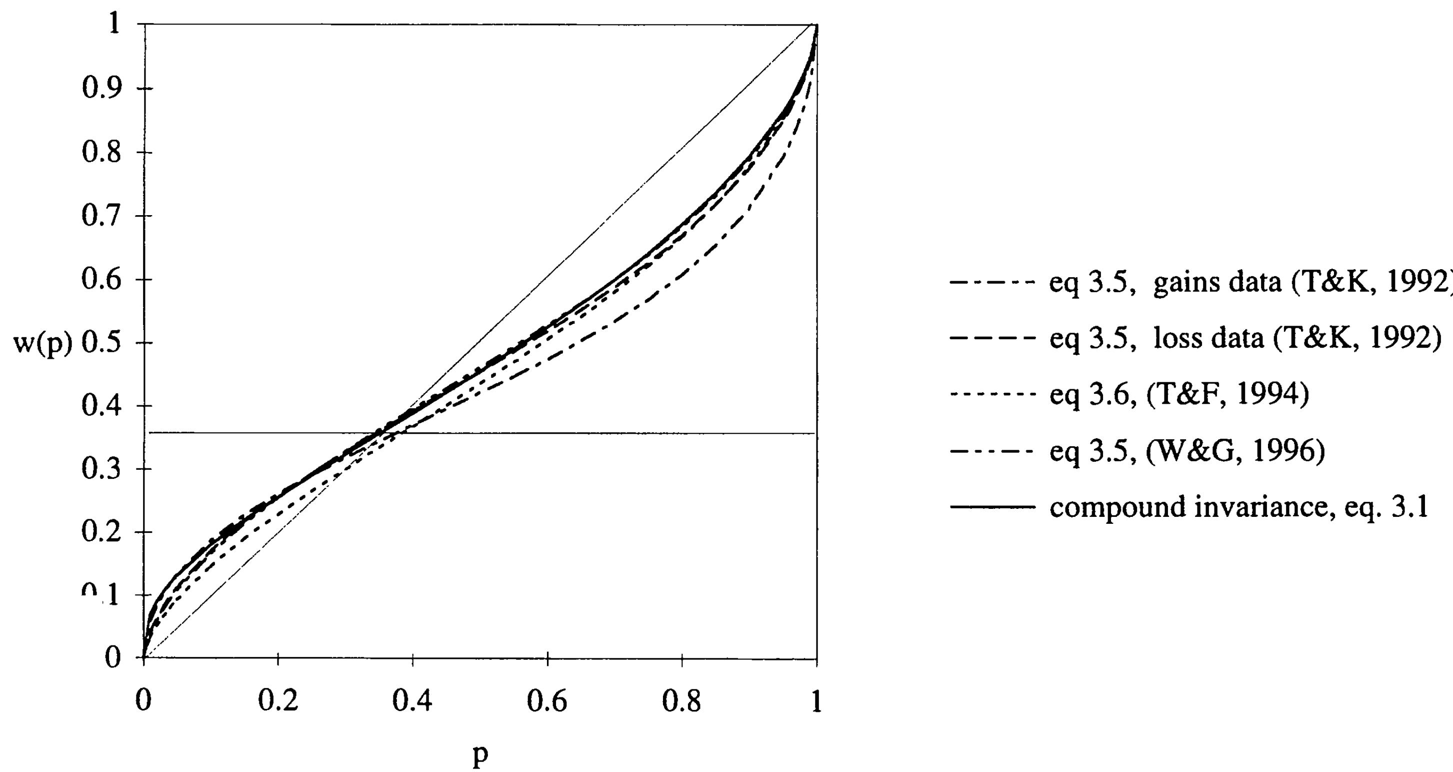
Ingredients: Prospect theory probability weighting function

Four-fold pattern of risk attitudes

regressive—intersecting the diagonal from above,
asymmetric—with fixed point at about 1/3,
s-shaped—concave on an initial interval and convex beyond that,
reflective—assigning equal weight to a given loss-probability as to a given gain-probability.



Ingredients: Prospect theory value function

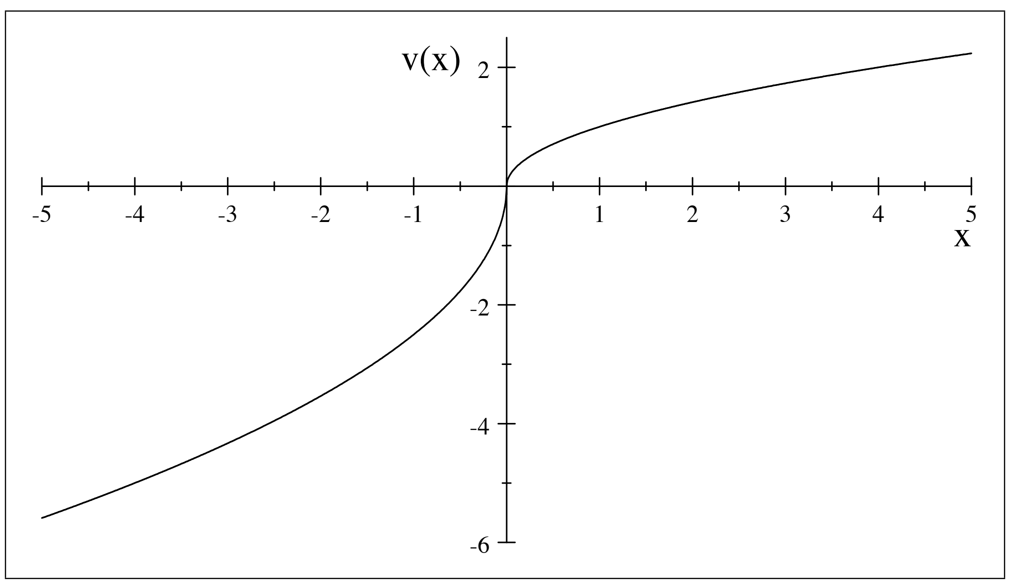
Definition *(Tversky and Kahneman, 1979). A utility function, $v(x)$, is a continuous, strictly increasing, mapping $v : \mathbb{R} \to \mathbb{R}$ that satisfies:*

1. $v(0) = 0$ (reference dependence). (*)

2. $v(x)$ is concave for $x \geq 0$ (declining sensitivity for gains).

3. $v(x)$ is convex for $x \leq 0$ (declining sensitivity for losses).

4. $-v(-x) > v(x)$ for $x > 0$ (loss aversion).



$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

$\alpha > 0$: degree of risk aversion in gains

$\beta > 0$: degree of risk seeking in losses

$\lambda > 0$: degree of loss aversion

(*) Note: reference point depends on context, so may be changed

Prospect theory: Value of a prospect

Given a prospect $b = (x, p; y, q)$

$$x, y \in \mathcal{R}, p, q \in [0, 1], p + q = 1$$

In EUT the aim is to maximise

$$U(b) = E[u(b)] = pu(x) + qu(y)$$

In PT the aim is to maximise

$$V(b) = w(p)u(x) + w(q)u(y)$$

- analogy to expectation
- not an expectation (distorted probabilities)
- can be generalised to more than two outcomes

Modelling: Prospect theory (PT)

Non-EUT behaviour as observed in empirical studies such as:

- *Certainty effect:* e.g. in Allais type problems
- *Reflection effect:* losses are associated with different behaviours than gains
- *Framing effect*

Question:

Can prospect theory explain observed decision making behaviour that was not explicable by EUT?

D Kahneman and A Tversky, *Prospect Theory: An Analysis of Decisions under Risk*, Econometrica, Vol. 47, March 1979, Number 2, pp. 263-291.

Certainty effect: Common consequence - empirical study

(Similar to Allais paradox)

PROBLEM 1: Choose between

A:	2,500 with probability	.33,	B:	2,400 with certainty.
	2,400 with probability	.66,		
	0 with probability	.01;		

PROBLEM 2: Choose between

C:	2,500 with probability	.33,	D:	2,400 with probability	.34,
	0 with probability	.67;		0 with probability	.66.

Is there a connection between P1 and P2?

Certainty effect: Common consequence - empirical study

(Similar to Allais paradox)

PROBLEM 1: Choose between

A:	2,500 with probability	.33,	B:	2,400 with certainty.
	2,400 with probability	.66,		
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PROBLEM 2: Choose between

C:	2,500 with probability	.33,	D:	2,400 with probability	.34,
	0 with probability	.67;		0 with probability	.66.

$\boxed{P2 = P1 - (2400, .66)}$

Note the shifting a 0.01 probability for 0 gain to 2,400 gain (from A to B and from C to D).

Certainty effect: Common consequence - empirical study

(Similar to Allais paradox)

PROBLEM 1: Choose between

A:	2,500 with probability	.33,	B:	$\boxed{\text{2,400 with certainty.}}$
	2,400 with probability	.66,		
	0 with probability	.01;		
$N = 72$	[18]	Subjects prefer B		[82]*

PROBLEM 2: Choose between

C:	2,500 with probability	.33,	D:	2,400 with probability	.34,
	0 with probability	.67;		0 with probability	.66.
$N = 72$	[83]*	Subjects prefer C		[17]	

$\boxed{\text{P2 = P1 - (2400, .66)}}$

- EUT: A<B equivalent to C<D
- Empirical evidence contradicts EUT
- PT can explain A<B & C>D (sheet 5)

Certainty effect: Common ratio - empirical study

(Also based on Allais)

PROBLEM 3:

A: (4,000,.80), or B: (3,000).

$N = 95$ [20] [80]*

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).

$N = 95$ [65]* [35]

Is there a connection between P3 and P4?

Certainty effect: Common ratio - empirical study

(Also based on Allais)

PROBLEM 3:

A: (4,000,.80), or B: (3,000).

$N = 95$ [20] [80]*

Subjects prefer B

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).

$N = 95$ [65]* [35]

Subjects prefer C

P4 is P3 but with all probabilities multiplied by 0.25

- EUT: A<B equivalent to C<D
- Empirical evidence contradicts EUT

Common ratio: EUT applied to example with P3 & P4

PROBLEM 3:

A:	(4,000,.80),	or	B:	(3,000).
$N = 95$	[20]			[80]*

PROBLEM 4:

C:	(4,000,.20),	or	D:	(3,000,.25).
$N = 95$	[65]*			[35]

$$x = 4000, y = 3000, \lambda = 4/5, p_1 = 1, p_2 = 0.25$$

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$\boxed{R_1 \succ S_1 \;\Leftrightarrow\; R_2 \succ S_2}$$

EUT

Common ratio: General form under EUT

Given a pair of prospects:

$$R = (x, \lambda p) \qquad S = (y, p) \qquad (x > y, 0 < \lambda < 1)$$

EUT says: $$R \succ S \;\Leftrightarrow\; u(x)\lambda p > u(y)p \;\Leftrightarrow\; \lambda > \frac{u(y)}{u(x)}$$

Note: This in **independent** of the probability p

Common ratio: General form under EUT

Given a pair of prospects:

$$R = (x, \lambda p) \qquad S = (y, p) \qquad (x > y, 0 < \lambda < 1)$$

EUT says: $R \succ S \;\Leftrightarrow\; u(x)\lambda p > u(y)p \;\Leftrightarrow\; \lambda > \dfrac{u(y)}{u(x)}$

(Assuming $u(0) = 0$ to keep argument concise)

Note: This in **independent** of the probability p

Hence, for two pairs of prospects:

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$R_1 \succ S_1 \;\Leftrightarrow\; R_2 \succ S_2$$

Common ratio: EUT applied to example with P3 & P4

PROBLEM 3:

A: (4,000,.80), or B: (3,000).

$N = 95$ [20] [80]*

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).

$N = 95$ [65]* [35]

$$x = 4000, y = 3000, \lambda = 4/5, p_1 = 1, p_2 = 0.25$$

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$\boxed{R_1 \succ S_1 \;\Leftrightarrow\; R_2 \succ S_2}$$

EUT

Common ratio: EUT applied to example P3 & P4

PROBLEM 3:

A: (4,000,.80), or B: (3,000).
$N = 95$ [20] [80]*

PROBLEM 4:

C: (4,000,.20), or D: (3,000,.25).
$N = 95$ [65]* [35]

Observed

$$R_1 \prec S_1$$

$$R_2 \succ S_2$$

$$x = 4000, y = 3000, \lambda = 4/5, p_1 = 1, p_2 = 0.25$$

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$\boxed{R_1 \succ S_1 \;\Leftrightarrow\; R_2 \succ S_2}$$

EUT

EUT does not describe the observed behaviour.

Common ratio: General form under PT

Given two prospects:

$$R = (x, \lambda p) \qquad S = (y, p) \qquad (x > y, 0 < \lambda < 1)$$

Prospect theory (PT) says:

$$R \succ S \;\Leftrightarrow\; v(x)w(\lambda p) > v(y)w(p) \;\Leftrightarrow\; \frac{w(\lambda p)}{w(p)} > \frac{v(y)}{v(x)}$$

Note: This in **not independent** of the probability p

Common ratio: PT applied to example P3 & P4

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$x = 4000, y = 3000, \lambda = 4/5, p_1 = 1, p_2 = 0.25$$

Observed $\quad R_1 \prec S_1 \quad$ and $\quad R_2 \succ S_2$

Common ratio: PT applied to example P3 & P4

$$R_i = (x, \lambda p_i) \qquad S_i = (y, p_i) \qquad i = 1, 2$$

$$x = 4000, y = 3000, \lambda = 4/5, p_1 = 1, p_2 = 0.25$$

Observed $\quad R_1 \prec S_1 \quad$ and $\quad R_2 \succ S_2$

In PT: $\quad \dfrac{w(\lambda p_1)}{w(p_1)} < \dfrac{v(y)}{v(x)} \quad$ and $\quad \dfrac{w(\lambda p_2)}{w(p_2)} > \dfrac{v(y)}{v(x)}$

In other words: $\quad \dfrac{w(0.8)}{w(1)} < \dfrac{v(y)}{v(x)} < \dfrac{w(0.2)}{w(0.25)}$

If there is a value function and a probability weighting function that fulfils this, then PT **can describe** the observed behaviour.

Common ratio: PT applied to example P3 & P4

Is there a value function and a probability weighting function such that the inequality below is correct?

$$\frac{w(0.8)}{w(1)} < \frac{v(y)}{v(x)} < \frac{w(0.2)}{w(0.25)} \qquad (*)$$

Common ratio: PT applied to example P3 & P4

Is there a value function and a probability weighting function such that the inequality below is correct?

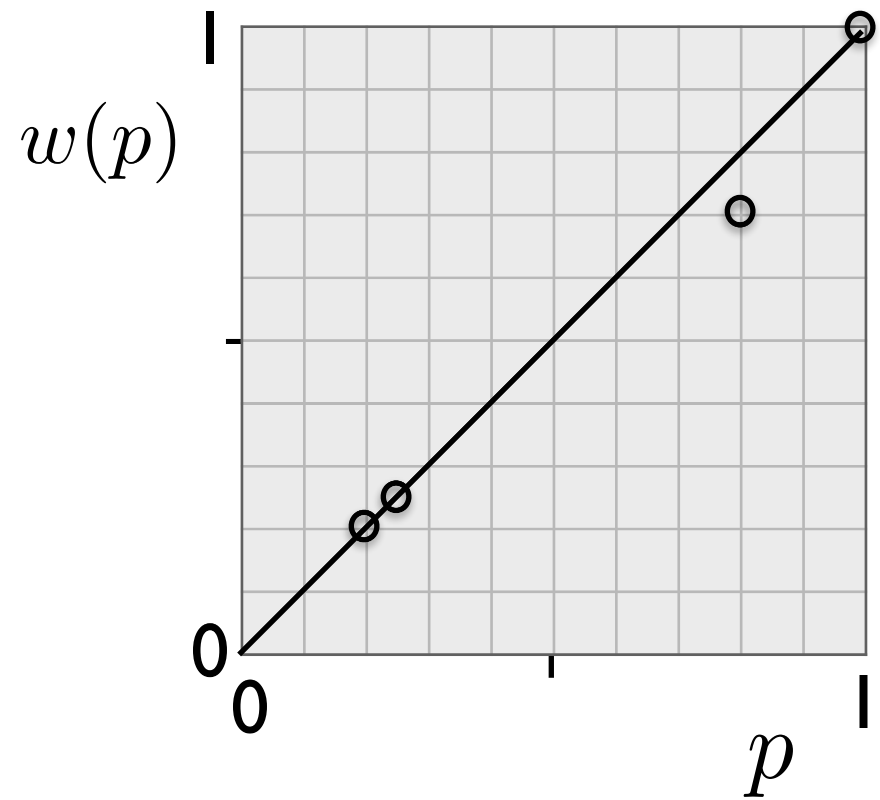
$$\frac{w(0.8)}{w(1)} < \frac{v(y)}{v(x)} < \frac{w(0.2)}{w(0.25)} \qquad (*)$$

Choose, for example: $v = Id$

$$w(1) = 1, w(0.8) = 0.7$$

$$w(0.25) = 0.25, w(0.2) = 0.2$$

- Probability weighting function with such values exists
- Only a small deformation (in this example)



Common ratio: PT applied to example P3 & P4

Is there a value function and a probability weighting function such that the inequality below is correct?

$$\frac{w(0.8)}{w(1)} < \frac{v(y)}{v(x)} < \frac{w(0.2)}{w(0.25)} \qquad (*)$$

Choose, for example: $v = Id$

$$w(1) = 1, w(0.8) = 0.7, w(0.25) = 0.25, w(2) = 0.2$$

Then (*) means $\frac{0.7}{1} < \frac{3000}{4000} < \frac{0.2}{0.25}$

In other words, $0.7 < 0.75 < 0.8$ which is true.

Common ratio: PT applied to example P3 & P4

Is there a value function and a probability weighting function such that the inequality below is correct?

$$\frac{w(0.8)}{w(1)} < \frac{v(y)}{v(x)} < \frac{w(0.2)}{w(0.25)} \qquad (*)$$

Choose, for example: $v = Id$

$$w(1) = 1, w(0.8) = 0.7, w(0.25) = 0.25, w(0.2) = 0.2$$

Then (*) means $\frac{0.7}{1} < \frac{3000}{4000} < \frac{0.2}{0.25}$

In other words, $0.7 < 0.75 < 0.8$ (which is true).

And these values can be realised by functions suitable for PT.

Hence, PT **can describe** the observed behaviour!

Modelling: Prospect theory (PT)

Question:

Can prospect theory explain behaviour deviating from EUT?

- Certainty effect

Answer: Yes, PT **can describe** the observed behaviour!

Used probability weighting.

Value function was not relevant for this.

Reflection effect: Observations

Observation:
For losses, people's risk aversion changes to risk seeking.

PREFERENCES BETWEEN POSITIVE AND NEGATIVE PROSPECTS

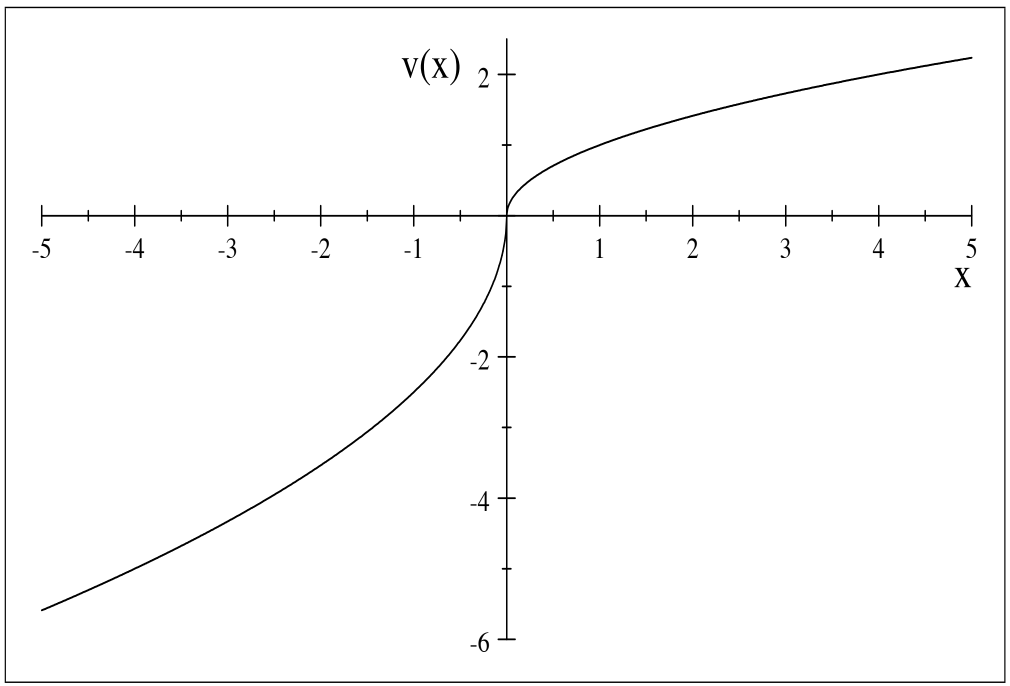
Positive prospects				Negative prospects			
Problem 3:	(4,000, .80)	<	(3,000).	Problem 3′:	(−4,000, .80)	>	(−3,000).
$N = 95$	[20]		[80]*	$N = 95$	[92]*		[8]
Problem 4:	(4,000, .20)	>	(3,000, .25).	Problem 4′:	(−4,000, .20)	<	(−3,000, .25).
$N = 95$	[65]*		[35]	$N = 95$	[42]		[58]
Problem 7:	(3,000, .90)	>	(6,000, .45).	Problem 7′:	(−3,000, .90)	<	(−6,000, .45).
$N = 66$	[86]*		[14]	$N = 66$	[8]		[92]*

Certainty effect implies *risk averse* preferences: **Sure lower gain preferred to higher probable gain.**

Certainty effect implies *risk seeking* preferences: **Probable higher loss preferred to sure lower loss.**

Reflection effect: Solution

PT replaces utility function by asymmetric value function:



$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

$\alpha > 0$: degree of risk aversion in gains

$\beta > 0$: degree of risk seeking in losses

$\lambda > 0$: degree of loss aversion

Framing effect: EUT vs PT

- EUT assumes that decision making is invariant to the manner of representation.
- Empirical evidence suggest this is incorrect (e.g. disease example with life vs death framings)
- PT is more flexible (e.g. gains and losses modelled differently)

Modelling: Prospect theory (PT)

Question:

Can prospect theory explain behaviour deviating from EUT?

- Reflection effect
- Framing effect

Answer:

Yes, through value function (convex in loss domain, concave for gains)

D Kahneman and A Tversky, *Prospect Theory: An Analysis of Decisions under Risk*, Econometrica, Vol. 47, March 1979, Number 2, pp. 263-291.

Modelling: Descriptive parts of prospect theory

Step 1: Editing phase

- coding
- combination (of similar prospects)
- segregation (of risky and non-risky parts)
- simplification (e.g. rounding)
- detection of dominance

Step 2: Evaluation phase

Examines, choses, combines and optimises…

Differences to EUT:

- Values attached to changes rather than final state
- Decision weights can be distorted probabilities

Heuristics & biases: How can fallacies be avoided?

Building blocks to increase rationality in decision making:

Collecting information	Combining probabilities and values
Editing phase	**Evaluation phase**

Question:

Is that enough to explain observed behaviour?

Heuristics & biases: Affect heuristic

Affect plays an important role in guiding judgments and decisions.

Here, *affect* means the specific quality of 'goodness' or 'badness'

- experienced as a feeling state (conscious or subconscious)
- demarcating a positive or negative quality of a stimulus

Affective responses occur rapidly and automatically

Reliance on such feelings: A*ffect heuristic* (P. Slovic)

Heuristics & biases: Affect heuristic

Affective responses occur rapidly and automatically

Potentially anyone may be subject to this, e.g.:

- political/ideological obsessions
- exam anxiety (quite common, see also (*))
- stress among traders (e.g. *https://en.wikipedia.org/wiki/Nick_Leeson*)
- jealousy (e.g. Hamlet)
- many more…

http://www2.warwick.ac.uk/fac/sci/statistics/courses/studying/examstress/

Example: Weather & stock market

"*The discovery that the* ***weather in New York City has a long history of significant correlation with major stock indexes*** *supports the view that investor psychology influences asset prices.*"

Saunders (1993)

Example: Weather & stock market

"Psychological evidence and casual intuition predict that sunny weather is associated with upbeat mood. This paper examines the relation between morning sunshine at a country's leading stock exchange and market index stock returns that day at 26 stock exchanges internationally from 1982-97. ***Sunshine is strongly significantly correlated with daily stock returns.***

After controlling for sunshine, rain and snow are unrelated to returns. There were positive net-of-transaction costs profits to be made from substantial use of weather-based strategies, but the magnitude of the gains was fairly modest. These findings are difficult to reconcile with fully rational pricesetting."

Hirshleifer and Shumway: *Good Day Sunshine: Stock Returns and the Weather,* The Journal of Finance react-text: 53 58(3), 2001

Further links and resources

Robert Grosse, (2012) "Bank regulation, governance and the crisis: a behavioral finance view", Journal of Financial Regulation and Compliance, Vol. 20 Iss: 1, pp.4 - 25

http://www.fca.org.uk/your-fca/documents/occasional-papers/occasional-paper-1
check out annex in particularly

Interview with Bank director Greg B Davies on the use of behavioural finance theory in real world banking:

www.seeitmarket.com/interview-greg-b-davies-barclays-behavioural-finance-13577/

Robert Shiller lecture on behavioural finance and prospect theory:

https://www.youtube.com/watch?v=chSHqogx2CI

"Behavioral Finance" (review) *http://www.sfb504.uni-mannheim.de/publications/dp03-14.pdf*

Markus Glaser, Markus Nöth, and Martin Weber (University of Mannheim)

"The Behavior of Individual Investors" (review) http://faculty.haas.berkeley.edu/odean/

Brad M. Barber and Terrance Odean (UC Davis, UC Berkeley)

Heuristics & biases: How can fallacies be avoided?

Building blocks to increase rationality in decision making:

Info quality	Training	Mental balance
Complete	Interpretation	Internal mood
Correct (source?)	Rules	External impact
Editing phase	**Evaluation phase**	**Interferes**

Affect heuristics infers with both editing phase and evaluation phase.

Does intelligence prevent affect heuristics?

Not always!

Examples: Exam stress,…

Example: 21st century information age

Increased amount of:

- data collection (reporting, imaging etc)
- networking

Large due to internet and high throughput technologies for measurements and images.

More data = more information = more knowledge

Or maybe not?

New Scientist 3rd December 2016:

"The most numerate people are better at distorting the data to fit their beliefs"

Heuristics & biases: How can fallacies be avoided?

Building blocks to increase rationality in decision making:

Info quality	Training	Mental balance
Complete	Interpretation	Internal mood
Correct (source?)	Rules	External impact
Editing phase	**Evaluation phase**	**Interferes**

Affect heuristics infers with both editing phase and evaluation phase.

Affect heuristics may dominate intelligence.

Mathematical decision making training alone is sufficient to ensure rational processes. Mind also needs to be in a functional state.

Capacity for cognitive dissonance (cause by data contradicting preexisting beliefs).

LISA METZ

This wisdom has been picked up by popular culture…

"People make bad choices when they're mad or scared or stressed"

Frozen (Disney movie)

"Inside Out" (Pixar)

"Be quiet. Calm yourself. Take up yoga."

Mr Bercow (Speaker of the House of Commons), comment made to MPs on a regular basis