Question from class test

Question: Consider a lottery in which, with probability p you win £100 and with probability $1 - p$ you lose £1. You may decide to enter or not to enter.

i. What is the maximin decision as a function of p?

ii. What is the expected monetary value decision as a function of p?

iii. What is the maximax decision as a function of p?

Solution:

i. Not to enter unless $p = 1$ – then the worst possible outcome is losing nothing. If $p = 1$ then the worst possible outcome of entering is to win £100 but for any other value you could lose £1.

ii. The expected reward of buying a ticket is $100p - 1(1 - p) = 101p - 1$. The expected reward of not buying a ticket is 0. The EMV decision, therefore, is to buy a ticket provided that $101p - 1 > 0$, in other words $p > 1/101$.

iii. For any $p > 0$ there is a possibility of winning £100 by buying a ticket whilst the best than can happen otherwise is no loss. Consequently, provided that $p > 0$ the maximax decision is to enter the lottery.

Practice question

Question: Tversky and Kahneman defined the probability weighting function

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \qquad (p \in [0,1])$$

for parameters $\gamma > 0$.

Show that w has the *subcertainty property* $\gamma \neq 1$, that is,

$$w(p) + w(1-p) < 1 \text{ for all } p \in (0,1).$$

Note:

For $\gamma = 1$ the inequality is not true, but this is the trivial case of w being the identity, so there is no probability weighting and the inequality becomes a trivial equality.

Practice question solution

$$
\begin{aligned}
w(p) + w(1-p) &= \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} + \frac{(1-p)^\gamma}{((1-p)^\gamma + (1-(1-p))^\gamma)^{1/\gamma}} \\
&= \frac{p^\gamma + (1-p)^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \\
&= \left(p^\gamma + (1-p)^\gamma\right)^{(\gamma-1)/\gamma}
\end{aligned}
$$

Hence,

$$w(p) + w(1-p) < 1 \Leftrightarrow \log(w(p) + w(1-p)) < 0 \Leftrightarrow \frac{\gamma - 1}{\gamma} \log\left(p^\gamma + (1-p)^\gamma\right) < 0$$

This is true if one of the factors is positive and the other is negative, but it turns out that this is always the case:

$$\frac{\gamma - 1}{\gamma} < 0 \Leftrightarrow \gamma < 1 \Leftrightarrow p\gamma + (1-p)^\gamma > 1 \Leftrightarrow \log\left(p^\gamma + (1-p)^\gamma\right) > 0.$$