Human perception of probability: Gambler's fallacy

Gambler's fallacy:

The confidence that after a long run of one kind of outcome the other kind of outcomes are more likely.

In random sequences that are actually composed of independent events this is *wrong* (e.g. coin tossing, many games).

Explanation for this wrong belief:

Erroneous conceptualisation of the law of large numbers, the belief that *small samples should be representative* for the distribution (which is generally not true).

Methodology: Local representativeness heuristics

Local representativeness assumption means that there was a ***law of small numbers***, whereby small samples are perceived to represent their population to the same extent as large samples (Tversky & Kahneman 1971).

Specifically, this would mean:

- A small sample which appears randomly distributed reinforces the belief that the population is randomly distributed.
- A small sample with a skewed distribution would weaken this belief.

For independent random sequences, this is wrong, because they have no memory.

Historical event: Monte Carlo

Monte Carlo Casino, August 18, 1913:

- Game of roulette, the ball fell in black 26 times in a row
- Probability for this is 1/67,108,863
- Gamblers lost millions for francs betting *against* black believing the *streak was causing an imbalance in the randomness of the wheel*
- Assumed that it had to be followed by a *streak of red*

Examples and non-examples of gambler's fallacy

- Joseph Jagger at Monte Carlo
- Black Jack
- Childbirth
- Evolutionary explanation
- Reverse gambler's fallacy

Practical applications: Detection of gambler's fallacy

Decision-Making under the Gambler's Fallacy: Evidence from Asylum Judges, Loan Officers, and Baseball Umpires *(NBER Working Paper No. 22026), D Chen, TJ Moskowitz, and K Shue*

Individuals have a slight bias against deciding the same way in successive cases in a number of areas:

- **Asylum judges in the US:** Odds that a judge rejects an asylum seeker are 3.3 percentage points higher if the judge has approved the previous case, all else being equal.
- **Loan officers in India:** Officers were eight percentage points less likely to approve the loan currently under review if they had approved the previous loan.
- **Baseball:** Umpires were 1.5 percentage points less likely to call a strike if the previous pitch was a called strike.

http://www.nber.org/digest/jun16/w22026.html

Example: Hot hand

Belief in hot hand:

The confidence that after a long run of one kind of outcome it's likely to obtain more of these.

Has occurred in descriptions of sports (basketball) and gambling (e.g. roulette). In random sequences that are actually composed of independent events this is wrong.

Contradiction? Hot hand vs gamber's fallacy

Hot hand belief can be seen as opposite fallacy of the gambler's fallacy.

Leading potentially to opposite conclusions.

There are many ways in which you can get something wrong, so that is not a contraction.

Whether/which people apply any of these depends on context and personality etc.

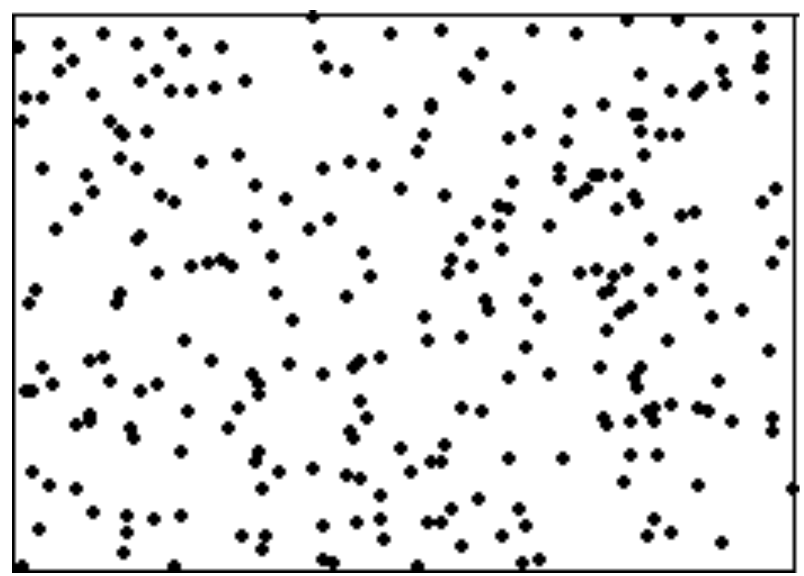
Look at more fallacies…

Concept: Clustering illusion

The tendency to erroneously consider the inevitable "streaks" or "clusters" arising in small samples from random distributions to be statistically significant.

Explanation: Underestimation of the amount of variability likely to appear in a small sample of random or semi-random data.

Examples:
Hot hand in basketball,
Seeing structure in
Poisson point patterns



Gilovich, Thomas; Robert Vallone & Amos Tversky (1985). "The hot hand in basketball: On the misperception of random sequences". *Cognitive Psychology* **17**: 295–314.

Example: Perception of randomness

From a study with over 800 Warwick UG students across subjects (2012)

"You are given a non-transparent box containing a large number of identical marbles, half are black ***(B)*** *and half are white* ***(W)****. Take out a marble and note its colour. Put it back and give the box a little shake. Take out another marble and note its colour. Do this repeatedly.*

*Write down a colour sequence (****B*** *or* ***W****) of* ***10*** *marbles you might have observed."*

Answer version 1: ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Answer version 2: Use attached small notepad

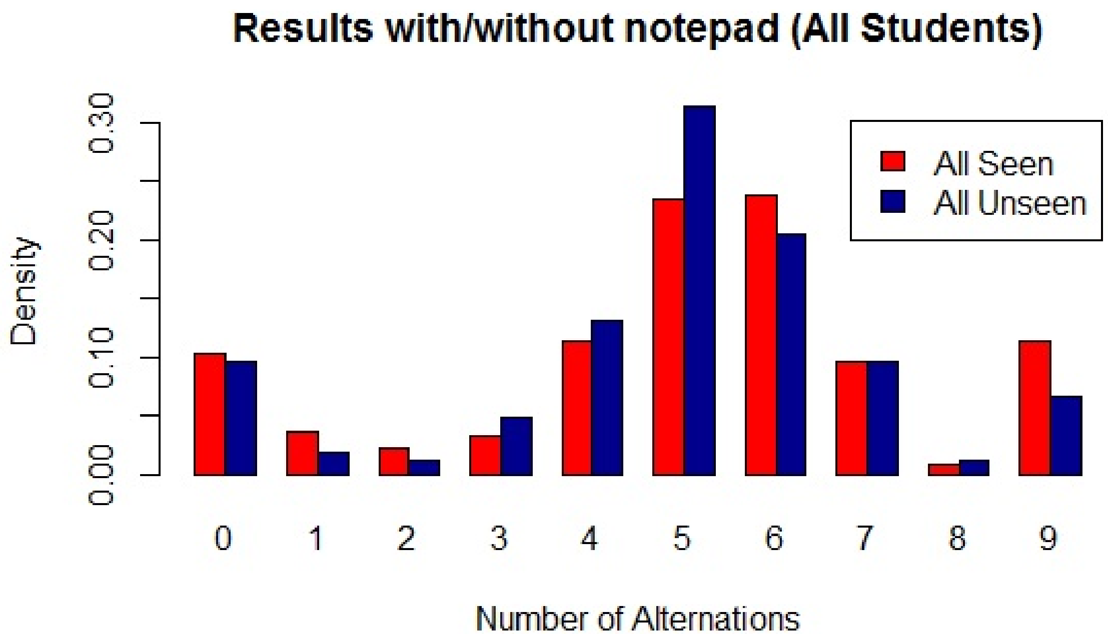
What do they put first? What is the number of alternations?

Source: MMORSE projects Mohan, Streather, Yip, supervised by Brettschneider 2012-2014

Example: Perception of randomness

What is the number of alternations?

Empirical distribution in study:



Theoretical answer:

Expected value of alternations in 10 independent fair Bernoulli trials is 4.5.
(Calculate that using indicators!)

Unimodal, some extreme values (0,1), mean about 5.
Difference between seen/unseen mainly in the centre, not significant.

Discussion: Overly alternating is consistent with previous findings.
Small difference between seen/unseen, though our sequences are shorter.

Example: Perception of random sequences

What did they put first?

About 90% put B first.

Explanation: ***Anchoring bias***

Information received at first dominates thinking.

Empirical studies from the literature: Anchoring bias

Group A

Is the Mississippi River more or less than 70 miles long? How long is it?

Group B

Is the Mississippi River more or less than 2000 miles long? How long is it?

Karen Jacowitz and Daniel Kahneman, *Measures of Anchoring in Estimation Tasks,* Pers Soc Psychol Bull November 1995 vol. 21 no. 11 1161-1166

Empirical studies from the literature: Anchoring bias

Group A

Is the Mississippi River more or less than 70 miles long? How long is it?

Mean answer: 300

Group B

Is the Mississippi River more or less than 2000 miles long? How long is it?

Mean answer: 1500

Karen Jacowitz and Daniel Kahneman, *Measures of Anchoring in Estimation Tasks,* Pers Soc Psychol Bull November 1995 vol. 21 no. 11 1161-1166

Anchoring bias: Priming influences answers.

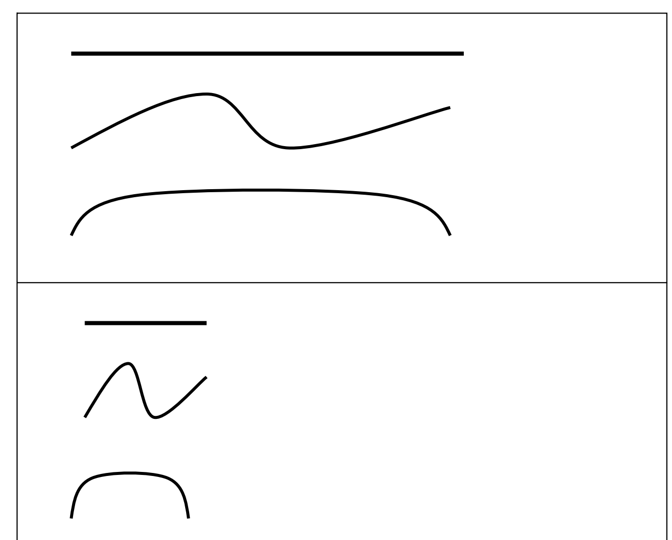
	A given	B given	A estim.	B estim.
Mississippi (mi)	70	2000	300	1500
Everest (ft)	2000	45500	8000	42550
Meat (lbs/year)	50	1000	100	500
SF to NY (mi)	1500	6000	2600	4000
Tallest Redwood (ft)	65	550	100	400
UN Members	14	127	26	100
Female Berkeley Profs	25	130	50	95
Chicago Population (mil.)	0.2	5.0	0.6	5.05
Telephone Invented	1850	1920	1870	1900
US Babies Born (per day)	100	50000	1000	40000

Anchoring bias with unrelated information

Methods

Participants: Seventy-one Stanford University undergraduates participated to fulfill part of a course requirement. The experiment consisted of two questionnaires in a packet of approximately 20 unrelated one-page questionnaires. Packets were randomly ordered and then distributed in class, and participants were given a week to complete the entire packet.

Design, stimuli, and procedure: Participants were presented with a set of three horizontal lines and were asked to replicate the lines as best as they could without using a ruler. The three lines were a straight line, a wavy line, and an inverted u. Participants in the short-anchor condition replicated 1-in. long lines, while participants in the long-anchor condition replicated 3.5-in. lines.



Anchors aweigh: A demonstration of cross-modality anchoring and magnitude priming, Daniel M. Oppenheimer , Robyn A. LeBoeuf , Noel T. Brewer, Cognition (2007)

On the next page, participants were presented with an ostensibly unrelated judgment task in which they were asked to estimate various quantities. The target quantity, the length of the Mississippi River, was always asked about first (only a simple question about how long it is, without the phrase "…is about … long" from the previous experiment). Several decoy questions followed to prevent participants from guessing the hypothesis.

Six participants who gave estimates falling more than 3.5 standard deviations from the mean were excluded as outliers.

Results and discussion:

Participants who drew long lines gave an average estimate of 1224 miles whereas participants who drew short lines gave an average estimate of 72 miles. This difference was statistically significant.

Participants who had been anchored by copying long lines reliably estimated the river to be longer than those anchored with short lines. In other words, not only can anchoring occur when no explicit comparison is made between an anchor and a target (cf. Wilson et al., 1996), it can even arise across modalities.

Variation of this experiment

Participants: Ninety-eight individuals recruited from arbitrarily chosen intersections in San Francisco participated in exchange for a candy bar.

Task: Estimate the average temperature in Honolulu in July in degrees Fahrenheit.

Results:

Participants who drew long lines gave an average estimate of 87.5 degrees.

Participants who drew short lines gave a lower average estimate of 84.0 degrees. Results were statistically significant.

Note that this is despite being from incompatible dimensions (length, temperature).

Anchoring bias in calculations

Group A

Within 5 seconds, estimate the product: $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Group B

Within 5 seconds, estimate the product: $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

First sequence median guess: 2250.

Second sequence median guess: 512.

Correct answer: 40,320.

Example: Framing effect

Key example from seminal paper on the framing effect:
The Framing of Decisions and the Psychology of Choice. Amos Tversky; Daniel Kahneman. Science, New Series, Vol. 211, No. 4481.

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

Which of the two programs would you favour?

So then the researchers asked the following version of the same question:

If Program C is adopted 400 people will die.

If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Which of the two programs would you favour?

Problem **1** *[N* = 1521*]*:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved. [72 percent]
If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28 percent]

Problem 2 [N = 1551]:

...

If Program C is adopted 400 people will die. [22 percent]
If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78 percent]
Which of the two programs would you favor?

Amos Tversky; Daniel Kahneman, *The Framing of Decisions and the Psychology of Choice*, Science, New Series, Vol. 211, No. 4481. (Jan. 30, 1981), pp. 453-458.

http://links.jstor.org/sici?sici=0036-8075%2819810130%293%3A211%3A4481%3C453%3ATFODAT%3E2.0.CO%3B2-3
Available also eg. at psych.hanover.edu/classes/cognition/papers/tversky81.pdf

If Program A is adopted, 200 people will be saved. [72 percent]
If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28 percent]

If Program C is adopted 400 people will die. [22 percent]
If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78 percent]
Which of the two programs would you favor?

Interpretation: ***Framing effect, risk aversion***

People behave risk-averse in the saving-lives formulation. They want to have certainty about saving lives.
In contrast, they because risk-seeking in the loosing-lives formulation.
The sure loss of 400 people (D) is not acceptable to them.
However, according to EUT it should all be the same.