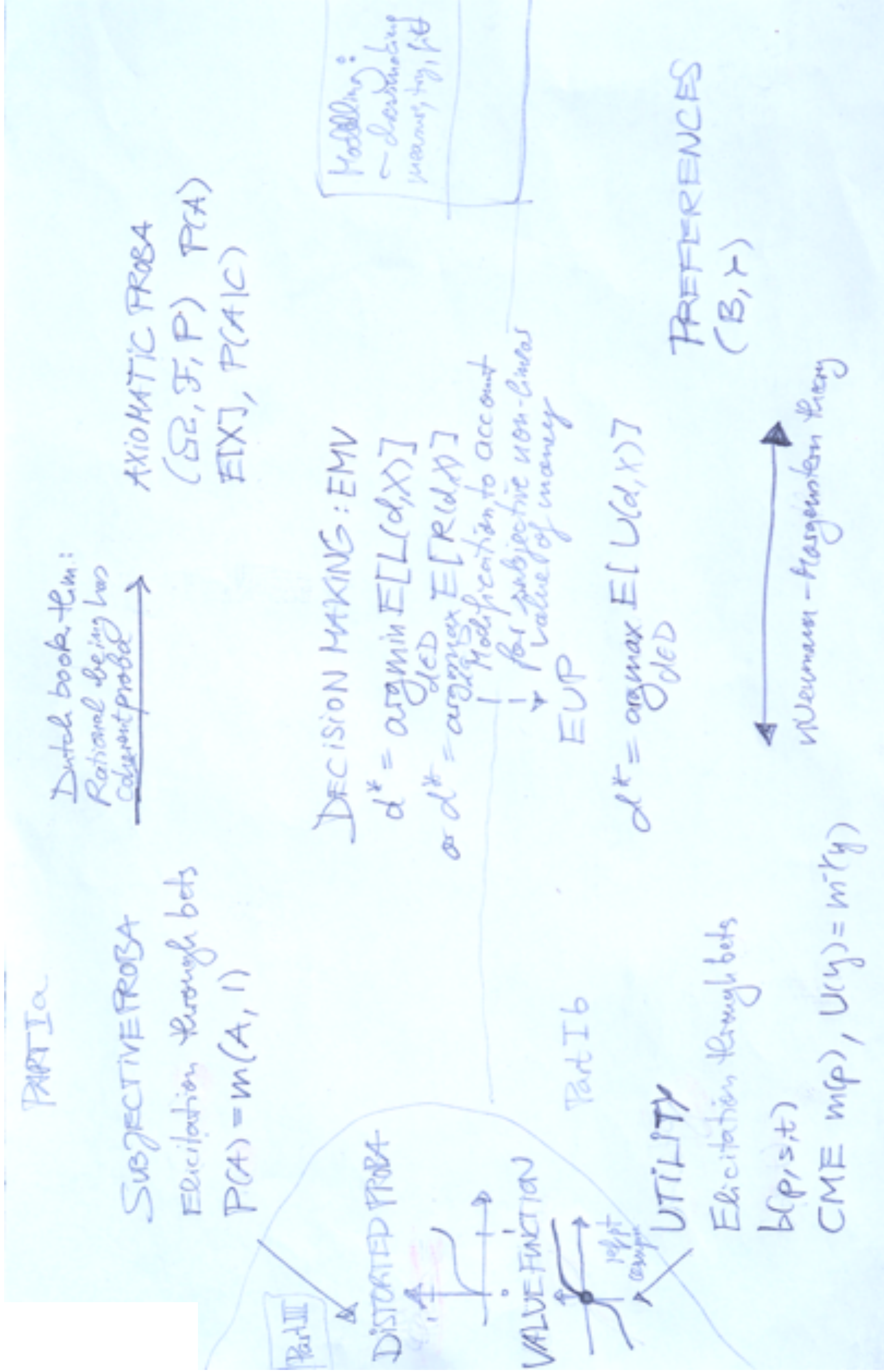

Preferences between actions (bets, lotteries etc)

Outcomes : $\Omega = \{\omega_1, \omega_2, \ldots \omega_n\}$

PMF $(p_i)_{i=1,\ldots,n}$ Action: x (with uncertain outcomes) — p_1: ω_1, p_2: ω_2, …, p_n: ω_3

$x = b(p_1, \omega_1;\ p_2, \omega_2, \ldots;\ p_n, \omega_n)$

$\mathcal{A}$ = action space

Use bivariate relation on $\mathcal{A}$ for (subjective) preferences

$x, y \in \mathcal{A}$

$x \succ y$ "x prefered to y"

$x \sim y$ "x indifferent to y"

$x \succsim y$ "x weakly preferred to y", also $\prec$, $\precsim$

$\alpha \geq 0$

$\alpha x + (1-\alpha) y$ combined action

Properties of bivariate relations:

What do these properties mean for preferences between actions?

Representation theorem

(Later: vN-M theory : More properties
Refined repr thm)

Properties of bivariate relations

$\wedge$ means AND
$\vee$ means OR
$\neg$ means NOT

$(B, \succ)$ also $\sim$, $\prec$

(C) Completeness: $\forall x,y \in B \quad x \succ y \vee x \sim y \vee x \prec y$

(A) Asymmetry: $\forall x,y \in B \quad x \succ y \Rightarrow \neg y \succ x$

(T) Transitivity: $\forall x,y,z \in B$

$$x \succ y \wedge y \succ z \Rightarrow x \succ z$$

(NT) Negative transitivity: $\forall x,y,z \in B$

$$\neg x \succ y \wedge \neg y \succ z \Rightarrow \neg x \succ z$$

Properties of preferences as bivariate relations and discussion

(C) Completeness:

- Forces people to "make up their minds". Often appropriate, but sometimes not realistic.

  e.g. choose between

  x_1 = "be stupid and satisfied"

  x_2 = "be smart and dissatisfied"

  (Original example by Savage is more dramatic: Choose whether you are hung until your health suffers or your reputation is damaged.)

  The choices in these examples are <u>incommensurable</u>.

  An assumption necessary to achieve completeness is that the actions in $\mathcal{A}$ are sufficiently <u>commensurable</u>.

  See www.merriam-webster.com/dictionary/commensurable

  (Can also demonstrate how to pronounce it!)

  Check also WiKi pages for this term under economics, philosophy of science and mathematics.

(C) (ct)

- Another assumption on $\mathcal{A}$ is that the choices are available at the same time in the same place.

  E.g.: x_1="eat foie gras" x_2="eat hotdog"

  Surely, there will be a restaurant in NYC that offers both French haute cuisine and streetfood, but that is unusual!

(T) Transitivity:

- What do you want in your coffee?

  People tend to be indifferent between small differences, e.g.

  no sugar $\sim$ 1 grain sugar $\sim$ 2 grains sugar $\sim$ ... $\sim$ 1 spoon sugar

  But this does not imply

  no sugar $\sim$ 1 spoon sugar

  (Similar issue to the fact that the world is locally flat, but still not globally flat.)

(T) (ct)

- Transitivity means "no cycles".

  A cycle would be this:

How realistic is the no-cycles-assumption?

Dutch Book type argument says that a rational person should not have cycles in their preferences, because otherwise one could exploit this by constructing a money pump as follows:

Assume Julia's preferences are

$$x \succ y \wedge y \succ z \wedge z \succ x$$

Then she would be happy to pay some amount $\alpha_1 > 0$ to swap z for x. She would also be willing to pay then $\alpha_2 > 0$ to swap z for y, and then $\alpha_3 > 0$ to swap y for x. So she is back to x and has lost $\alpha_1 + \alpha_2 + \alpha_3$! Continue this and she will end up loosing any arbitrary amount of money.

(T) (ct)

An example for how this can be exploited by sellers of whatever goods is this:

Purchase of a smart phone

Evaluate preferences using ranks (1 best)

	Design	Function	Price
iphone	1	2	3
jphone	2	3	1
kphone	3	1	2

and say $x \succ y$ iff majority of criteria in x is better than in y. This yields

$i \succ j \;\wedge\; j \succ k \;\wedge\; k \succ i$

which is cyclic.

Hence, smart phone preferences may not be transitive.

This explains why (most) people buying smart phones can be exploited using money pump techniques explained above.