cdot))] \qquad (*)$$

Using algebra,

$$\begin{aligned}(*) \quad &\Leftrightarrow p\log(C+100) + (1-p)\log(C-200) > \log C \\ &\Leftrightarrow p \log \frac{C+100}{C-200} > \log \frac{C}{C-200}\end{aligned}$$

Which yields a threshold of

$$p_0 = \log \frac{C}{C-200} \Big/ \log \frac{C+100}{C-200}$$

Now, p_0 does depend on C, because of the nonlinear utility of money.

(b) **[6 points: Recalling and applying definition of seperability, derivation with matrix algebra similar to question on exercise sheet.]**

i. **[2 pts]** Separability for this 2×2 zero-sum game means that there are u, v, x and y such that

$$\begin{pmatrix} \text{a} & \text{b} \\ 0 & \text{d} \end{pmatrix} = \begin{pmatrix} \text{u+x} & \text{u+y} \\ \text{v+x} & \text{v+y} \end{pmatrix}$$

ii. **[4 pts]** The condition above can be expressed as a system of linear equations with 4 variables:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{u} \\ \text{v} \\ \text{x} \\ \text{y} \end{pmatrix} = \begin{pmatrix} \text{a} \\ \text{b} \\ 0 \\ \text{d} \end{pmatrix}$$

Swapping second with third row, subtracting first from former second row, and subtracting third from fourth row yields

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} \text{u} \\ \text{v} \\ \text{x} \\ \text{y} \end{pmatrix} = \begin{pmatrix} \text{a} \\ 0 \\ \text{b-a} \\ \text{d} \end{pmatrix}$$

If $d = b - a$, the system has infinitely many solutions. Hence, the game it is separable.
If $d \neq b - a$, the system has no solutions. Hence, the game is not separable.

(c) [**3 points: Fallacy involving conditional probabilities, example from lecture.**]

i. [**2 pts**] This is the conjunction fallacy. It means that people overrate the probability of a conjunction of two events relative to the probability of one of them, that is, that judge $P(A \cap B) > P(A)$. But $A \cap B \subset A$, and hence $P(A \cap B) \leq P(A)$.

ii. [**1 pt**] Conjunctions involving a (hypothetical) cause are particularly prone to fallacies. The experts may intuitively asses the probability of the effect given the cause rather than the joint probability, e.g.:

$$P(\text{suspension of USA-SU relationship} \,|\, \text{Russian invasion of Poland})$$

instead of

$$P(\text{suspension of USA-SU relationship and Russian invasion of Poland})$$

2. (a) [**7 points: Example from lecture/homework using normative probability and linking this to perception of probability.**]

i. [**5 pt**] Let $Z_r^{(1)}$ be the number or runs of $1's$ and $Z_r^{(0)}$ be the number or runs of $0's$. Considering the coin is fair, by symmetry, $Z_r^{(0)} = Z_r^{(1)}$ and

$$E[Z_r] = E[Z_r^{(1)} + Z_r^{(0)}] = E[Z_r^{(1)}] + E[Z_r^{(0)}] = 2 \cdot E[Z_r^{(1)}]$$

The case $r = n$ is trivial:
Using independence of the coin tosses and $P(X_i = 1) = P(X_i = 0) = 0.5$,

$$E[Z_n^{(1)}] = E[1_{\{X_1 = X_2 = \ldots = X_n = 1\}}] = P(X_1 = X_2 = \ldots = X_n = 1) = 2^{-n}$$

and therefore

$$E[Z_n] = 2^{-n+1}$$

Now consider the case $r \leq n - 1$:
Splitting the event of a run into three categories (at the beginning, at the end, somewhere in the middle),

$$\begin{aligned}
E[Z_r^{(1)}] = E\Big[& 1_{\{X_1 = X_2 = \ldots = X_r = 1, X_{r+1} = 0\}} \\
& + 1_{\{X_{n-r} = 0, X_{n-r+1} = X_{n-r+2} = \ldots = X_n = 1\}} \\
& + \sum_{i=2}^{n-r} 1_{\{X_i = X_{i+1} = \ldots = X_{i+r-1} = 1, X_{i-1} = X_{i+r} = 0\}}\Big] \\
= P(X_1 & = X_2 = \ldots = X_r = 1, X_{r+1} = 0) \\
& + P(X_{n-r} = 0, X_{n-r+1} = X_{n-r+2} = \ldots = X_n = 1) \\
& + \sum_{i=2}^{n-r} P(X_i = X_{i+1} = \ldots = X_{i+r-1} = 1, X_{i-1} = X_{i+r} = 0)
\end{aligned}$$

Using independence and $P(X_i = 1) = P(X_i = 0) = 0.5$,

$$= 2^{-(r+1)} + 2^{-(r+1)} + (n - r - 2 + 1) \cdot 2^{-(r+2)}$$
$$= 4 \cdot 2^{-r-2} + (n - r - 1) \cdot 2^{-r-2}$$
$$= (n - r + 3) \cdot 2^{-r-2}$$

Hence,

$$E\big[Z_r\big] = 2 \cdot (n - r + 3) \cdot 2^{-r-2} = (n - r + 3) \cdot 2^{-r-1}$$

ii. **[2 pts]** Due to the Gambler's fallacy, people assume there need to be more alternations than is actually typical for a sequence of independent random variables. In other words, they underestimate the length of runs, in particularly the length of the longest run. It is possible to compare the occurrences of runs with what is expected for a sequence of independent random variables to draw conclusions about how likely it is that a given sequence was generated from independent random variables.

(b) **[7 points: Fallacies about random sequences related to concepts and examples discussed in lecture.]**

i. **[2 pts]** This is the common belief that after a long sequence of one kind it is becomes more likely that the next element of the sequence is of the other kind. If the elements of the sequence are independent, this is incorrect, because for the outcome of the last element it is irrelevant what the elements were before.

ii. **[2 pts]** People may conclude, for example, that after a runs of $1's$ a 0 "is due" to insure that the average stays close to the limiting constant 0.5. However, the Law of Large Numbers makes an asymptotic statement and imbalances in finite patterns, no matter how long, do not have to be corrected in a short timeframe.
Alternative ways of describing the issue will be accepted. For example:
People may incorrectly concluded from the Law of Large Numbers that small samples are representative of the sequence (which is not generally true).

iii. **[1 pt]** Situations where people make a lot of binary decisions sequentially and where there is *a priori* no dependency, e.g. cancer screening, mortgage approval, asylum judges, trading decisions.

iv. **[2 pts]** Two bullet points are enough as long as they make sense and are properly reasoned. Here are some examples.

- People use different heuristics in different contexts (framing effect), despite mathematicians making them look alike.
- Hot hand may actually be a correct belief because independence assumption may be wrong (e.g. reinforcement of confidence, positive feedback from team)
- Confusion between different abilities between players with the empirical sampling of these through observing shots during the game potentially resulting in an incorrect perception that shots are dependent. (An example for model uncertainty).

(c) **[6 points: Solving zero sum game, from exercise sheet.]**

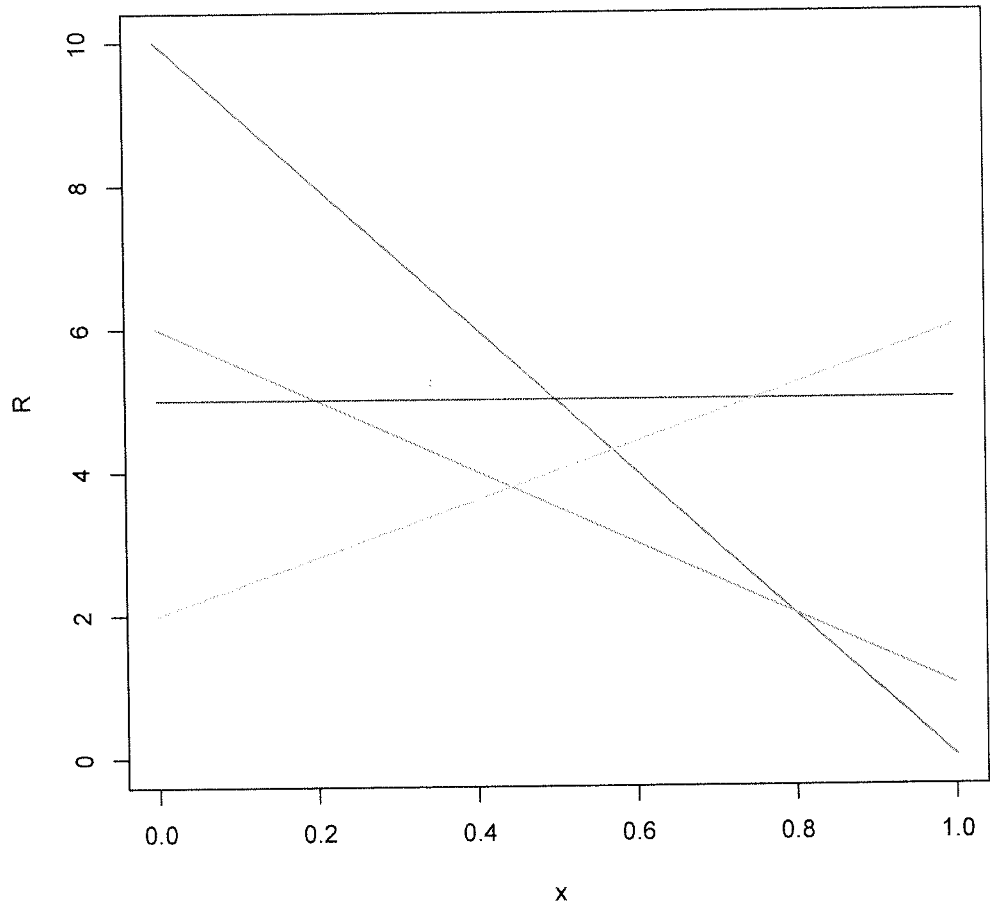
i. **[3 pts]** First, notice that δ_5 is dominated by δ_4, so it does not need to be considered for optimal strategies. A mixed strategy for player 1 denoted by $\overline{x} = (x, 1-x)$ means d_1 is played with probability x and d_2 is played with probability $1-x$. The maximin strategy has associated probabilities $\overline{x}^\star = (x^\star, 1-x^\star)$ with $x^\star$ chosen to maximise the expected return obtained if player 2 makes the worst possible move.
The expected return R_k for player 1 against a pure strategy δ_k for player 2 is:

$$R_k = E[R(\overline{x}, \delta_k)] = xR(d_1) + (1-x)R(d_2)$$

The relevant strategies for player 2 are $k = 1, 2, 3, 4$ and the returns are:

$$\begin{aligned}
R_1 &= 10(1-x) = -10x + 10 \\
R_2 &= x + 6(1-x) = -5x + 6 \\
R_3 &= 5x + 5(1-x) = 5 \\
R_4 &= 6x + 2(1-x) = 4x + 2
\end{aligned}$$

Plotting expected return against x for each of player 2's possible moves using red for δ_1, violet for δ_2, blue for δ_3 and green for δ_4 yields the figure below.
From the figure below it's clear that δ_2 and δ_4 have associated lines which intersect at the maximin point. Checking by calculation, it is clear that this intersection occurs at $x = 4/9$ at which point player 1 has an expected reward of $34/9$.



ii. **[2 pts]**
Player 2 may use the fact that the value of the game is the expected reward of player 1 at their maximin strategy (i.e. 34/9) and that player 1 only has to consider moves δ_2 and δ_4 (knowing that they will do better if player 2 plays any other strategy). Thus their maximin mixed strategy is $(0, y^\star, 0, 1 - y^\star)$ with $y^\star$ chosen to achieve a reward for player 1 of at most 34/9. Thus, $y^\star + 6(1 - y^\star) \leq 34/9$ and $6y^\star + 2(1 - y^\star) \leq 34/9$. And we must have $y^\star = 4/9$.

iii. **[1 pt]** The value of the game is the expected reward of the first player with both adopting their maximin mixed strategy: 34/9.
Note: The weights in the mixed strategies happen to be the same in player 1 and player 2, but that is not generally true.

3. (a) **[10 points: Typical EMV and EUT decision making task, in new context.]**

i. **[2 pts]** After her first round with stake x she has $35x + x = 36x$ for her second stake obtaining $35 \cdot 36x + 36x = 36^2x = 1,296x$. So the money she raised is $1,296x - x = 1,295x$. As $x = 100$ marks, this is 129,500 marks.
Alternatively, write this more explicitely:

Her first stake is 100 marks and her return is $35 \cdot 100$ marks = 3500 marks. Her second stake is all she has, which is 3600 marks. Her second return is $35 \cdot 3,600$ marks =126,000 marks, so she now has 126,000+3,600 marks = 129,600 marks. Subtracting the initial stake of 100 marks means she raised 129,500 marks.

ii. **[2 pts]** There are two different options:

- d_1 : She does not enter the game. The outcome is that she does not gain anything and keeps her 100 marks. The expected value for d_1 is 100 marks.
- d_2 : She enters the game. There are three possible outcomes:
  - ω_1 : She wins in both rounds. She has 129,600 marks (see above).
  - ω_2 : She wins the first round and loses the second. As she staked everything she now has 0 marks.
  - ω_3 : She loses the first round. That is the end of her game, as she loses her stake and has 0 marks.

  $P(\omega_1) = 1/37^2$, so her expected value for d_2 is

$$\frac{1}{37^2} \cdot 36^2 \cdot 100 \text{ marks} + \left(1 - \frac{1}{37^2}\right) \cdot 0 \text{ marks} = \left(\frac{36}{37}\right)^2 \cdot 100 \text{ marks.}$$

Since $\frac{36}{37} < 1$, the expected value for d_2 is smaller than the expected outcome for d_1. Hence the EMV approach recommends to not enter the game.

iii. **[2 pt]** No. One would think that Lola prefers to safe her boyfriend's life even if entering the game is an expected loss.

Alternative answers showing the student got the point will also be accepted. For example, students may feel tempted to make sarcastic comments about relationships...

iv. **[2 pts]** Lola needs to win at least 99,900 marks to keep her boyfriend alive and any other amount is worthless. This suggests $u(x) = 0$, for $x < 99,900$ marks and $u(x) = 1$, for $x \geq 99,900$ marks. (*Alternative answers are possible.*)

v. **[2 pts]** The expected utility for d_1 is 0.

The expected value for d_2 is

$$\frac{1}{37^2} \cdot 1 + \left(1 - \frac{1}{37^2}\right) \cdot 0 = \frac{1}{37^2}.$$

This is larger than the expected utility for d_1.

Hence the EUT based decision for Lola is to enter the game.

(b) **[6 points: Alternative decision making strategies, slightly changed question from class test.]**

i. **[2 pts]** For any $p > 0$ there is a possibility of winning £100 by buying a ticket whilst the best than can happen otherwise is no loss. Consequently, provided that $p > 0$ the maximax decision is to enter the lottery.

ii. **[2 pts]** The expected reward of buying a ticket is $100p - 1(1-p) = 101p - 1$. The expected reward of not buying a ticket is 0. The EMV decision, therefore, is to buy a ticket provided that $101p - 1 > 0$, in other words $p > 1/101$.

iii. **[2 pts]** Not to enter unless $p = 1$. Then the worst possible outcome is losing nothing. If $p = 1$ then the worst possible outcome of entering is to win £100 but for any other value you could lose £1.

(c) [**4 points: Related to examples and concepts discussed in lecture.**]

i. **[2 pts]** GRGRRR is the conjunction of RGRRR and another event, hence the set of sequences A that contain GRGRRR is a strict subset of the set of sequences B that contain RGRRR and are therefore less likely to be observed than just RGRRR. More specifically, using independence,

$$P(A) = P(GRGRRR) = P(G) \cdot P(RGRRR) = 2/3 \cdot P(RGRRR) < P(RGRRR).$$

Also,

$$P(GRRRRR) = 2/3 \cdot (1/3)^5 < 2/3)^2 \cdot (1/3)^4 = P(GRGRRR).$$

Hence $RGRRR$ hast the highest chance of winning.

ii. **[2 pts]** Subjects perceive RGRRR as imbalanced, as it contains only one G, even though G is more likely than R. They notice that GRGRRR has two Gs and overlook that the pattern is longer, or at least do not realise that as a result it is less likely to obtain. They confused the longer pattern being *representative* of the die, with it being more *probable*.

4. (a) [**8 points: Example from lecture presented here as exercise in slightly modified and more comprehensive way.**]

i. **[2 pts]** Choices under EMV (Expected Monetary value) approach are based on expected value of the payoff X. Below we compute these expectations and then choose the option with the higher one.

A: $E[X(a)] = \$7.5, E[X(b)] = \$9$, hence choose (b).

B: $E[X(a)] = \$30, E[X(b)] = \$36$, hence choose (b).

C: $E[X(a)] = 0.25 \cdot \$30, E[X(b)] = 0.25 \cdot 0.8 \cdot \$45 = \$9$, hence choose (b).

ii. **[3 pts]**

A: The majority chooses (b) based on the EMV approach.

B: The majority chooses (a) against the EMV approach, and instead based on the certainty principle.

C: The majority again chooses (a), despite the EMV approach suggesting (b). It has been suggested that this is due to the subjects ignoring the first step in C, which is an example for the disjunction effect.

iii. **[3 pts]** According to Expected Utility Theory (EUT), (b) is preferred to (a) if and only if $E[u(X(a))] > E[X(b)]$. For both, A and C, that condition is equivalent to

$$0.2 \cdot u(\$45) > 0.25 \cdot u(\$30) \quad (*)$$

For B, (b) is preferred to (a) if and only if

$$0.8 \cdot u(\$45) > 1 \cdot u(\$30) \quad (*)$$

which is also equivalent to $(*)$ (divide by 4). That means, subjects who behave consistently with EUT should either prefer (a) to (b) in all three choices or they should prefer (b) to (a) in all three choices.

(b) **[7 points: Recall of definitions, simple proofs.]**

i. **[2 pts]** *Completeness:* Need to show that for each pair $x, y \in \mathcal{R}^2$ one of the following is true: $x \succ y$, $y \succ x$ or $x \sim y$.
If $x_1 > y_1$ then $x \succ y$. If $x_1 < y_1$ then $y \succ x$. In the remaining case, $x_1 = y_1$, we proceed with comparing x_2 and y_2. If $x_2 > y_2$ then $x \succ y$. If $x_2 < y_2$ then $y \succ x$. Finally, if $x_2 = y_2$ then $x \sim y$.

ii. **[2 pts]** *Transitivity:* Let $x, y, z \in \mathcal{R}^2$ with $x \succ y$ and $y \succ z$. Need to show $x \succ z$.
If $x_1 > y_1$ or $y_1 > z_1$ then $x_1 > z_1$ and thereby $x \succ z$. Otherwise, $x_1 = y_1 = z_1$ and we proceed with the second coordinate. Since $x \succ y$, $x_2 \succ y_2$ and, since $y \succ z$, $y_2 \succ z_2$. Hence $x_2 > z_2$, which in this case implies $x \succ z$.

iii. **[3 pts]** *Archimedian:* It is not true. For example, $x = (0,1), y = (0,0), z = (-1,0)$ fulfills $x \succ y \succ z$. Assume there were $\alpha, \beta \in (0,1)$ with

$$\alpha x + (1-\alpha)z \prec y \prec \beta z + (1-\beta)z \qquad (*)$$

That means,

$$(\alpha \cdot 0 + (1-\alpha)\cdot(-1), \alpha \cdot 1 + (1-\alpha)\cdot 0) \succ (0,0) \succ (\beta \cdot 0 + (1-\beta)\cdot(-1), \beta \cdot 1 + (1-\beta)\cdot 0)$$

which simplifies to

$$(\alpha - 1, \alpha) \succ (0,0) \succ (\beta - 1, \beta).$$

For the first relation to be true $\alpha - 1 \geq 0$. But that implies $\alpha \geq 1$, which is a contradiction to the assumptions on α.

(c) **[5 points: Similar to examples in class and to question on exercise sheet.]**

i. **[4 pts]** Let $+$ be the event that the applicant's test was positive. Let U be the event the applicant uses the drugs tested for.
The question asks us to calculate $P(U \mid +)$.

$$P(U \mid +) = P(+ \mid U) \cdot \frac{P(U)}{P(+)}$$

Using that

$$\begin{aligned} P(+) &= P(+ \mid U)P(U) + P(+ \mid U^c)P(U^c) \\ &= 1 \cdot 0.01 + 0.1 \cdot 0.99 = 0.01 + 0.099 = 0.109 \end{aligned}$$

we obtain

$$P(U \mid +) = 1 \cdot \frac{0.01}{0.109} = \frac{1}{10.9}$$

This is smaller than 10%.

ii. **[1 pt]** Base rate neglect.

End